MA 121-001

Thursday, December 6

- 6.3 today
- review for final exam
- FINAL EXAM: Thursday, December 13
  8:00 - 11:00 am SAS 2203
- do class evaluation, please
  (before 12/10 at 8:00 am)

MAX / MIN / SADDLE POINT FOR $f(x, y)$:

1.) find f_x, f_y, f_{xx}, f_{yy}, f_{xy} : f_{yx} =

2.) set $f_x = 0$ & $f_y = 0$ $\Longrightarrow$ (a, b)

3.) D TEST (2nd DERIV TEST) $(a, b, f(a,b))$

$$D = f_{xx}(a,b) \cdot f_{yy}(a,b) - \left[f_{xy}(a,b) \right]^2$$

a.) if $D > 0$ & $f_{xx}(a,b) < 0$
$\longrightarrow$ $(a, b, f(a,b))$ is a MAX

b.) if $D > 0$ & $f_{xx}(a,b) > 0$
$\longrightarrow$ $(a, b, f(a,b))$ is a MIN

c.) if $D < 0$ $\longrightarrow$ $(a, b, f(a,b))$ is a SADDLE POINT

d.) if $D = 0$ $\longrightarrow$ TEST FAILS

$$f_x = 2x + y(1) + 0 - 7$$

$$\boxed{f_x = 2x + y - 7}$$

1 $f_{xx} = 2 + 0 - 0 = 2$

2 $f_{xy} = 0 + 1 - 0 = 1$

$$f_y = 0 + x + 4y - 0 = x + 4y$$

$$\boxed{f_y = x + 4y}$$

1 $f_{yy} = 0 + 4 = 4$

2 $f_{yx} = 1 + 0 = 1$

example:

$$f(x,y) = x^2 + xy + 2y^2 - 7x$$

1.) find f_x, f_y, f_{xx}, f_{xy}, f_{yy}, f_{yx}:

$\boxed{f_x = 2x + y - 7}$ $\qquad 2x + y - 7 = 0$

$\boxed{f_y = x + 4y}$ $\qquad x + 4y = 0$

$f_{xx} = 2 \qquad f_{yy} = 4 \qquad f_{xy} = f_{yx} = 1$

2.) solve $f_x = 0$ and $f_y = 0$

$2x + y - 7 = 0 \qquad x + 4y = 0$

$\qquad\qquad\qquad\qquad x = -4y$

SUBST.

$2(-4y) + y - 7 = 0$

$-8y + y - 7 = 0$

$-7y = 7$

$\boxed{y = -1}$

$x = -4y = -4(-1) = 4$

$(4, -1)$ actually $(4, -1, f(4,-1))$

is a possible max / min / saddle

$(4, -1, ?)$ with x and y labels

$f(4,-1) =$

z

3.) D-TEST:

$$D = f_{xx}(4,-1) \cdot f_{yy}(4,-1) - [f_{xy}(4,-1)]^2$$

$$D = (2) \cdot (4) - 1^2$$

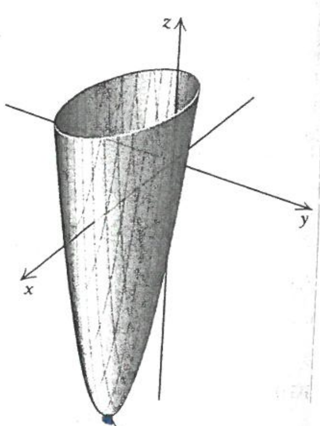
$$D = 7$$

4.) $D = 7$ and $f_{xx}(4,-1) = 2$

(since $D > 0$ and $f_{xx}(4,-1) > 0$, this is a relative MINIMUM.

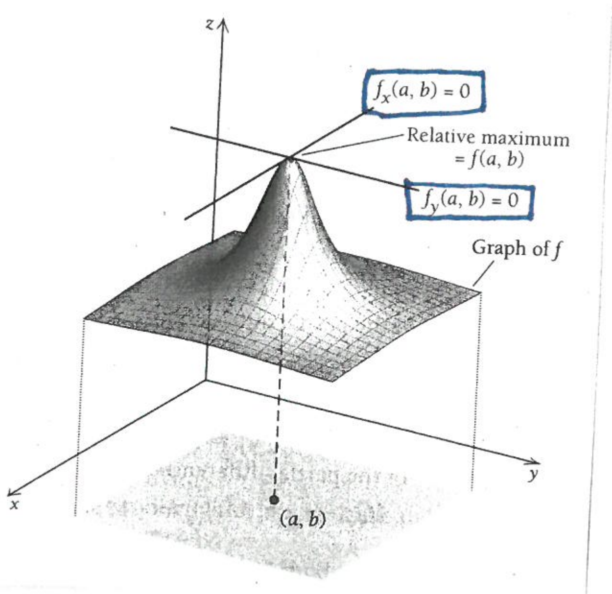
$$f(+4,-1) = 4^2 + 4(-1) + 2(-1)^2 - 7 \cdot 4$$
$$= 16 - 4 + 2 - 28$$
$$= -14$$

thus $(+4,-1,-14)$ is a relative MIN

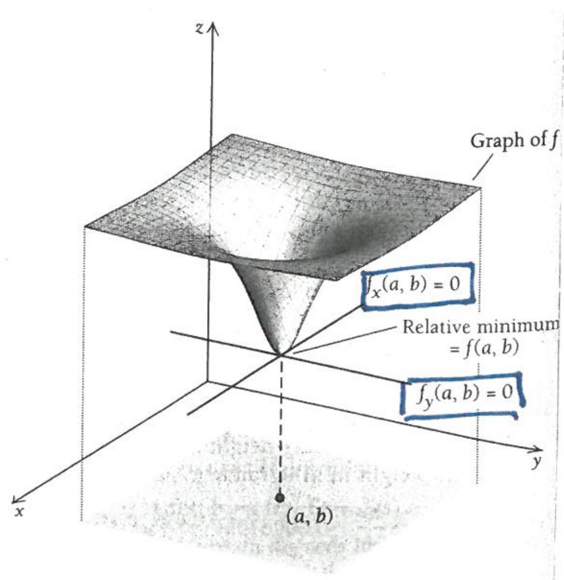


Relative minimum: $(4, -1, -14)$
$z = f(x, y) = x^2 + xy + 2y^2 - 7x$

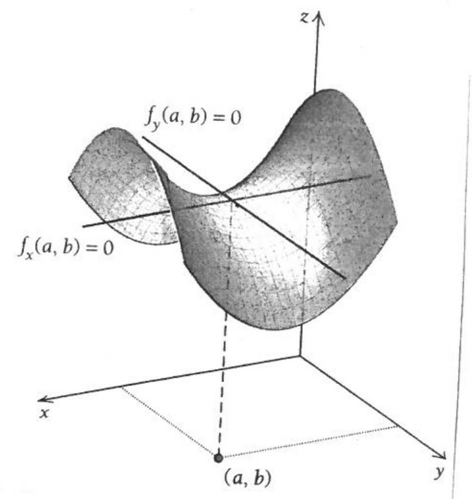
RELATIVE MAX:

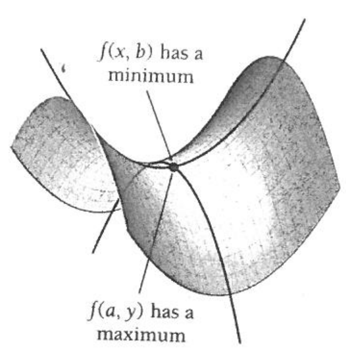


RELATIVE MIN:



SADDLE POINT:





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TEST #4 RESULTS

A's 75 (40.5%)

B's 51 (27.6%)

68.1%

C's 24 (12.9%)

D's 16 (8.6%)

F's 19 (10.3%)

18.9%

AVE: 81.49