MA 121-001

Tuesday, October 23

$\begin{cases} 10/23: & 3.3;\ 3.4 \\ 10/25: & 3.5;\ 4.1 \end{cases}$

$\begin{cases} 10/30: & 4.2;\ 4.3 \\ 11/1: & 4.3;\ \text{review} \end{cases}$

11/6: TEST #3

$$y = \frac{\ln x}{x^2} \quad \text{find } y':$$

$$y' = \frac{(x^2)\left(\frac{1}{x}\right) - (\ln x)(2x)}{[x^2]^2}$$

$$y' = \frac{x - 2x \cdot \ln x}{x^4} = \frac{\cancel{x}(1 - 2 \cdot \ln x)}{\cancel{x} \cdot x^3}$$

$$y' = \frac{1 - 2\ln x}{x^3}$$

$$y = \ln\left(\frac{x^2+x}{x+3}\right) \quad \text{find } y':$$

rewrite using: $\underbrace{\ln\left(\frac{a}{b}\right)} = \underbrace{\ln a - \ln b}$

$$y = \underbrace{\ln(x^2+x)} - \underbrace{\ln(x+3)}$$

$$y' = \frac{1}{x^2+x} \cdot (2x+1) - \frac{1}{x+3} \cdot (1)$$

$$y' = \frac{2x+1}{x^2+x} - \frac{1}{x+3}$$

$$y = \ln\left(\frac{x^2+x}{x+3}\right)$$

$$y' = \frac{1}{\frac{x^2+x}{x+3}} \cdot \left[\frac{(x+3)(2x+1) - (x^2+x)(1)}{(x+3)^2}\right]$$

$$e^{.08t} = 143$$

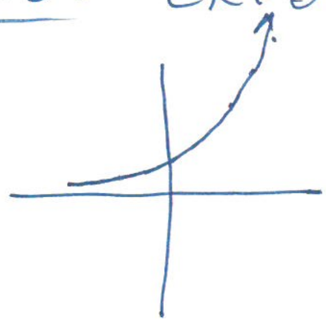
*(take the LN of both sides)

ln 8

$$\ln \boxed{e}^{.08t} = \ln 143$$

$$\frac{\cancel{.08}t}{\cancel{.08}} = \frac{\ln 143}{.08} \approx \_\_\_\_\_\_$$

3.3: EXPONENTIAL GROWTH



rate of growth is directly proportional to the amount present at any time "t"

$$\frac{dy}{dt} = k \cdot y$$

ch5

$$\boxed{y = C \cdot e^{kt}}$$ *

(verify by taking the deriv.)

$$\frac{dy}{dt} = C \cdot e^{kt} \cdot k$$

$$\frac{dy}{dt} = k\left[C e^{kt}\right]$$

$$\frac{dy}{dt} = k \cdot y$$

$$y = C \cdot e^{kt} \longrightarrow \boxed{y = \boxed{y_0} e^{kt}}$$

initial y-value

$t = 0 \quad y = y_0$

$$y_0 = C \cdot \boxed{e^{k \cdot 0}}$$

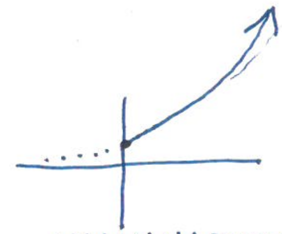
$$y_0 = C \cdot 1$$

$$y = y_0 \cdot e^{kt}$$

$$P = P_0 \cdot e^{kt}$$

$$\boxed{A = P \cdot e^{rt}} \leftarrow \text{contin. comp. interest}$$

$\uparrow$ initial #



UNINHIBITED GROWTH

** $\boxed{y = y_0 \cdot e^{kt}}$

"standard" exp. growth

k = growth rate

t = time

y_0 : initial amount

y : future amount

pop. growth

SPEEDWAY, INDIANA:

(1) (t=0) 2000: 10,882

(2) (t=10) 2010: 12,461

predict pop. in 2025:

(1) $y = \underbrace{10{,}882}_{} \cdot e^{kt}$

(2) $12{,}461 = 10{,}882 \cdot e^{k(10)}$

solve for k ...

$$\frac{12{,}461}{10{,}882} = \frac{\cancel{10{,}882} \cdot e^{10k}}{\cancel{10{,}882}}$$

$$\frac{12{,}461}{10{,}882} = e^{10k}$$

$$\frac{\ln\left(\frac{12{,}461}{10{,}882}\right)}{10} = \frac{\cancel{10}k}{\cancel{10}} \approx \underline{+.0135}$$

GROWTH $\downarrow$

$$y = [10,882] \cdot e^{.0135t}$$

predict pop in 2025 $(t = 25)$

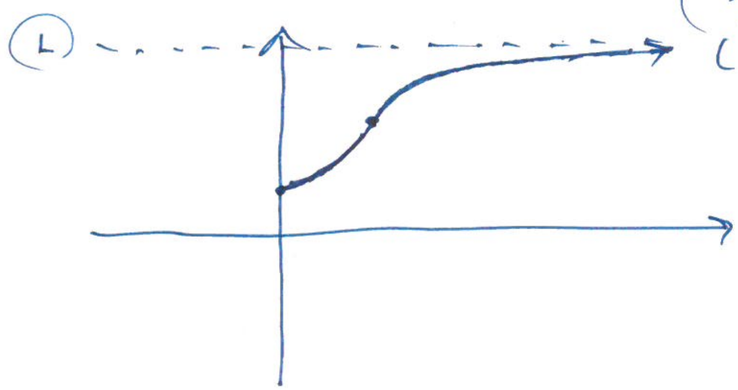
$$y = [10,882] \cdot e^{.0135(25)}$$

$$y \approx 15,250$$

LOGISTIC GROWTH (long-term model)
(k is NEG)

$$y = \frac{L}{1 + b \cdot e^{kt}}$$

L: limiting value
(max. supp. pop)



ex: RALEIGH, NC ① 2010: 403,892 $(t=0)$
2018: 464,758
L: 700,000

① $$y = \frac{700,000}{1 + b \cdot e^{k(0)}}$$

$$\frac{403,892}{1} = \frac{700,000}{1 + b}$$

$$\frac{\cancel{403,892}(\cancel{1} + b)}{\cancel{403,892}} = \frac{700,000}{403,892} - 1$$

$$b = \frac{\overset{L}{700{,}000}}{\underset{y_0}{403{,}892}} - 1 \approx \underline{.7331}$$

$$y = \frac{700{,}000}{1 + (.7331)\cdot e^{kt}}$$

2018 ($t=8$)
464,758

$$\frac{464{,}758}{1} = \frac{700{,}000}{1 + (.7331)\cdot e^{k(8)}}$$

find k ...

$$\frac{700{,}000}{464{,}758} - 1 = \frac{\cancel{464{,}758}\left[\cancel{1} + .7331\, e^{8k}\right]}{\cancel{464{,}758}}$$

$$\frac{\frac{700{,}000}{464{,}758} - 1}{.7331} = \frac{\cancel{.7331}\, e^{8k}}{\cancel{.7331}}$$

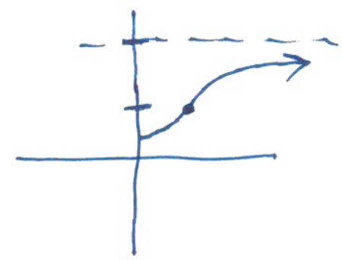
$$\frac{\frac{700{,}000}{464{,}758} - 1}{.7331} = e^{8k}$$

$$\frac{\ln\left[\frac{\frac{700{,}000}{464{,}758} - 1}{.7331}\right]}{8} = \frac{\cancel{8}k}{\cancel{8}} \approx \underline{-.0463}$$

$$y = \frac{700{,}000}{1 + (.7331)\cdot e^{-.0463t}}$$

predict pop in 2020 $t=10$

$$\underline{\underline{y}} = \frac{700{,}000}{1+(.7331)\,e^{-.0463(10)}}$$



$$y \approx \underline{478{,}990}$$

3.4: EXPONENTIAL DECAY

$$y = y_0 \cdot e^{kt} \qquad (k \text{ is NEG})$$

radioactive substance: CESIUM 100 lbs.

(HALF-LIFE: 2 yr.)

when will there (t = ??) be 5 lbs of CESIUM.

$$y = 100 \cdot e^{kt}$$

$$\frac{50}{100} = \frac{\cancel{100} \cdot e^{k(2)}}{\cancel{100}}$$

$$\frac{1}{2} = e^{2k}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{2} = \frac{\cancel{2}k}{\cancel{2}} \approx \underline{-.3466} \quad \cancel{-.1386}$$

100 (2) 50 (2) 25 (2) $12\frac{1}{2}$ (2) $\underline{6\frac{1}{4}}$

$$y = \underline{100} \cdot e^{-\cancel{.1386}\,.3466t}$$

$$\frac{5}{100} = \frac{\cancel{100} \cdot e^{-.3466t}}{\cancel{100}}$$

$$\frac{5}{100} = e^{-.3466t}$$

$$\frac{\ln\left(\frac{5}{100}\right)}{-.3466} = \frac{-.3466t}{-.3466}$$

$$t \approx \underline{8.64 \text{ yrs.}}$$

NEWTON'S LAW OF COOLING:

$$\frac{dT}{dt} = k(T - m)$$

temp of object

surrounding medium

T : Temp
t : time

$$\boxed{T = a \cdot e^{kt}} + m$$

not memor.

$$\begin{cases} t = 0 & T = 400^\circ \\ t = 10\text{ min} & T = 350^\circ \\ \underline{t = ??} & \underline{T = 150^\circ} \end{cases}$$