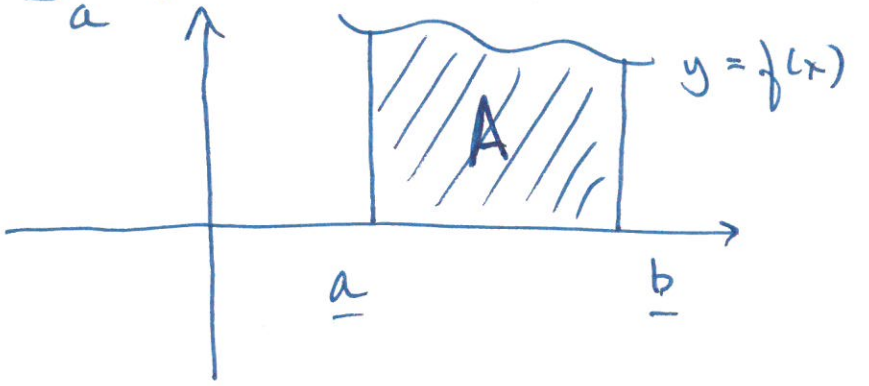
MA121-001

Tuesday, November 20

5.3: IMPROPER INTEGRALS

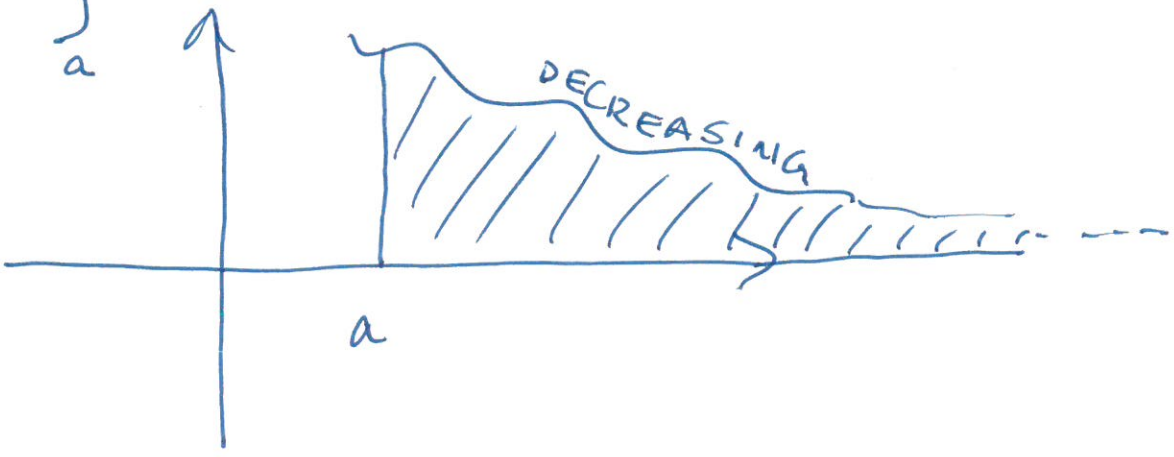
proper:

$$A = \int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$



improper:

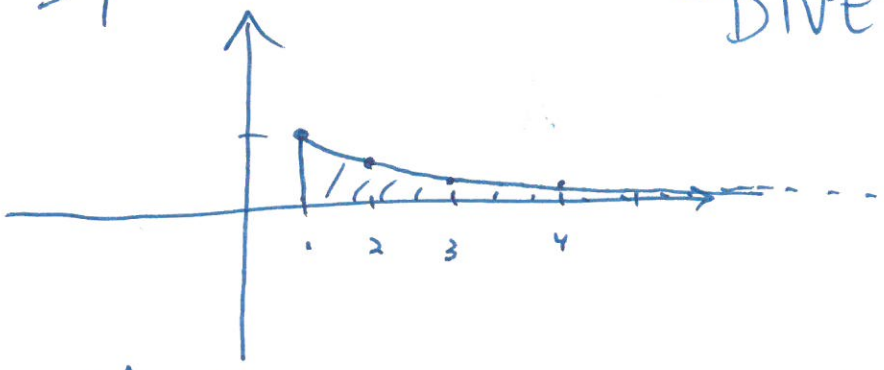
$$A = \int_a^\infty f(x)\,dx = F(x)\Big]_a^\infty \quad ??? \quad \ldots$$



ex: $y = \frac{1}{x}$ find the area under this curve from $x = 1$ to $x = \infty$

$$\int_1^{\infty} \frac{1}{x} \cdot dx$$

integral DIVERGES



$$\lim_{A \to \infty} \int_1^A \frac{1}{x} dx = \lim_{A \to \infty} \left[\ln|x| \right]_1^A$$

$$= \lim_{A \to \infty} \left[\ln A - \ln 1 \right] = \lim_{A \to \infty} \left[\ln A \right]^{\infty}$$

$\ln 1 = 0$

no $\lim$

$A = 100$ | $A = 10{,}000$ | $A = 100{,}000{,}000$

$\ln 100$ | $\ln 10{,}000$ | $\ln 100{,}000{,}000$ ... larger

ex: $y = \frac{1}{x^2}$ find the area from $x=1$ to $x=\infty$

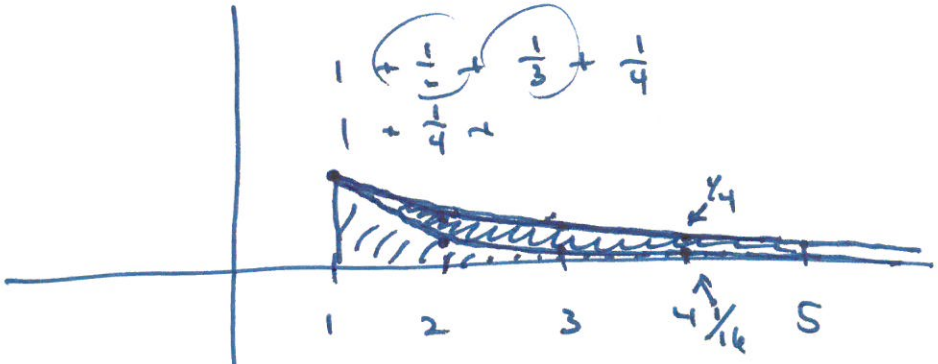
$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{B\to\infty} \int_1^B x^{-2}\,dx$$

$$= \lim_{B\to\infty} \left[\frac{x^{-1}}{-1}\right]_1^B = \lim_{B\to\infty} \left[\frac{-1}{x}\right]_1^B$$

$$= \lim_{B\to\infty} \left[\left(\frac{-1}{B}\right) - \left(\frac{-1}{1}\right)\right]$$

$$= \lim_{B\to\infty} \left(1 + \frac{1}{B}\right) \to 0 = 1$$

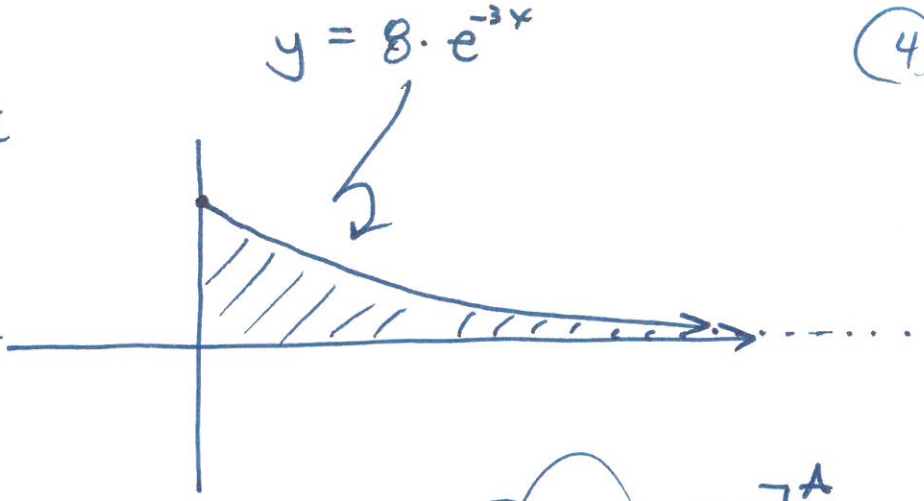
$\therefore$ integral CONVERGES



$$\int_1^{672{,}985} \frac{1}{x^2}\,dx = .999998613$$

(less than 1)

$$A = \int_0^\infty 8 \cdot e^{-3x}\, dx$$



$$\int e^{bx}\, dx = \frac{1}{b} \cdot e^{bx} + C$$

$$\lim_{A \to \infty} 8 \int_0^A e^{-3x}\, dx = 8 \cdot \lim_{A \to \infty} \left[\frac{1}{-3} e^{-3x} \right]_0^A$$

$$= \frac{8}{-3} \lim_{A \to \infty} \left[\frac{1}{e^{3x}} \right]_0^A = -\frac{8}{3} \cdot \lim_{A \to \infty} \left[\frac{1}{e^{3A}} - \frac{1}{e^{3(0)}} \right]$$

$$= \left(\frac{-8}{3} \right) \lim_{A \to \infty} \left[\frac{1}{e^{3A}} - 1 \right] = \frac{-8}{3}(-1) = \frac{8}{3}$$

$\therefore$ integral CONVERGES

$$\int_1^\infty \frac{1}{x^1}dx = \text{DIVERGES}$$

$$\int_1^\infty \frac{1}{x^{1.0000001}}dx \Rightarrow \text{CONV.}$$

5.6: VOLUMES OF SOLIDS OF REVOLUTION

$$\int_a^b \underbrace{f(x)\cdot dx}_{\text{AREA}} \longrightarrow \int_a^b \underbrace{\pi\,(f(x))^2}_{\text{VOL}}\cdot \underbrace{(dx)}_{h}$$

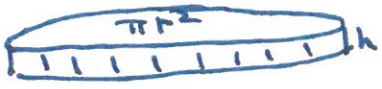
HT. WDTH

r^2

VOLUME OF ONE DISK

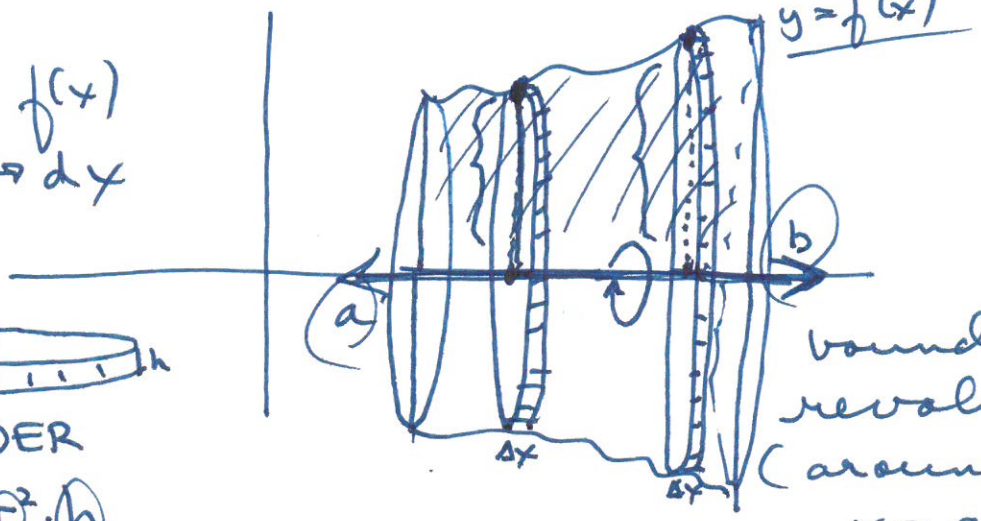
$r = y = f(x)$

$h = \Delta x \to dx$



CYLINDER

$V = \pi r^2 \cdot h$

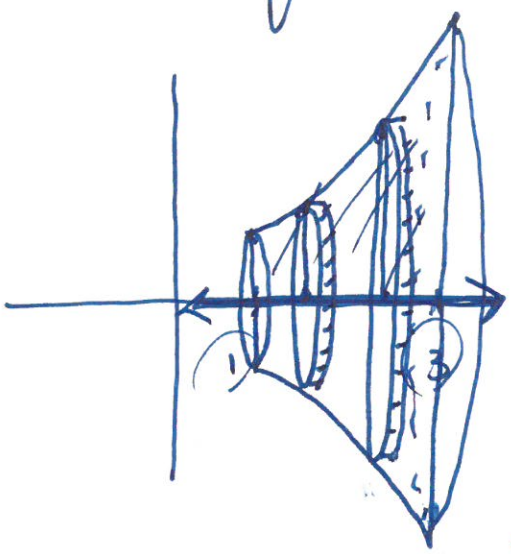


bounded region revolved about (around) the x-axis

$$V = \int_a^b \pi\,(f(x))^2\,dx$$

(6)

$f(x) = x^2$ find the volume of this:



region bounded by $y = x^2$ from $x = 1$ to $x = 3$ revolved about the x-axis

$$V = \int_1^3 \pi (f(x))^2 \cdot dx$$

$$V = \pi \cdot \int_1^3 (x^2)^2 \cdot dx$$

$$V = \pi \int_1^3 x^4 \, dx$$

$$V = \pi \left[\frac{x^5}{5} \right]_1^3 = \frac{\pi}{5} \cdot \left[3^5 - 1^5 \right]$$

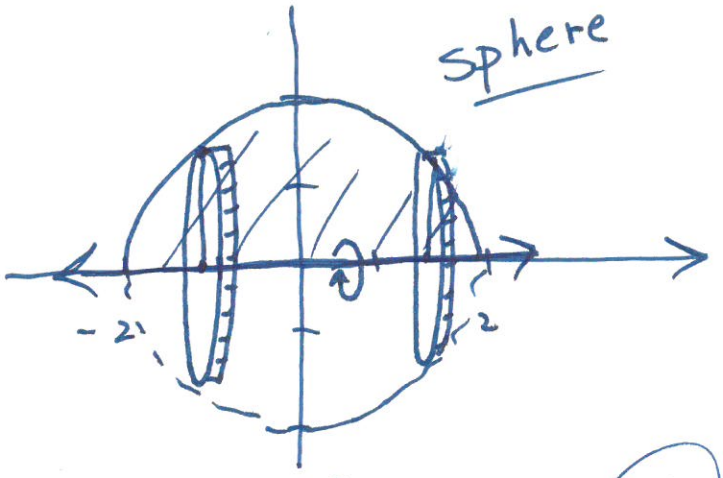
$$V = \frac{\pi}{5} \cdot \left[243 - 1 \right] = \frac{242\pi}{5}$$

$f(x) = \sqrt{4-x^2}$ upper semicircle

from $x=-2$ to $x=2$

revolve about the x-axis

find VOLUME:



$y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2+y^2 = 4$

circle; $c(0,0)$ $r=2$

$$V = \int_{-2}^{2} \pi (f(x))^2 \cdot dx$$

$$V = \int_{-2}^{2} \pi \left(\sqrt{4-x^2}\right)^2 \cdot dx$$

$$V = \pi \int_{-2}^{2} (4-x^2) \cdot dx$$

sphere:

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (2)^3 = \frac{32\pi}{3}$

$$V = \pi \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \pi \left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$V = \pi \left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right] = \pi \left[16 - \frac{16}{3}\right]$$

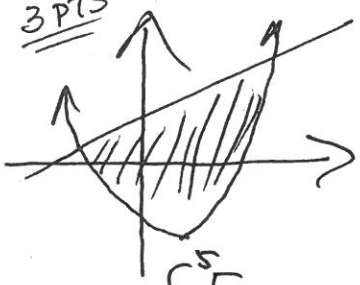
$$V = \pi \left[\frac{48}{3} - \frac{16}{3}\right] = \pi \left[\frac{32}{3}\right] = \frac{32\pi}{3}$$

NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA121 Quiz #2 — J. Griggs

Three points per question; you are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet.

1.) Find the area bounded by the two curves: $f(x)=x^2-x-5$ *and* $g(x)=x+10$. (3 PTS)



pts of int: $x^2-x-5=x+10$

$x^2-x-5-x-10=0$

$x^2-2x-15=0$

$(x+3)(x-5)=0$

$x=-3$ and $x=5$

$$A=\int_{-3}^{5}\left[(x+10)-(x^2-x-5)\right]dx$$

$$=\int_{-3}^{5}(-x^2+2x+15)\,dx$$

$$=\left[\frac{-x^3}{3}+x^2+15x\right]_{-3}^{5}$$

$$=\left[\frac{-5^3}{3}+5^2+15(5)\right]-\left[\frac{-(-3)^3}{3}+(-3)^2+15(-3)\right]$$

$$=\left(\frac{-125}{3}+25+75\right)-\left(\frac{27}{3}+9-45\right)$$

$$=145-18-\frac{125}{3}$$

$$=\frac{256}{3}=A=85\tfrac{1}{3}$$

2.) Find the average value of the function $f(x)=x^2-x+1$ *on* $[0,2]$. (3 PTS)

$$\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{2-0}\int_0^2(x^2-x+1)\,dx=\frac{1}{2}\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^2$$

$$=\frac{1}{2}\left[\left(\frac{2^3}{3}-\frac{2^2}{2}+2\right)-(0)\right]=\frac{1}{2}\left[\frac{8}{3}-2+2\right]=\frac{4}{3}=f_{AVE}$$

3.) Integrate using substitution: $\int(2t^5-3)^2\,t^4\,dt$ (3 PTS)

let $u=2t^5-3$

$du=\left[2(5t^4)-0\right]dt$

$du=10t^4\,dt$

$$\frac{1}{10}\int\underbrace{(2t^5-3)^2}_{u^2}\cdot\underbrace{t^4\cdot dt\cdot 10}_{du}$$

$$\frac{1}{10}\int u^2\cdot du=\frac{1}{10}\cdot\frac{u^3}{3}+C$$

$$=\frac{1}{30}(2t^5-3)^3+C$$

NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-001 Quiz #3 Due Tuesday, November 27, 2018 (at the beginning of class) J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

1.) Evaluate the improper integral $\int_2^\infty 7x^{-2}\,dx$. Doe this integral converge or diverge?

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5 \ \ for -5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of $200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?