

Thursday, October 25

$$\frac{dT}{dt} = k(T - m)$$

T = temp. of object

t = time

m = temp of surr. medium

"the rate at which an object cools is directly proportional to the difference in temp. between the object and the surrounding med."

$$\downarrow T = a \cdot e^{kt} + m$$

① $T = a \cdot e^{kt} + 70$

② $t = 0 \quad T = 400^\circ$

$$400 = a \cdot e^{k(0)} + 70$$

$$400 = a + 70$$

$$\begin{array}{r} -70 \\ \hline 330 = a \end{array}$$

$$T = 330 e^{kt} + 70$$

③ $350 = 330 e^{k(10)} + 70$

$$\begin{array}{r} -70 \\ \hline 280 = 330 e^{10k} \end{array}$$

$$\frac{280}{330} = \frac{330 e^{10k}}{330}$$

$$\left\{ \begin{array}{l} t=0 \quad T=400^\circ \\ t=10 \text{ min} \quad T=350^\circ \\ \underline{m=70^\circ} \\ t=? \quad T=150^\circ \end{array} \right.$$

radio carbon dating (CARBON DATING)

C₁₄ half-life: 5730 years

$$y = y_0 \cdot e^{kt}$$

$$\frac{\frac{1}{2} y_0}{y_0} = \frac{y_0 \cdot e^{k(5730)}}{y_0}$$

$$\frac{1}{2} = e^{5730k}$$

$$\frac{\ln(\frac{1}{2})}{5730} = \frac{5730(k)}{5730}$$

$$k \approx \frac{-0.00012097}{-0.00012097 t}$$

$$y = y_0 \cdot e^{kt}$$

how old is —
that has lost 40%
of its C₁₄ . . .

$$\frac{60\%}{100\%} = \frac{100\%}{100\%} e^{-0.00012097 t}$$

$$.6 = 1 \cdot e^{-0.00012097 t}$$

$$\ln e^{10k} = 10k$$

$$\frac{280}{330} = e^{10k}$$

$$\frac{\ln\left(\frac{280}{330}\right)}{10} = \frac{10k}{10} \approx \underline{\underline{-0.0164}}$$

$$T = 330 e^{-0.0164t} + 70$$

t=??
T=150° ✓

$$150 = 330 \cdot e^{-0.0164t} + 70$$

$$\frac{150 - 70}{330} = \frac{330 e^{-0.0164t} - 70}{330}$$

$$\frac{80}{330} = e^{-0.0164t}$$

$$\frac{\ln\left(\frac{80}{330}\right)}{-0.0164} = \frac{-0.0164t}{-0.0164} \approx \underline{\underline{86.4 \text{ min}}}$$

3.5:

$$\left(\begin{array}{l} y = e^x \\ y' = e^x \end{array} \right)$$

$$\left\{ \begin{array}{l} y = e^{2x^2+x} \\ y' = e^{2x^2+x} \cdot (4x+1) \end{array} \right.$$

$$\left(\begin{array}{l} y = \ln(x) \\ y' = \frac{1}{x} \end{array} \right)$$

$$\left\{ \begin{array}{l} y = \ln(3x+5) \\ y' = \frac{1}{3x+5} \cdot 3 = \frac{3}{3x+5} \end{array} \right.$$

$$y = \underline{2}^x ; y = \underline{8}^x ; y = \underline{10}^x$$

in general:

$$e^{\ln a^x} = a^x$$

$$y = a^x$$

rewrite:

$$y = \ln e^{a^x}$$

$$y = e^{x \cdot \ln a}$$

$$y = e^{x \cdot \ln a}$$

$$\ln e^u = u$$

$$\ln e^{a^x} = a^x$$

$$y = e^x$$

$$y' = e^x$$

$$y = a^x$$

$$y' = a^x \cdot \ln a$$

- x · 17
- x · ln 5
- x · ln 8

take DERIV:

$$y' = e^{x \cdot \ln a} \cdot d(x \cdot \ln a)$$

$$y' = e^{x \cdot \ln a} \cdot \ln a$$

$$y' = \underline{a^x} \cdot \underline{\ln a}$$

$$y = 2^x$$

$$y' = 2^x \cdot \ln 2$$

$$y = e^x$$

$$y' = e^x \cdot \ln(e)$$

$$y' = e^x \cdot 1 = e^x$$

$$y = 5^{3x+7}$$

$$y' = 5^{3x+7} \cdot \ln 5 \cdot 3$$



$$y = \ln x$$

$$y = \log_4 x$$

$$y = \log_{10} x$$

$$y = \log_a x$$

$$f(x) \equiv \log_a x$$

rewrite: (in exp. form)

$$a^{f(x)} = x$$

take DERIV:

$$a^{f(x)} \cdot \ln a \cdot f'(x) = 1$$

$$\frac{a^{f(x)} \cdot \ln a \cdot f'(x)}{a^{f(x)} \cdot \ln a} = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$\left. \begin{aligned} y &= \ln x \\ y' &= \frac{1}{x} \end{aligned} \right\}$$

$$f'(x) = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{x \cdot \ln a}$$

$$\left. \begin{aligned} y &= \log_a x \\ y' &= \frac{1}{x \cdot \ln a} \end{aligned} \right\}$$

ex: $y = \log_5(x)$

$$y' = \frac{1}{x \cdot \ln 5}$$

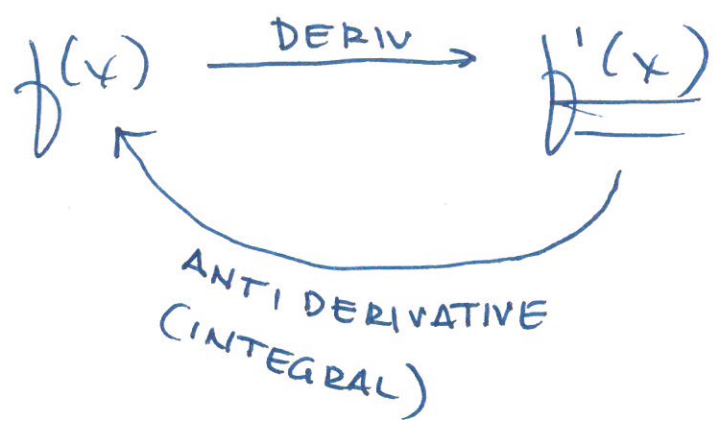
$$y = \log_8(\underline{x^2 + 5x})$$

$$y' = \frac{1}{(x^2 + 5x) \cdot \ln 8} (2x + 5)$$

↙ chain rule

$$y' = \frac{2x + 5}{(x^2 + 5x) \cdot \ln 8}$$

4.1:

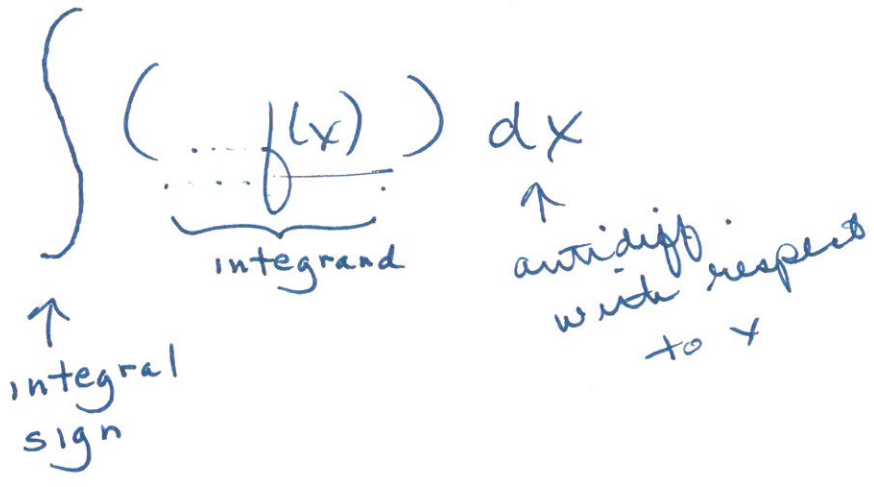


$$f(x) = \underline{4x^2} - \underline{11x} + \underline{3}$$

$$f'(x) = \underline{8x - 11}$$

$$f'(x) = \underline{8x} - \underline{11}$$

$$f(x) = \underline{4x^2} - \underline{11x} + \underline{C}$$



① power rule:

$$\int a \cdot x^n \cdot dx = \frac{a \cdot x^{n+1}}{n+1} + C$$

$$\int 7x^4 dx = 7 \cdot x^5 + C$$

check: $d(7x^5 + C)$
 $= 7 \cdot 5x^4$

$$\int 7x^4 dx = \frac{7 \cdot x^5}{5} + C$$

$$= \frac{7}{5} x^5 + C$$

check: $d\left(\frac{7}{5}x^5 + C\right)$
 $= \frac{7}{5} \cdot 5x^4 = 7x^4$

(9)

$$\int \underline{11 \cdot x^{-3}} dx$$

$$= \frac{11 \cdot x^{-2}}{-2} + C$$

$$= -\frac{11}{2} x^{-2} + C$$

.....

$$\int \underline{47 \cdot x^{1/2}} dx$$

$$= \frac{47 \cdot x^{3/2}}{3/2} + C$$

$$= 47 \cdot \frac{2}{3} x^{3/2} + C$$

$$= \underline{\underline{\frac{94}{3} x^{3/2} + C}}$$

$$\int a \cdot x^n \cdot dx = \frac{ax^{n+1}}{n+1} + C$$

for $n \neq -1$

$$\textcircled{2} \int a \cdot x^{-1} dx = \int a \cdot \left(\frac{1}{x}\right) dx$$

$$= a \cdot \ln|x| + C$$

$$\textcircled{3} \int e^x dx = e^x + C$$

$$\int e^{5x} dx \stackrel{??}{=} \boxed{e^{5x} + C}$$

$$d(e^{5x} + C) \stackrel{??}{=} e^{5x} \cdot 5$$

rule: $\int e^{kx} \cdot dx = \frac{e^{kx}}{k} + C$

$$\int e^{5x} dx = \frac{e^{5x}}{5} + C$$

$$d\left(\frac{1}{5} \cdot e^{5x} + C\right) = \frac{1}{5} \cdot e^{5x} \cdot 5$$

$$\int 4 dx = 4x + C$$

and

$$\int 4 dr = 4r + C$$