MA 121-001

Thursday, October 25

$$\frac{dT}{dt} = \underline{k}(\underline{T-m})$$

T = temp. of object
t = time
m = temp of surr. medium

"the rate at which an object cools is directly proportional to the difference in temp. between the object and the surrounding med."

$$T = a \cdot e^{kt} + m$$

t = 0 T = 400°
t = 10 min T = 350°
m = 70°
t = ?? T = 150°

(1) $T = a \cdot e^{kt} + 70$

(2) $t = 0 \quad T = 400°$

$$400 = a \cdot e^{k(0)} + 70$$

$$400 = a + \cancel{70}$$

$$-70 \qquad -\cancel{70}$$

$$330 = a$$

$$T = 330 e^{kt} + 70$$

(3) $350 = 330 e^{k(10)} + \cancel{70}$

$$-70 \qquad -\cancel{70}$$

$$\frac{280}{330} = \frac{\cancel{330} e^{10k}}{\cancel{330}}$$

radiocarbon dating

(CARBON DATING)

C_{14} half-life: 5730 years

$$y = \underline{y_0} \cdot e^{kt}$$

$$\frac{\frac{1}{2}\cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} \cdot e^{k(5730)}}{\cancel{y_0}}$$

$$\frac{1}{2} = e^{5730k}$$

$$\frac{\ln(\frac{1}{2})}{5730} = \frac{\cancel{5730}\,k}{\cancel{5730}}$$

$$k \approx -\underline{.00012097}$$

$$y = y_0 \cdot e^{-.00012097t}$$

how old is ____ that has lost 40% of its C_{14}?

$$\underline{60\%} = \underline{100\%}\, e^{-.00012097t}$$

$$.6 = 1 \cdot e^{-.00012097t}$$

3

$\ln e^{10k} = 10k$

$\frac{280}{330} = e^{10k}$

$\frac{\ln\left(\frac{280}{330}\right)}{10} = \frac{\cancel{10}k}{\cancel{10}} \approx -.0164$

$T = 330\, e^{-.0164t} + 70$

$t = ??$
$T = 150°$ ✓

$150 = 330 \cdot e^{-.0164t} + \cancel{70}$
$-70 \qquad -\cancel{70}$

$\frac{80}{330} = \frac{\cancel{330}\, e^{-.0164t}}{\cancel{330}}$

$\frac{80}{330} = e^{-.0164t}$

$\frac{\ln\left(\frac{80}{330}\right)}{-.0164} = \frac{\cancel{-.0164}t}{\cancel{-.0164}} \approx 86.4$ min

3.5:

$$y = e^x \qquad y' = e^x$$

$$y = e^{2x^2+x} \qquad y' = e^{2x^2+x} \cdot (4x+1)$$

$$y = \ln(x) \qquad y' = \frac{1}{x}$$

$$y = \ln(3x+5) \qquad y' = \frac{1}{3x+5} \cdot 3 = \frac{3}{3x+5}$$

$y = 2^x$; $y = 8^x$; $y = 10^x$

in general:

$$\boxed{y = a^x}$$

$$\ln e^u = u$$

$$\boxed{\ln e^{a^x} = a^x}$$

$$e^{\ln a^x} = a^x$$

rewrite:

$$y = \cancel{\ln e^{a^x}}$$

$$y = e^{x \ln a^x}$$

$$y = e^{x \cdot \ln a}$$

$x \cdot 17$

$x \cdot \ln 5$

$x \cdot \ln 8$

$$y = e^x \qquad y' = e^x$$

$$y = a^x \qquad y' = a^x \cdot \ln a$$

take DERIV:

$$y' = e^{x \cdot \ln a} \cdot d(x \cdot \ln a)$$

$$y' = e^{x \ln a} \cdot \ln a$$

$$y' = a^x \cdot \ln a$$

$y = 2^x$

$y' = 2^x \cdot \ln 2$

$y = e^x$

$y' = e^x \cdot \ln e$

$y' = e^x \cdot 1 = e^x$

$y = 5^{3x+7}$

$y' = 5^{3x+7} \cdot \ln 5 \cdot 3$

$y = \ln x$

$y = \log_4 x$

$y = \log_{10} x$

$\boxed{y = \log_a x}$

$\boxed{f(x) = \log_a x}$

rewrite: (in exp. form)

$\boxed{a^{f(x)} = x}$

take DERIV:

$a^{f(x)} \cdot \ln a \cdot f'(x) = 1$

$$\frac{\cancel{a^{f(x)}} \cdot \cancel{\ln a} \cdot f'(x)}{\cancel{a^{f(x)}} \cdot \cancel{\ln a}} = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{x \cdot \ln a}$$

$$\left.\begin{aligned} y &= \ln x \\ y' &= \frac{1}{x} \end{aligned}\right\}$$

$$\left.\begin{aligned} y &= \log_a x \\ y' &= \frac{1}{x \cdot \ln a} \end{aligned}\right\}$$

ex: $y = \log_5 (x)$

$$y' = \frac{1}{x \cdot \ln 5}$$

$$y = \log_8 (x^2 + 5x)$$

chain rule

$$y' = \frac{1}{(x^2+5x) \cdot \ln 8} \quad (2x+5)$$

$$y' = \frac{2x+5}{(x^2+5x) \cdot \ln 8}$$

4.1:

$$
\begin{tikzcd}
f(x) \arrow[r, "\text{DERIV}"] & f'(x) \arrow[l, bend left, "\text{ANTIDERIVATIVE (INTEGRAL)}"]
\end{tikzcd}
$$

$$f(x) = 4x^2 - 11x + 3$$

$$f'(x) = 8x - 11$$

$$f'(x) = 8x - 11$$

$$f(x) = 4x^2 - 11x + C$$

$\int \quad \Sigma$ ⑧

$$\int \big(\ldots f(x) \big)\, dx$$

$\int$ ↑ integral sign

$\underbrace{(\ldots f(x))}_{\text{integrand}}$

dx ↑ antidiff. with respect to x

① power rule:

$$\int a \cdot x^n \cdot dx = \frac{a \cdot x^{n+1}}{n+1} + C$$

$$\int 7x^4\, dx = 7 \cdot x^5 + C$$

check: $d(7x^5 + C)$

$= 7 \cdot 5x^4$

$$\int 7x^4\, dx = \frac{7 \cdot x^5}{5} + C$$

$= \frac{7}{5} x^5 + C$

check: $d\left(\frac{7}{5}x^5 + C\right)$

$= \frac{7}{\cancel{5}} \cdot \cancel{5} x^4 = 7x^4$

$$\int \underline{11 \cdot x^{-3}}\, dx$$

$$= \frac{11 \cdot x^{-2}}{-2} + C$$

$$= -\frac{11}{2} x^{-2} + C$$

$$\int \underline{47 \cdot x^{1/2}}\, dx$$

$$= \frac{47 \cdot x^{3/2}}{3/2} + C$$

$$= 47 \cdot \frac{2}{3}\, x^{3/2} + C$$

$$= \underline{\frac{94}{3} x^{3/2} + C}$$

$$\int a \cdot x^{n} \cdot dx = \frac{a x^{n+1}}{n+1} + C$$

for $n \neq -1$

② $\int a \cdot x^{-1}\, dx = \int a \cdot \frac{1}{x}\, dx$

$= a \cdot \ln|x| + C$

③ $\int e^{x}\, dx = e^{x} + C$

$\int e^{5x}\, dx \overset{??}{=} \boxed{e^{5x} + C}$

$d(e^{5x} + c) \overset{??}{=} e^{5x} \cdot 5$

rule: $\boxed{\int e^{kx} \cdot dx = \frac{e^{kx}}{k} + C}$

$\int e^{5x}\, dx = \frac{e^{5x}}{5} + C$

$d\left(\frac{1}{5} e^{5x} + c\right) = \cancel{\frac{1}{5}} \cdot e^{5x} \cdot \cancel{5}$

$\int 4\, dx$ and $\int 4\, dr$

$= 4x + C$ $\quad$ $= 4r + C$