Thursday, September 20

1.6: prod rule; quot. rule
(T2)

1.7: CHAIN RULE

power rule:

$$f(x) = 3x^4 - 8x^3 + 5x^2 - 11x + 4$$

$$f'(x) = 3(4 \cdot x^3) - 8(3x^2) + 5(2x) - 11(1) + 0$$

won't work here:

$$g(x) = (3x+1)^3$$

find $g'(x)$??

need chain rule:

(1) EXTENDED POWER RULE:

$$y = [f(x)]^n \quad y' = ?$$

$$y' = n \cdot [f(x)]^{n-1} \cdot f'(x)$$

chain rule

$y = (3x+1)^3$

chain rule

$y' = 3(3x+1)^{3-1} \cdot 3$

$y' = 9(3x+1)^2$

$y = \cancel{(3x+1)(3x+1)(3x+1)}$

$=$

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(2) composition of functions:

$y = (f \circ g)(x) = f(g(x))$

$y' = f'(g(x)) \cdot g'(x)$

$\boxed{\begin{array}{l} f(x) = x^3 \\ g(x) = 3x+1 \end{array}}$

ex:

$y = (f \circ g)(x)$

$y = f(g(x)) = f(3x+1) = (3x+1)^3$

$y' = 3(g(x))^2 \cdot 3$

$y' = 9(3x+1)^2$

(3) $\left\{ \begin{array}{l} y \text{ in terms of } u \longrightarrow \dfrac{dy}{du} \\ u \text{ in terms of } x \longrightarrow \dfrac{du}{dx} \end{array} \right.$

$$\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = y' = f'(x)$$

{ y in terms of u:

$y = 8 + u^3$

$\frac{dy}{du} = 0 + 3u^2$

$\frac{dy}{du} = 3u^2$

{ u in terms of x:

$u = x^2 + 5$

$\frac{du}{dx} = 2x + 0$

$\frac{du}{dx} = 2x$

$\frac{dy}{dx} = ???$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

x's only

$\frac{dy}{dx} = 3u^2 \cdot 2x$

$\frac{dy}{dx} = 3(x^2+5)^2 \cdot 2x$

$\frac{dy}{dx} = 6x(x^2+5)^2$

$y = 8 + u^3 \qquad u = x^2+5$

y in terms of x??

$y = 8 + (x^2+5)^3$

$y' = 0 + 3(x^2+5)^2 \cdot 2x$

$y' = 6x(x^2+5)^2$

$$\frac{dy}{du} \cdot \frac{du}{dr} \cdot \frac{dr}{dw} \cdot \frac{dw}{dx} = \frac{dy}{dx}$$

find the equation of the tangent line to the graph of $y = \sqrt{x^2 + 3x}$ at the point $(1, 2)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = m(x - 1)$$

$$m_{TAN} = y'$$

$$y = (x^2 + 3x)^{1/2}$$

$$y' = \frac{1}{2}(x^2 + 3x)^{-1/2} \cdot (2x + 3)$$

$$y' = \frac{(2x+3)}{2 \cdot \sqrt{x^2 + 3x}}$$ when $x = 1$

(at $x=1$) $$y' = \frac{2(1)+3}{2 \cdot \sqrt{1^2 + 3(1)}} = \frac{5}{2 \cdot 2} = \frac{5}{4}$$

$$\boxed{y - 2 = \frac{5}{4}(x-1)}$$

1.8: HIGHER ORDER DERIV.

$f(x) =$ ______ $y =$ ______

y', y'', y'''

$f'(x) =$ ______

$f''(x) =$ ______

$f'''(x) =$ ______

$\vdots$

$f^{(4)}(x) \quad \ldots\ldots \quad f^{(n)}(x) =$ ______

$$\frac{dy}{dx} \;;\; \frac{d^2y}{dx^2} \;;\; \frac{d^3y}{dx^3}$$

$y = 3x^2 - 8x + 11$

$y' = 6x - 8$

$y'' = 6$

$y''' = 0$

$\vdots$

$f(x) = (5x^2+4)^8$

$f'(x) = 8(5x^2+4)^7 \cdot (10x)$

$f'(x) = 80x \cdot (5x^2+4)^7$

$f''(x) \Rightarrow$ prod rule

$f''(x) = (\cancel{80}\,x) \left[7(5x^2+4)^6 \cdot (10x) \right]$

$+ (5x^2+4)^7 \cdot (80)$

$f''(x) = 80(5x^2+4)^6 \left[70x^2 + (5x^2+4) \right]$

$f''(x) = 80(5x^2+4)^6 \left[75x^2+4 \right]$

$0 = 80(5x^2+4)^6 \left[75x^2+4 \right]$

$g(x) = (2x-1)^4 \cdot (x^2+x+1)^7$

$g'(x) =$ prod. rule

$g'(x) = (2x-1)^4 \cdot \left[7(x^2+x+1)^6 (2x+1) \right]$

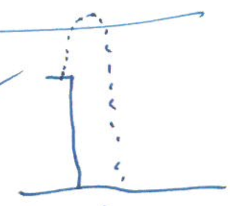
$+ (x^2+x+1)^7 \cdot \left[4(2x-1)^3 \cdot 2 \right]$

$$j(x) = \sqrt[3]{\frac{x-3}{x+4}} = \left(\frac{x-3}{x+4}\right)^{1/3}$$

$$j'(x) = \frac{1}{3}\left(\frac{x-3}{x+4}\right)^{-2/3} \cdot \left[\frac{(x+4)(1) - (x-3)(1)}{(x+4)^2}\right]$$

deriv of "inside" function

$$\boxed{s(t) = -16t^2 + 64t + 100}$$



free falling object (GRAVITY)

position; height; distance

$s(t)$ = height (ft)

t = time (sec)

$t=0$:

$$s(0) = -16(0)^2 + 64(0) + 100 = 100 \text{ ft.}$$

$$s(1) = -16(1)^2 + 64(1) + 100 = 148 \text{ ft.}$$

$\vdots$

$s'(t) = v(t)$ VELOCITY

$$s'(t) = -32t + 64 = v(t)$$

$$v(0) = -32(0) + 64 = 64 \text{ ft/sec}$$

$$v(1) = -32(1) + 64 = 32 \text{ ft/sec}$$

$$v(2) = -32(2) + 64 = 0 \text{ ft/sec}$$

$$s''(t) = v'(t) = a(t)$$

$$v'(t) = -32$$

$$v(0) = -32 \text{ ft/sec/sec} \quad \underline{\underline{\text{GRAV}}}$$

$$v(1) = -32$$

$$v(2) = -32$$