MA 121-001:

Tuesday, November 27

- today: <u>5.7</u>, QUIZ #3 collected / graded
- TEST #4: Thursday, November 29

<u>5.7</u>: SEPARABLE DIFFERENTIAL EQUATIONS (D.E.'s) — has a DERIV. in it.

#1 separate x's & dx's from the y's & dy's.

already done:

$y' = 11x$ find y such that $y(2) = 4$

$x = 2$

$y = 4$

$y = \int 11x\,dx$

$y = 11 \cdot \frac{x^2}{2} + C$

$4 = \frac{11}{2}(2)^2 + C$

$4 = \frac{11}{\cancel{2}_1}(\cancel{4}^2) + C$

$4 = 22 + C$

$C = -18$

$y = \frac{11}{2}x^2 - 18$

ex:

$$\cancel{dx}\boxed{\frac{dy}{\cancel{dx}} = \frac{2x}{y}}\cdot dx$$ (1st order D.E.)

$$y \cdot dy = \frac{2x}{\cancel{y}} \cdot dx \cdot \cancel{y}$$

$$y \cdot dy = 2x\,dx$$

$$\int y\,dy = \int 2x\,dx$$

① sep. x's & dx's from y's & dy's
② integrate both sides
③ solve for y

$$\frac{y^2}{2} + C_1 = \cancel{2} \cdot \frac{x^2}{\cancel{2}} + C_2$$

$$\frac{y^2}{2} = x^2 + \underbrace{C_2 - C_1}$$

$$\frac{y^2}{2} = x^2 + C$$

$$y^2 = 2x^2 + K$$

$$y = \pm\sqrt{2x^2 + K}$$ ✓

ex: $$\frac{dy}{dx} = 5x^4 \cdot y$$

$$\cancel{dx}\,\frac{dy}{\cancel{dx}} = 5x^4 \cdot y \cdot dx$$

$$\frac{1}{y}\,dy = 5x^4 \cdot \cancel{y} \cdot dx \cdot \frac{1}{\cancel{y}}$$

$$\frac{1}{y}\,dy = 5x^4 \cdot dx$$

$$\int \frac{1}{y}\,dy = \int 5x^4\,dx$$

$$\ln y = \cancel{5} \cdot \frac{x^5}{\cancel{5}} + C$$

$$\underline{\ln y} = \underline{x^5 + C}$$

$$e^{\ln y} = e^{x^5 + C} \quad \text{exponentiating}$$

$$y = e^{x^5 + C}$$

$$y = e^{x^5} \cdot e^{c} \qquad e^{c} = A$$

$$\boxed{y = A \cdot e^{x^5}}$$

$$\frac{dP}{dt} = K \cdot P \cdot$$ exponential growth/ decay

is

rate of change of Population

$$\cancel{dt}\,\frac{dP}{\cancel{dt}} = K \cdot P \cdot dt$$

$$\frac{1}{P} \cdot dP = K \cdot \cancel{P} \cdot dt\,\frac{1}{\cancel{P}}$$

$$\frac{1}{P}\,dP = K \cdot dt$$

$$\int \frac{1}{P}\,dP = \int K \cdot dt$$

$$\ln P = Kt + C$$

$$e^{\ln P} = e^{Kt + C}$$

$$P = e^{Kt} \cdot e^{c} \qquad e^{c} = B$$

$$P = B \cdot e^{Kt}$$

$t = 0$ ✓

$P = P_0$ ✓

$$P_0 = B \cdot e^{K(0)}$$

$$P_0 = B$$

$$P = P_0 \cdot e^{Kt}$$

ex:

$$y' = 2x + xy$$

$$\cancel{dx}\,\frac{dy}{\cancel{dx}} = x(2+y)\cdot dx$$

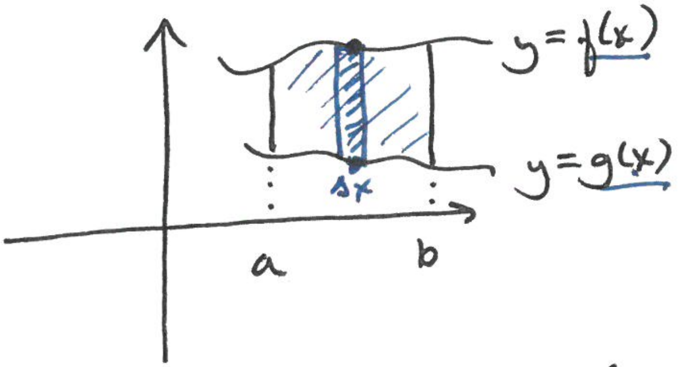
$$\frac{1}{2+y}\,dy = x\cdot\cancel{(2+y)}\,dx\,\cancel{\frac{1}{2+y}}$$

$$\int \frac{1}{2+y}\,dy = \int x\,dx$$

$$\vdots$$

MA121 — TEST #4:

4.4: • area between 2 curves:



$$A = \int_a^b [f(x) - g(x)]\,dx$$

• average value of a function

$$\text{AVE HT} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

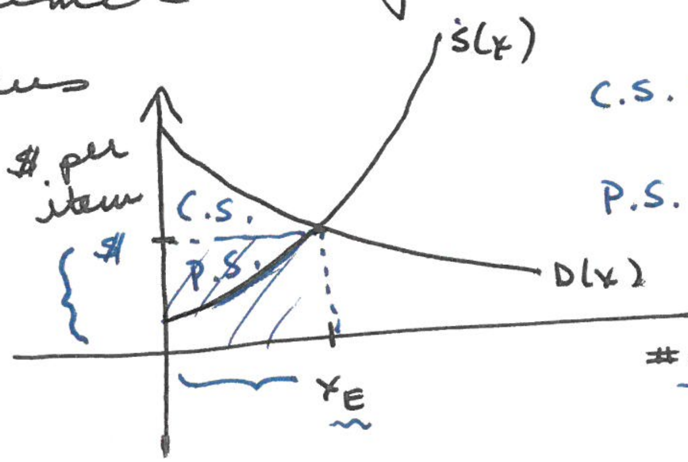
4.5: • integration using SUBSTITUTION

let $u =$ ______

$du = 8t^4\,dt$

$$\left(\frac{1}{8}\right)\int \underbrace{t^4 \cdot dt\, 8}_{du}$$

5.1: • consumer surplus and producer surplus



$$C.S. = \int_0^{x_E} D(x)\,dx - \$\text{ box}$$

$$P.S. = \$\text{ box} - \int_0^{x_E} S(x)\,dx$$

EQ. PT: $D(x) = S(x)$

$x_E =$ ______

$D(x_E) = S(x_E)$

5.2: • accumulation models

① one-time contribution:

$$y = y_0 \cdot e^{kt} \quad *$$

② multiple (yearly) contributions:

$$\int_{T_0}^{T_1} y_0 e^{kt}\, dt \qquad \text{ACCUM F.V.} = \int_0^{20} 5000 \cdot e^{(.031)t}\, dt$$

(accumulated future value of a continuous income stream)

5.3: • improper integrals

$$\int_3^{\infty} f(x)\,dx \rightarrow \lim_{A\to\infty} \int_3^A f(x)\,dx$$

① delay ∞ 'til end of prob. (lim …)

② integrate

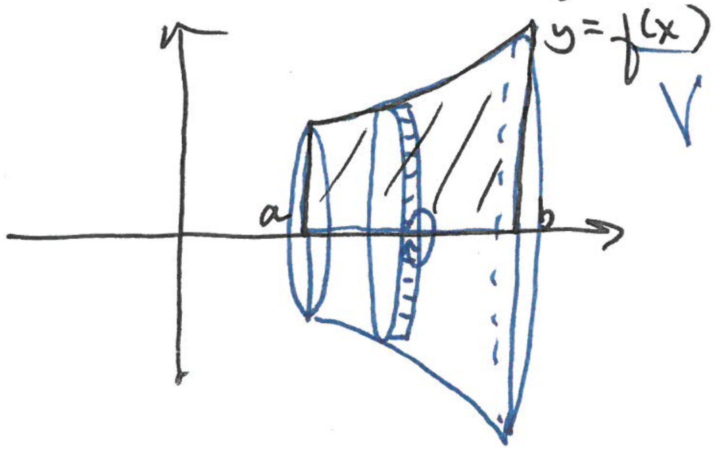
③ evaluate

④ simplify

⑤ take limit *.

⑥ converge or diverge

5.6: • volumes of solids of revolution



$$V = \int_a^b \pi \, [f(x)]^2 \cdot dx$$

$$\pi \cdot r^2 \cdot h$$

5.7: • separable differential equations

1. separate x's & dx's from y's & dy's.
2. integrate both sides
3. if possible, solve for y

QUIZ #3

NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA121 — J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

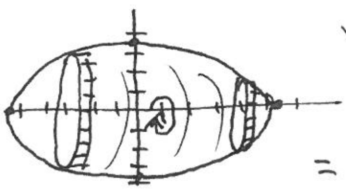
1.) Evaluate the improper integral $\int_2^{\infty} 7x^{-2}\,dx$. Does this integral converge or diverge?

$$\lim_{A\to\infty} 7\int_2^A x^{-2}\,dx = 7\cdot\lim_{A\to\infty}\left[\frac{x^{-1}}{-1}\right]_2^A = 7\cdot\lim_{A\to\infty}\left[\frac{-1}{x}\right]_2^A$$

$$= 7\cdot\lim_{A\to\infty}\left[\left(\frac{-1}{A}\right)-\left(\frac{-1}{2}\right)\right] = 7\cdot\lim_{A\to\infty}\left[\frac{1}{2}-\frac{1}{A}\right] \to 0 = 7\cdot\frac{1}{2} = \frac{7}{2}$$

∴ integral converges

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5 \;\; for -5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.



$$\text{VOL} = \int_{-5.5}^{5.5} \pi\,(f(x))^2\cdot dx = \pi\int_{-5.5}^{5.5}\left(-.116x^2+3.5\right)^2\cdot dx$$

$$= \pi\int_{-5.5}^{5.5}\left(.013456x^4 - .812x^2 + 12.25\right)dx$$

$$= \pi\left[.013456\frac{x^5}{5} - .812\frac{x^3}{3} + 12.25x\right]_{-5.5}^{5.5} = \pi\left[\left(\frac{.013456}{5}(5.5)^5 - \frac{.812}{3}(5.5)^3 + (12.25)(5.5)\right) - \left(\frac{.013456}{5}(-5.5)^5 - \frac{.812}{3}(-5.5)^3 + (12.25)(-5.\right.\right.$$

$$\approx 71.774\pi \approx 225.48 \text{ in}^3$$

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of $200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?

$$\int_0^{34} 200{,}000\cdot e^{.029t}\,dt = 200{,}000\left[\frac{1}{.029}e^{.029t}\right]_0^{34}$$

$$= \frac{200{,}000}{.029}\left[e^{.029(34)} - e^{.029(0)}\right] = \frac{200{,}000}{.029}\left[2.68049 - 1\right]$$

$$\approx \$11{,}589{,}593.^{35}$$