MA121-001

Tuesday, October 30

TEST #3: TUES, NOV. 6th

2.5 ; 3.1 → 3.5 ; 4.1 → 4.3

CH 4:

4.1: ANTIDERIVATIVES (INTEGRALS)

$$\int \underbrace{f(x) \cdot dx}_{\text{integrand}} = \underbrace{F(x) + C}_{\text{indef. integral}}$$

Family of curves

(where $F'(x) = f(x)$)

① $\int a \cdot x^n \, dx = \dfrac{a \cdot x^{n+1}}{n+1} + C$ (for $n \neq -1$)

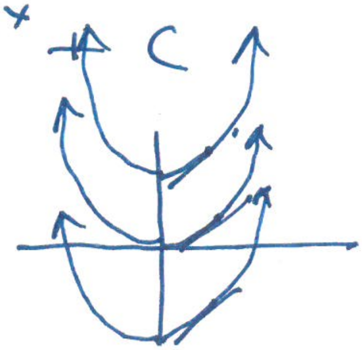
② $\int a \cdot x^{-1} \cdot dx = \int a \cdot \dfrac{1}{x} dx = a \cdot \ln|x| + C$

③ $\int a \cdot e^{bx} \cdot dx = a \cdot \dfrac{1}{b} e^{bx} + C$

ex: $\int 2x \, dx = \cancel{2} \cdot \dfrac{x^2}{\cancel{2}} + C$

$y = x^2 + C$

$(y' = 2x)$

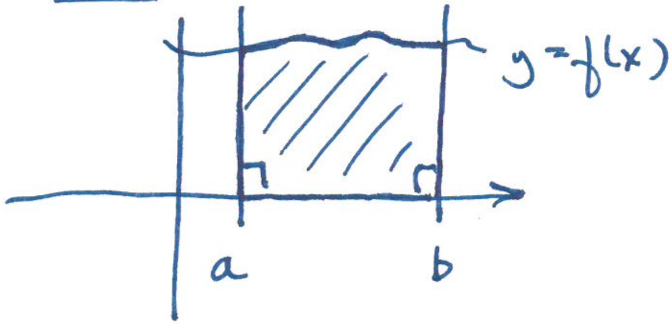


$$\int 11\, e^{7x}\, dx = 11 \cdot \frac{1}{7} \cdot e^{7x} + C$$

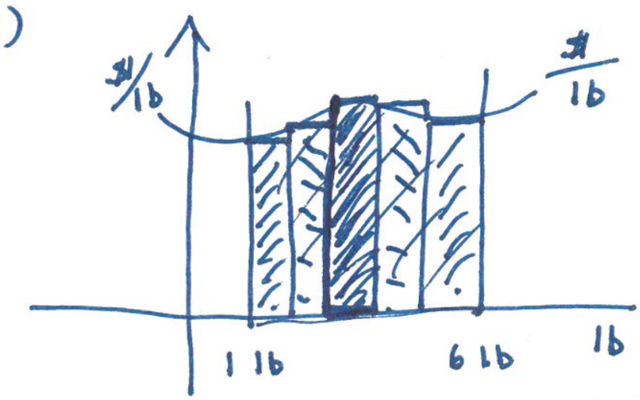
check:
$$d\left(\frac{11}{7} e^{7x} + C\right) = \frac{11}{\cancel{7}} \cdot e^{7x} \cdot \cancel{7}$$

$$\int 7 \cdot x^{-1}\, dx = \int 7 \left(\frac{1}{x}\right) dx$$
$$= 7 \cdot \ln|x| + C$$

4.2: AREAS



curve = rate

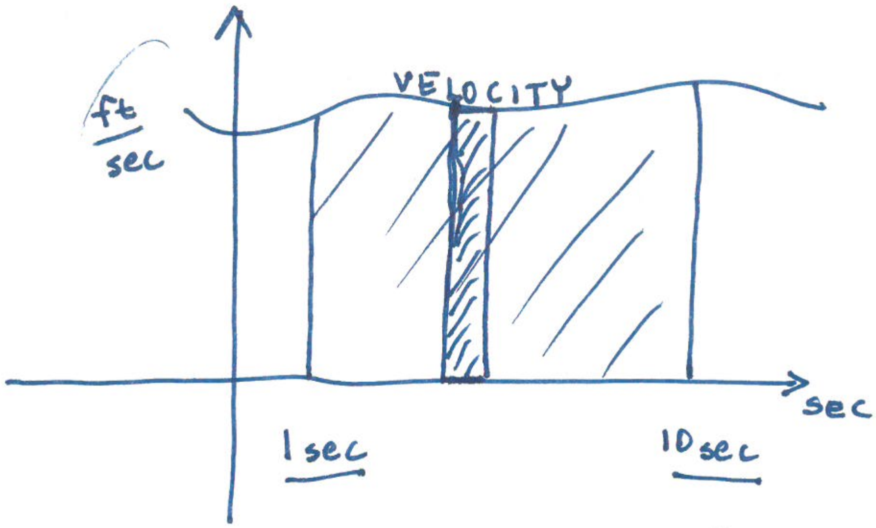


$$A = b \cdot h$$
$$A = (\sim \text{lb})(\sim \#/\text{lb})$$
$$= \sim \#$$

APPROX THE AREA USING RECTANGLES

$(A = b \cdot h)$

more rectangles → higher accuracy

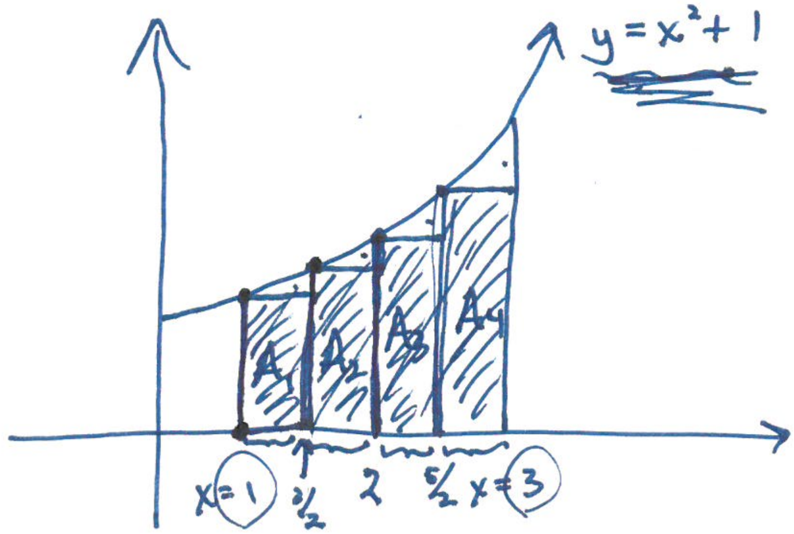


$$A = (\sim \tfrac{ft}{\cancel{SEC}})(\sim \cancel{SEC})$$

$$= \sim \underline{ft}.$$

the area under a VELOCITY curve is DIST, HT, POS.

APPROX. AREA USING RECTANGLES:



area under $y = x^2+1$ from $x=1$ to $x=3$

WIDTH: $\frac{3-1}{4} = \frac{2}{4}$

$\Delta x = \frac{1}{2}$

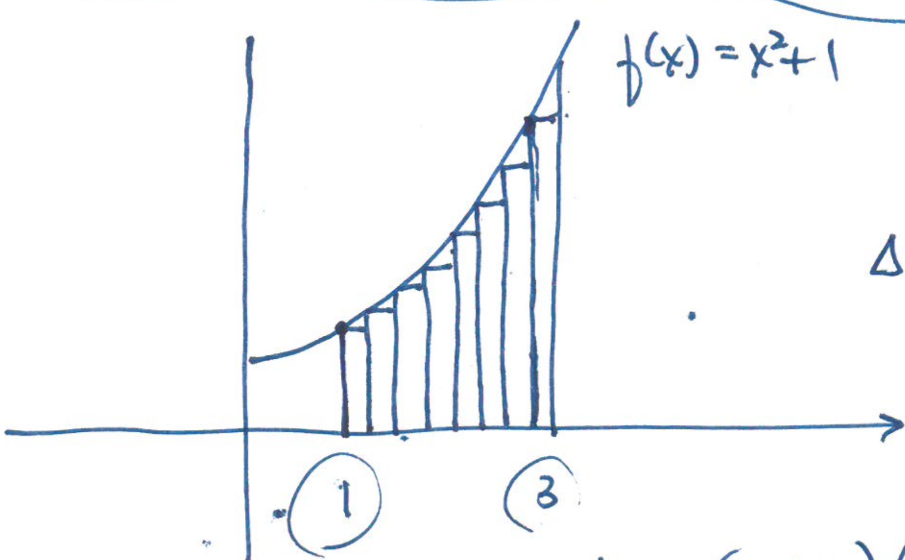
$$A \approx A_1 + A_2 + A_3 + A_4$$

$A_1 = (1^2+1)(\frac{1}{2})$ $A_2 = ((\frac{3}{2})^2+1)(\frac{1}{2})$ $A_3 = (2^2+1)(\frac{1}{2})$ $A_4 = ((\frac{5}{2})^2+1)(\frac{1}{2})$

rectangles
$$\begin{cases} A_1 = (2)(\tfrac{1}{2}) \\ A_2 = (\tfrac{13}{4})(\tfrac{1}{2}) \\ A_3 = (5)(\tfrac{1}{2}) \\ A_4 = (\tfrac{29}{4})(\tfrac{1}{2}) \end{cases}$$

$$A \approx 1 + \frac{13}{8} + \frac{5}{2} + \frac{29}{8} = \frac{8}{8} + \frac{13}{8} + \frac{20}{8} + \frac{29}{8}$$

$$A \approx \frac{70}{8} = \frac{35}{4} = 8\tfrac{3}{4}$$ (underestimate)



$$\Delta x = \frac{3-1}{8} = \frac{1}{4}$$

LESS ERROR

$A_1 = (f(1))(\tfrac{1}{4})$

$A_2 =$

$\vdots$

$A_8 = (f(2\tfrac{3}{4}))(\tfrac{1}{4})$

$$A_1 = (f(1))(1/4) = f(x_1) \cdot \Delta x$$

$$\vdots \qquad\qquad f(x_5) \cdot \Delta x$$

$$\vdots$$

$$A_8 = (f(2\tfrac{3}{4}))(1/4) = f(x_8) \cdot \Delta x$$

$$A \approx A_1 + \cdots + A_8$$

$$A \approx \sum_{i=1}^{8} \left[\overset{HT}{f(x_i)} \cdot \overset{WDTH}{\Delta x} \right]$$

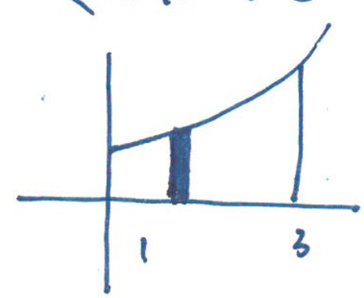
summation

$$A \approx (f(x_1)) \cdot (\Delta x) + (f(x_2)) \cdot (\Delta x) + \cdots + (f(x_8)) \cdot (\Delta x)$$

$$A \approx \sum_{i=1}^{700} f(x_i) \cdot \Delta x$$

$$A \approx \sum_{i=1}^{1,000,000} f(x_i) \cdot \Delta x$$

$$\begin{cases} n \to \infty \\ \Delta x \to 0 \end{cases}$$



$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

\# of rectangles to approach INF.

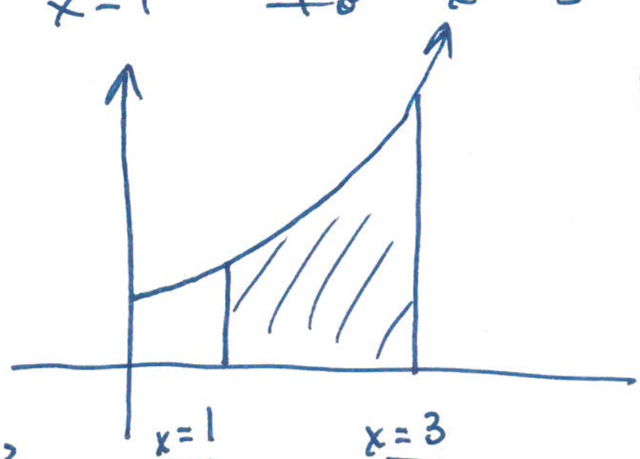
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x = A = \int_a^b \underbrace{f(x)}_{\text{HT}} \cdot \underbrace{dx}_{\text{WDTH}}$$

DEFINITE INTEGRAL

$$\text{AREA} = \int_a^b f(x)\cdot dx = F(x)\Big]_a^b = F(b) - F(a)$$

(where $F'(x) = f(x)$)

ex: find the area under the curve $f(x) = x^2 + 1$ from $x = 1$ to $x = 3$



Indef integ: family of curves

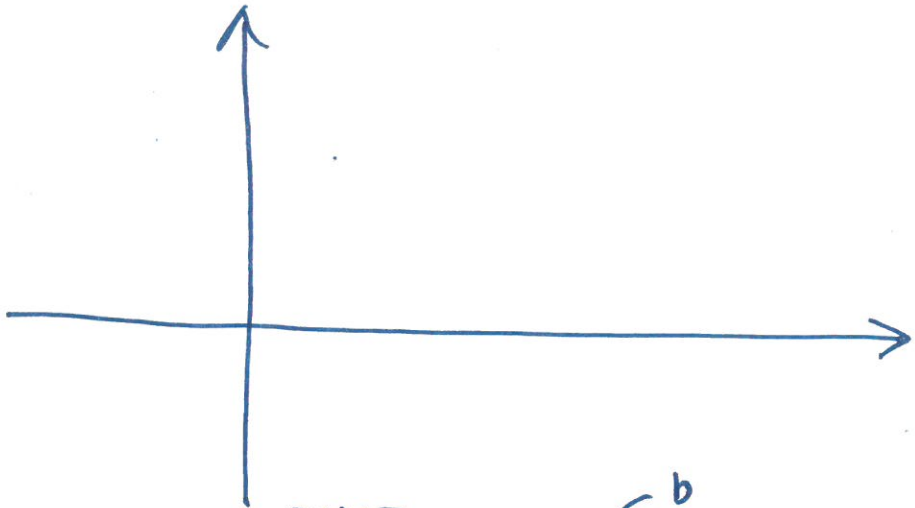
def integ: #

$$A = \int_1^3 (x^2+1)\,dx = \frac{x^3}{3} + x \Big]_1^3$$

$$= \left[\frac{3^3}{3} + 3\right] - \left[\frac{1^3}{3} + 1\right]$$

$$= 9 + 3 - 1/3 - 1 = 10\tfrac{2}{3} \text{ or } 32/3$$

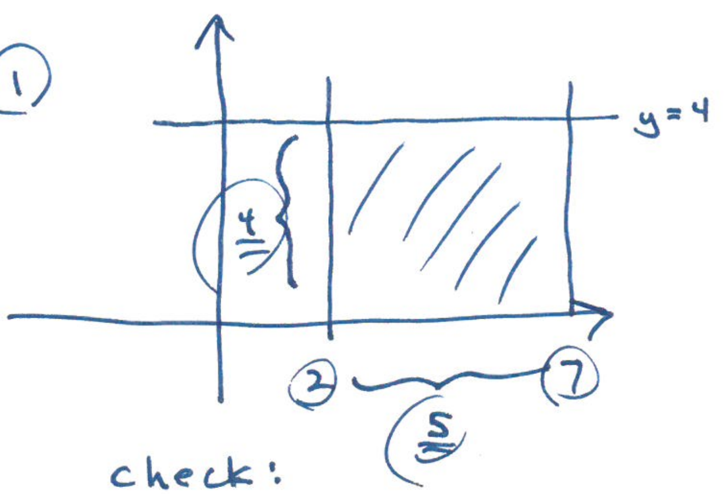
4.3:



$$\text{EXACT AREA} = \int_a^b f(x)\cdot dx = F(x)\Big]_a^b$$

$$= F(b) - F(a)$$

1



area under $f(x)=4$ from $x=2$ to $x=7$

$$A = \int_2^7 4\,dx = 4x\Big]_2^7$$

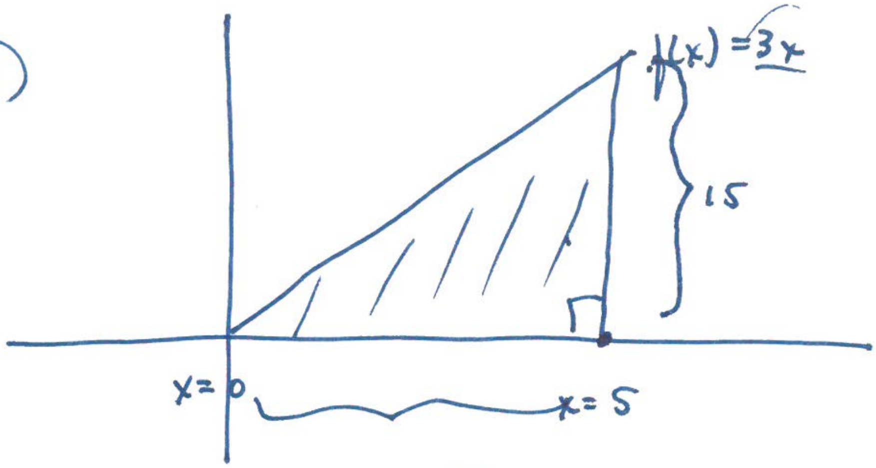
$$A = 4(7) - 4(2)$$

$$A = 28 - 8 = 20$$

check:

$$A = 5\cdot 4 = 20$$

(2)



area under $f(x) = 3x$ from $x = 0$ to $x = 5$

$$A = \int_0^5 (3x)\,dx = 3 \cdot \frac{x^2}{2}\Big]_0^5$$

$$= \frac{3}{2} \cdot x^2 \Big]_0^5 = \frac{3}{2}(5)^2 - \cancel{\frac{3}{2}(0)^2}$$

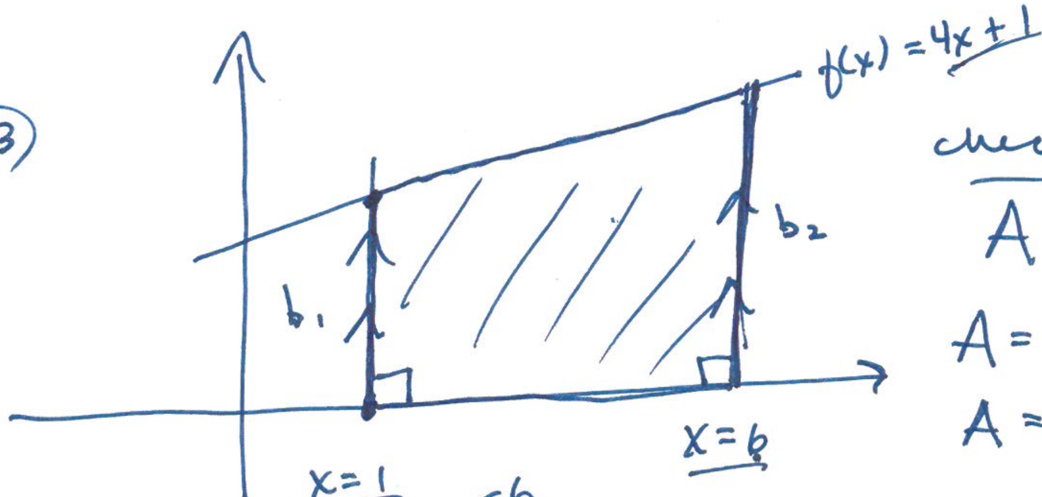
$$= \frac{75}{2}$$

check:

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot 5 \cdot 15 = \frac{75}{2}$$

(3)



check: TRAPEZOID

$$A = \underbrace{\frac{1}{2}(b_1 + b_2)}\cdot h$$

$$A = \frac{1}{2}(5 + 25) \cdot 5$$

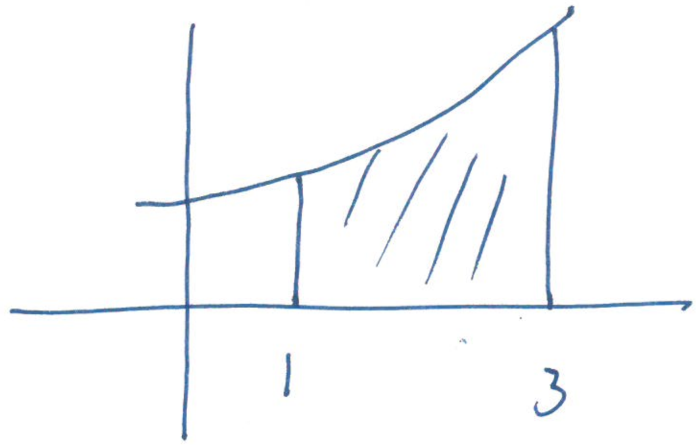
$$A = 15 . 5 = 75$$

$$A = \int_1^6 (4x + 1)\,dx = 4 \cdot \frac{x^2}{2} + x \Big]_1^6$$

$$A = 2x^2 + x \Big]_1^6$$

$$A = \left[2(6)^2 + 6\right] - \left[2(1)^2 + 1\right]$$

$$A = 78 - 3 = 75$$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