121-001 Thurs, 10/18 (continued from TUESDAY, 10/16)

MIN. INVENTORY COST:

let $x$ = LOT SIZE

2.) COST = (ON SITE) + (REORDER COST)

$$C = (\$20)\left(\frac{x}{2}\right) + (16x + 40)\left(\frac{100}{x}\right)$$

ASSUME

pool tables on hand

# of times to order

$$\boxed{100 = N \cdot x}$$

# of times to reorder

solve for N:

$$N = \frac{100}{x}$$

$4000\,x^{-1}$

$$C = 10x + 1600 + \frac{4000}{x}$$

$$C' = 10 + 0 + 4000(-1 \cdot x^{-2})$$

$$C' = \boxed{10 - \frac{4000}{x^2} = 0}$$

$(-2)(-4000)x^{-3}$

$$\frac{10}{1} = \frac{4000}{x^2}$$

$10x^2 = 4000$

$x^2 = 400$

$x = 20.$

$$\frac{100}{x} = \frac{100}{20} = 5$$ ← reorder 5 times

MIN? (VALIDATION) $C''(x) = \frac{8000}{x^3} = +$

∴ concave up

∴ MIN

Since $r > 0$ we consider only the positive root.

$$r_1 = \sqrt{\frac{A_0}{6\pi}}$$

The second derivative $V''(r) = -6\pi r$ is negative on the interval $(0, \infty)$. Since this interval is our domain of interest, we can apply Theorem 1 to conclude that $r_1$ is the only critical number of interest and that it is the location of the global maximum. Hence the global maximum volume is

$$V_{max} = V(r_1) = -\pi(r_1)^3 + \frac{1}{2}A_0 r_1 = -\pi\left(\sqrt{\frac{A_0}{6\pi}}\right)^3 + \frac{1}{2}A_0\sqrt{\frac{A_0}{6\pi}}$$

Using this value we find the corresponding $h_1$ to be

$$h_1 = \frac{A_0 - 2\pi r_1^2}{2\pi r_1} = \frac{A_0 - 2\pi\left(\sqrt{\frac{A_0}{6\pi}}\right)^2}{2\pi\sqrt{\frac{A_0}{6\pi}}} = \sqrt{\frac{2A_0}{3\pi}} = \sqrt{\frac{2}{2}\frac{2A_0}{3\pi}} = \sqrt{\frac{4A_0}{6\pi}} = 2\sqrt{\frac{A_0}{6\pi}} = 2r_1$$

Thus the volume of a cylinder is maximized for a fixed surface area when the height is twice the radius.

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**Example 25.** *When the ticket price is $40, the average attendance at the football game is 40,000 people. It has been determined that for every $1 decrease in the ticket price, an additional 2000 people will purchase tickets and attend the game. Under this arrangement, what price should be charged per ticket to maximize the revenue for the university? How many fans will attend the game at this price? What is the maximum revenue?*

Let $x$ = # of times the price is DECREASED

$(40)(40{,}000) = 1{,}600{,}000$

ticket price — people attending

$\text{REV} = ((40 - 1\cdot x))(40{,}000 + 2{,}000x)$

$R(x) = 1{,}600{,}000 + 80{,}000x - 40{,}000x - 2000x^2$

$R(x) = 1{,}600{,}000 + 40{,}000x - 2000x^2$

$R'(x) = 0 + 40{,}000 - 4000x = 0$

$\boxed{40{,}000 - 4000x} = 0$

$\frac{40{,}000}{4000} = \frac{4000x}{4000}$

$10 = x$

$R''(x) = -4000$ ∴ concave down ∴ MAX

ticket price: $(40 - 1\cdot 10) = \$30^{00}$

people attending: $(40000 + 2000(10)) = 60{,}000$

REV: $(30)(60{,}000) = \$1{,}800{,}000^{00}$

MA 121-001

Thursday, October 18

- return TEST #2
- begin Chapter 3 (3.1; 3.2)

3.1:

$y = 2^x$; $y = 5^x$; $y = e^x$; $y = 10^x$

~~$e = \lim_{K \to 0} (1 + \frac{1}{1}$~~

$$e = \lim_{K \to \infty} \left(1 + \frac{1}{K}\right)^K$$

$K = 100$

$$\left(1 + \frac{1}{100}\right)^{100} \approx 2.705$$

$K = 10{,}000$

$$\left(1 + \frac{1}{10{,}000}\right)^{10{,}000} \approx 2.71814$$

$K = 100{,}000{,}000$

$$\left(1 + \frac{1}{100{,}000{,}000}\right)^{100{,}000{,}000} \approx \boxed{2.718}2$$

$$\boxed{e \approx 2.718}$$

$y = e^x$

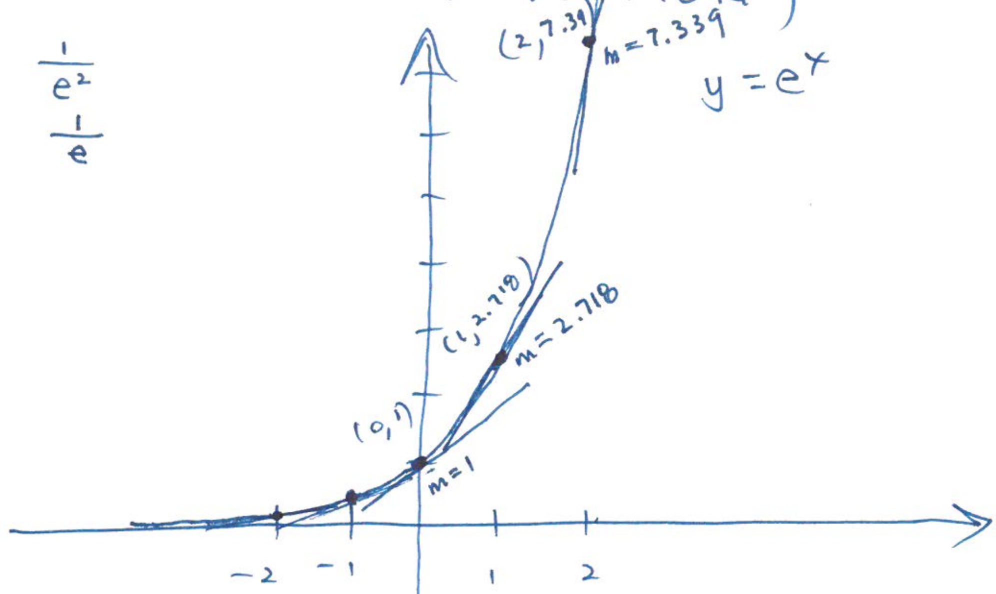
$y = e^x$ (NATURAL EXPONENTIAL FUNCTION)

| $x$ | $y$ |
|---|---|
| $-2$ | $e^{-2} \approx .135$ |
| $-1$ | $e^{-1} \approx .368$ |
| $0$ | $e^0 = 1$ |
| $1$ | $e^1 \approx 2.718$ |
| $2$ | $e^2 \approx 7.39$ |

$\frac{1}{e^2}$

$\frac{1}{e}$

$y = e^x$




$y = e^x$

$y' = m_{TAN} = e^x$

DERIVATIVE: $\boxed{y = e^x}$ $\boxed{y' = e^x}$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y' = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$y' = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$y' = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$$y' = e^x \cdot \boxed{\lim_{h \to 0} \frac{e^h - 1}{h}}$$

$h = .01$

$$\frac{e^{.01} - 1}{.01} \approx 1.005$$

$h = .00001$

$$\frac{e^{.00001} - 1}{.00001} \approx 1.000005$$

$1$

$y' = e^x \cdot 1$

$y' = e^x$

$y = e^x$

$y' = e^x$

$$y = e^{5x}$$
$$y' = e^{5x} \cdot d(5x)$$
$$y' = e^{5x} \cdot 5$$
$$y' = 5 \cdot e^{5x}$$

$$y = e^{3x^2 - 5x + 1}$$
$$y' = e^{3x^2-5x+1} \cdot (6x-5)$$
$$y' = (6x-5) \cdot e^{3x^2-5x+1}$$

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3.2: INVERSE OF $y = e^x$ NAT. EXP.

(switch $x$ & $y$)

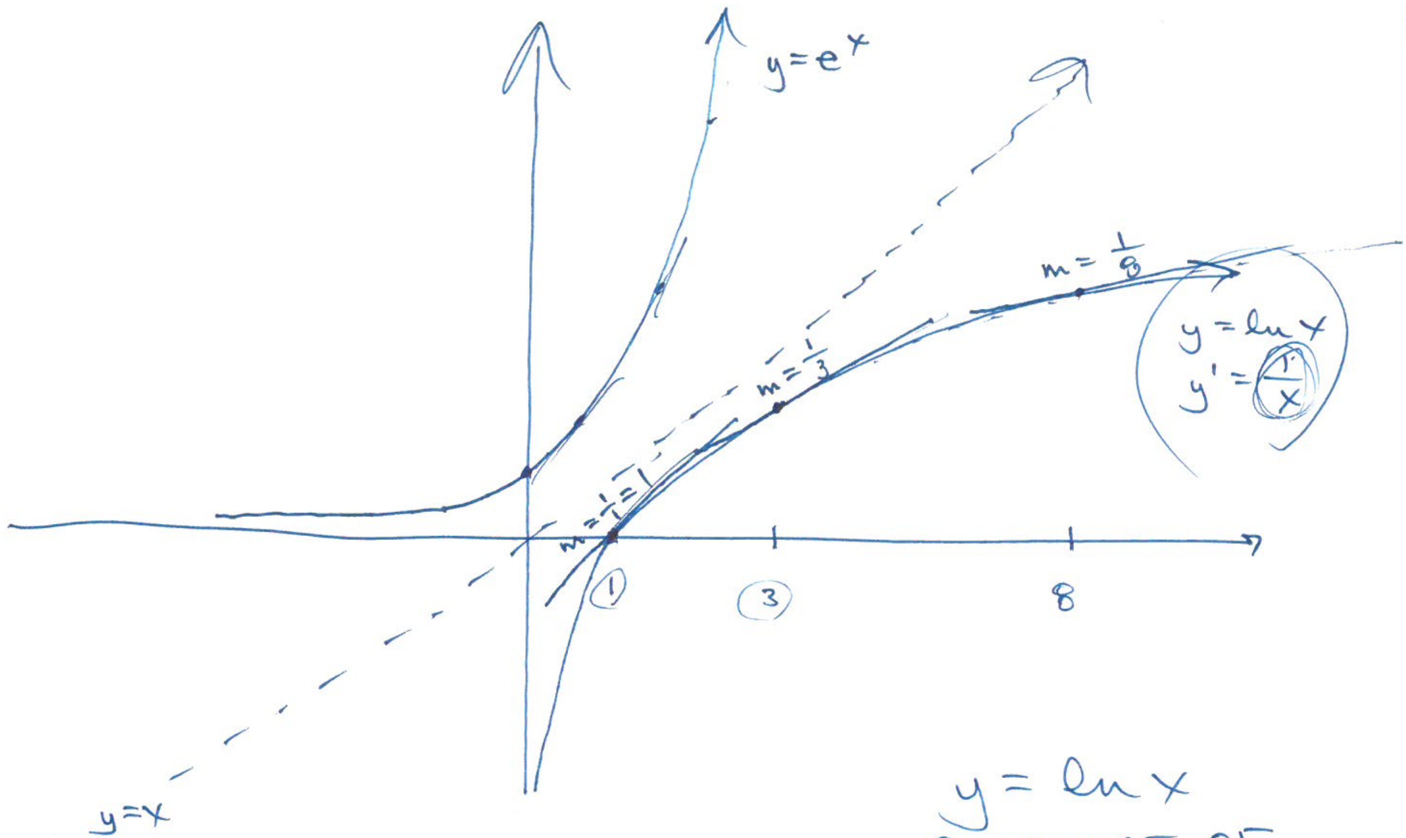
* $\boxed{x = e^y}$ logarithm = power; exp.

(solve for $y$ .... )

$\boxed{y}$ is the power to which "e" is raised to get $x$

* $y = \log_e x$

$y = \ln x$ (NAT. LOG. FUNCTION)




$y = \ln x$
(INVERSE OF $y = e^x$)

$f(x) = \ln_e x$

rewrite: $e^{f(x)} = x$ (find $f'(x)$)

DERIV:

$e^{f(x)} \cdot f'(x) = 1$

$f'(x) = \dfrac{1}{e^{f(x)}} = \dfrac{1}{x}$

$f(x) = \ln x$

$f'(x) = \dfrac{1}{x}$

chain rule: $y = \ln(a \cdot b)$

$y = \ln a + \ln b$

$y = \ln(x)$

$y' = \frac{1}{x}$

$y = \ln(4 \cdot x)$

$y' = \frac{1}{(\cancel{4}x)} \cdot \cancel{4} = \frac{1}{x}$

$y = \ln 4 + \ln x$

$y' = 0 + \frac{1}{x} =$

$y = \ln(3x^2 + 5x - 11)$

$y' = \frac{1}{3x^2 + 5x - 11} \cdot (6x + 5)$

$y' = \frac{6x + 5}{3x^2 + 5x - 11}$

$\ln x \cdot x$

$y = x^2 \cdot e^x$

$y' = x^2 \cdot e^x + e^x(2x)$

$y' = e^x(x^2 + 2x)$

$y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{1}{x \cdot \ln x}$

MA121-001 10/18/18

TEST #2 RESULTS:

| | | | |
|---|---|---|---|
| A's | 79 | (38.9%) | 60.0% |
| B's | 43 | (21.2%) | |
| C's | 21 | (10.3%) | |
| D's | 27 | (13.3%) | 29.6% |
| F's | 33 | (16.3%) | |

AVE: 79.291