MA 121-001

Thursday, November 8

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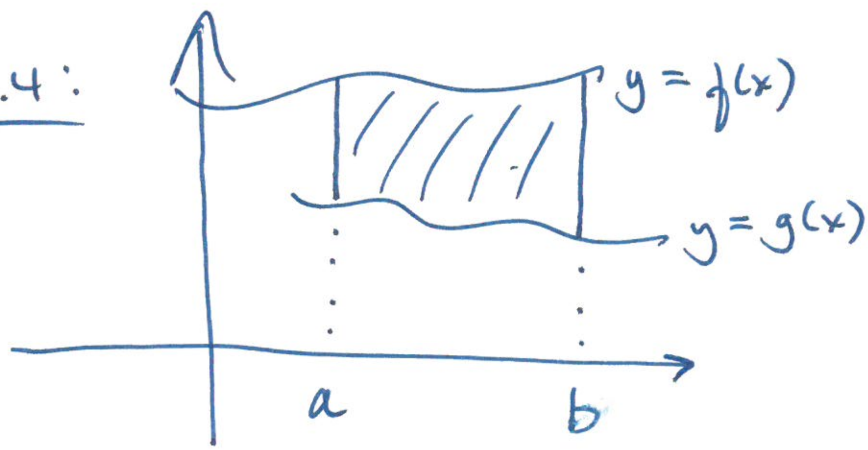
4.4: • area between 2 curves

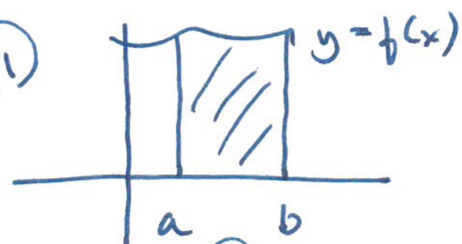
• average value of a function

4.5: • integration using SUBSTITUTION

4.4:



(1)



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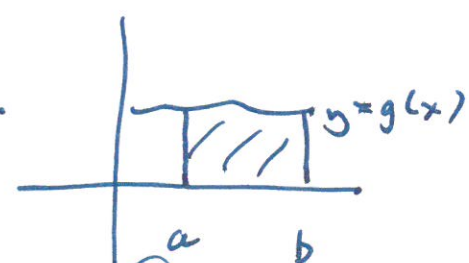
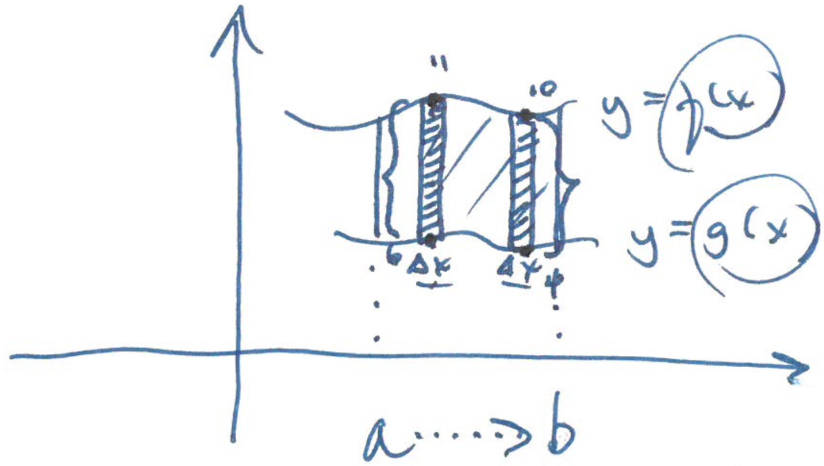


$$\int_a^b f(x)\,dx \quad - \quad \int_a^b g(x)\,dx$$

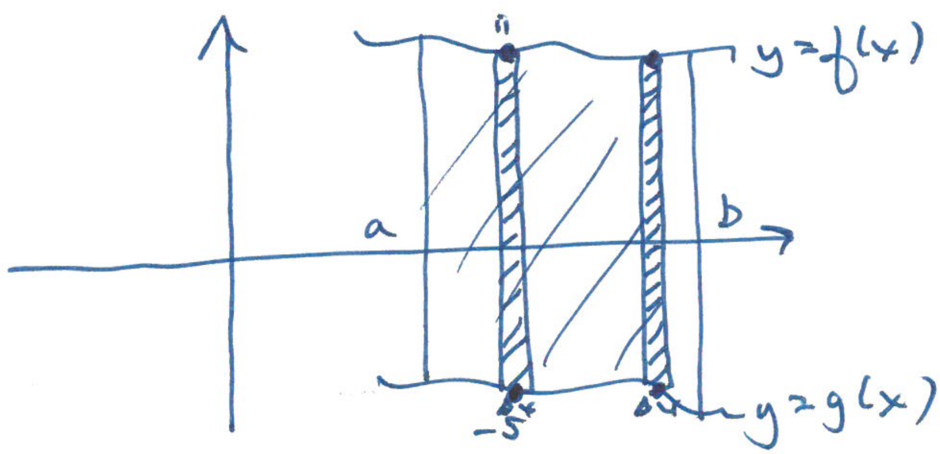
$$A = \int_a^b \left(f(x) - g(x) \right) dx$$

2



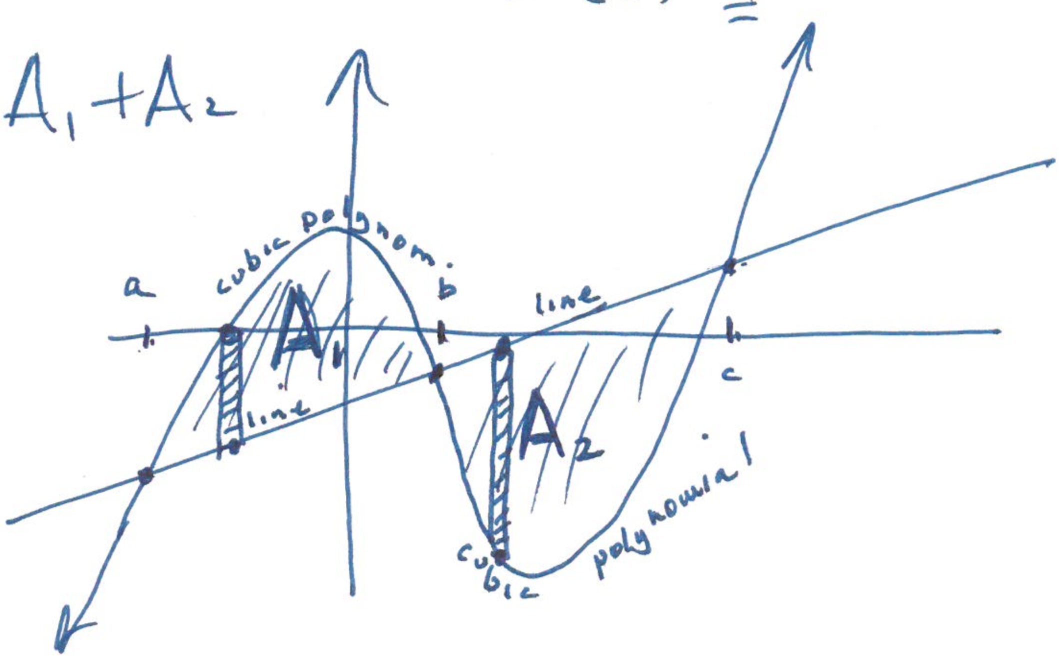
$$A = \int_a^b \underbrace{[f(x) - g(x)]}_{\text{HEIGHT}} \cdot \underbrace{dx}_{\text{WIDTH}} \;*$$

or:



$11 - (-5) = \underline{\underline{16}}$

$A = A_1 + A_2$



$$A_1 = \int_a^b \left[(\text{cubic poly}) - (\text{line})\right] dx$$

$$A_2 = \int_b^c \left[(\text{line}) - (\text{cubic poly})\right] dx$$

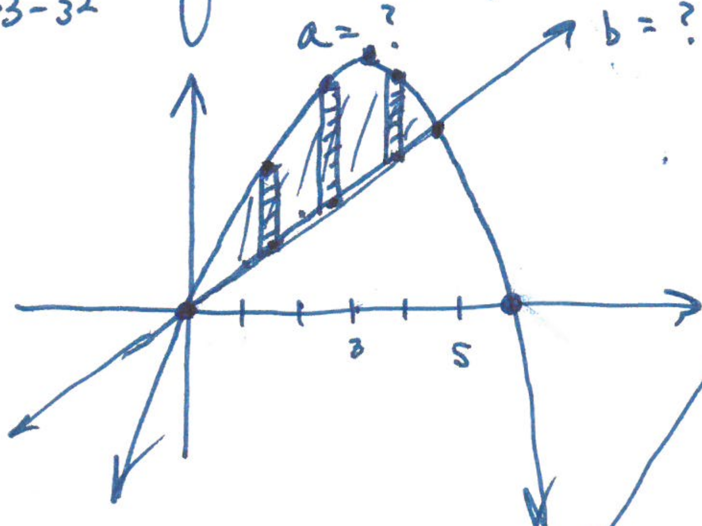
v(3,9)

$f(x) = x(6-x)$ (0,0) (6,0)

ex: $f(x) = \boxed{6x - x^2}$ $\quad g(x) = \boxed{x}$

$f(3) = 6 \cdot 3 - 3^2$

a = ? b = ?



$$\int_0^5 \left[\underbrace{(6x - x^2) - (x)}_{\text{HT.}}\right] \cdot dx$$

pts of int:

$$6x - x^2 = x$$

$$0 = x + x^2 - 6x$$

$$0 = x^2 - 5x$$

$$0 = x(x-5)$$

$x = 0 \qquad x - 5 = 0$

$\qquad\qquad x = 5$

$$\int_0^5 (5x - x^2)\, dx$$

$$\left[5 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_0^5$$

$$\left[\frac{5}{2}(5)^2 - \frac{5^3}{3}\right] - \left[\cancel{\frac{5}{2}(0)^2 - \frac{(0)^3}{3}}\right]$$

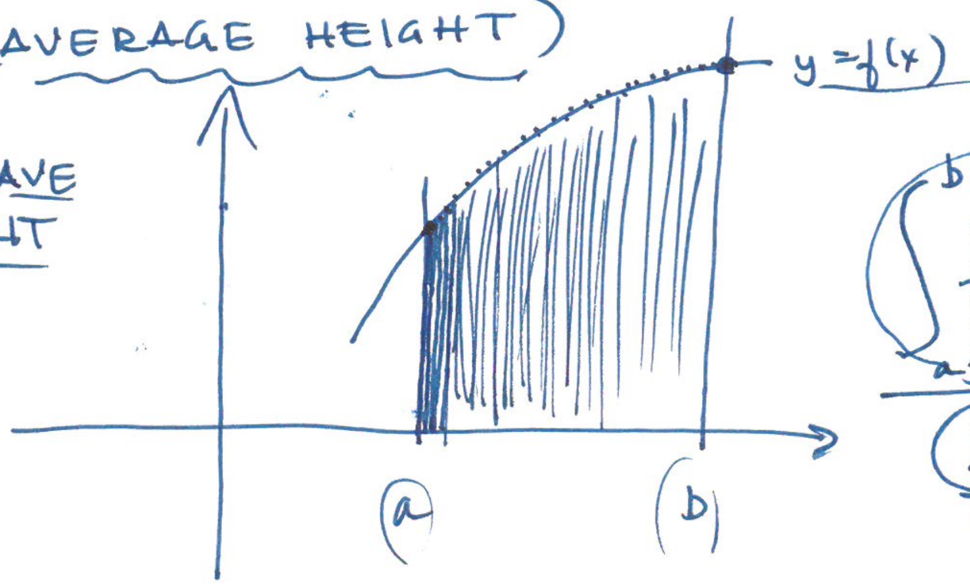
$$\frac{3\cdot 125}{3\cdot 2} - \frac{125\cdot 2}{3\cdot 2} = \frac{375}{6} - \frac{250}{6} = 125/6 = A$$

AVERAGE $\underbrace{\text{VALUE OF A FUNCTION}}_{\text{y-values}}$:

(AVERAGE Y-VALUE)

(AVERAGE HEIGHT)

$$\frac{A}{w} = \text{AVE HT}$$

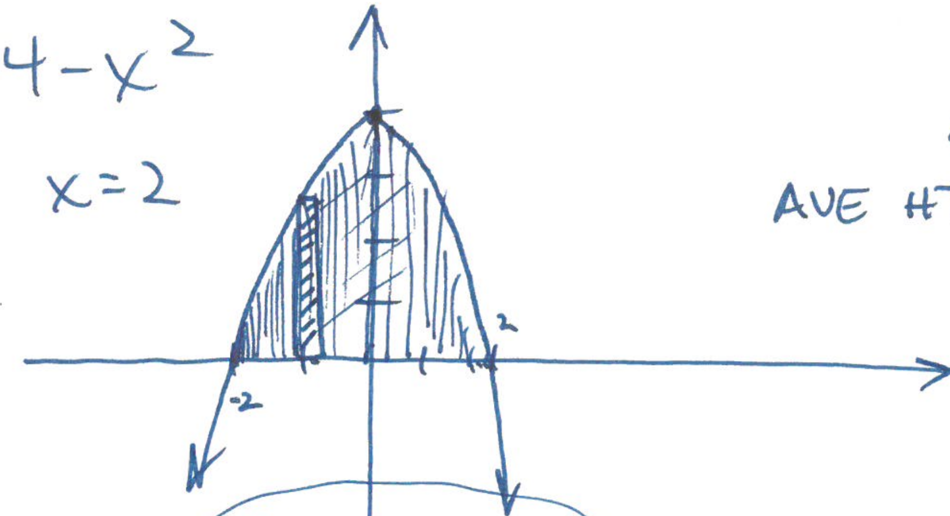


$$\frac{\int_a^b f(x)\,dx = A}{\underbrace{b-a}_{\text{WIDTH}}}$$

AVE VALUE OF THE FUNCTION $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

$f(x) = 4 - x^2$

$x = -2$ to $x = 2$



GUESS?

AVE HT: $\underline{\underline{2}}$

$$\text{AVE HT} = \frac{\int_{-2}^{2} (4 - x^2)\,dx}{\boxed{(2)-(-2)}} = \frac{1}{4}\left(\frac{32}{3}\right) = \frac{8}{3}$$

$$\int_{-2}^{2} (4 - x^2)\,dx = \underline{\underline{A}}$$

$$\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(4 \cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3}$$

$$= \frac{48}{3} - \frac{16}{3} = \frac{32}{3} = \underline{\underline{A}}$$

4.5: INTEGRATION USING SUBSTITUTION

① $\int a \cdot x^n \cdot dx = a \cdot \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)

$\int a \cdot u^n \cdot du = a \cdot \frac{u^{n+1}}{n+1} + C$ (for $n \neq -1$)

(makes it LOOK easier)

② $\int a \cdot x^{-1} dx = a \cdot \ln|x| + C$

$\int a \cdot u^{-1} \cdot du = a \cdot \ln|u| + C$

③ $\int a \cdot e^{bx} \cdot dx = a \cdot \frac{1}{b} \cdot e^{bx} + C$

$\int a \cdot e^{u} \cdot du = a \cdot e^{u} + C$

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ex: $\frac{1}{12} \int (3t^4+2)^5 \cdot \boxed{t^3 \cdot dt \cdot 12}$

let $\boxed{u = 3t^4 + 2}$

$\cancel{dt} \cdot \frac{du}{\cancel{dt}} = 12t^3 \cdot dt$

$\boxed{du = 12t^3 \cdot dt}$

$\boxed{\frac{1}{12} \int u^5 \cdot du}$

$\frac{1}{12} \cdot \frac{u^6}{6} + C = \frac{1}{72} \cdot (3t^4+2)^6 + C$

$$\int x \sqrt[5]{1-x^2} \cdot dx$$

$$-\frac{1}{2}\int (1-x^2)^{1/5} \boxed{x \cdot dx\,(-2)}$$

$$\text{let } \boxed{u = 1-x^2}$$

$$\cancel{dx} \cdot \frac{du}{\cancel{dx}} = -2x \cdot dx$$

$$\boxed{du = -2x \cdot dx}$$

$$-\frac{1}{2}\int u^{1/5} \cdot du = -\frac{1}{2}\left[\frac{u^{6/5}}{6/5}\right] + C$$

$$= -\frac{1}{2} \cdot \frac{5}{6}(1-x^2)^{6/5} + C$$

$$= -\frac{5}{12}(1-x^2)^{6/5} + C$$

$$\int_{x=1}^{x=4} \frac{2x+1}{x^2+x+1}\,dx$$

let $u = x^2+x+1$

$du = (2x+1)\,dx$

$u = x^2+x+1$

$x=1 \rightarrow u=3$

$x=4 \rightarrow u=21$

$$\int_{3}^{21} \frac{du}{u} = \int \frac{1}{u}\cdot du = \ln|u| \Big]$$

$$= \ln|x^2+x+1| \Big]_1^4$$

$$= \ln|4^2+4+1| - \ln|1^2+1+1|$$