NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121-001 Test #2 Form B Thursday, October 11, 2018 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided; one problem per page (the back of a page constitutes a new page). Calculators can be used; however, graphing calculators (nor any other calculator that does calculus) cannot be used. Please put your **name, row letter** and **seat number** on the blue book – upper right corner. Seven questions – 14 points each; 2 points for following directions; 5 point bonus question. Simplify all derivatives completely. Turn in the test copy with the blue book.

1.) Find all intercepts and asymptotes; use $f'(x)$ to determine where the function is increasing and decreasing; graph it: $f(x)=\dfrac{4x+1}{x-3}$

2.) Find and use $f'(x)$ *and* $f''(x)$ in order to find all critical points, points of inflection, areas where the function is increasing, decreasing, concave up and concave down, and all relative maximum and minimum values of the function. Graph the function: $f(x)=x^3-3x+6$

3.) A stone is projected upward from a platform 55 feet above the ground with an initial velocity of 130 feet per second. The height *s(t)* above the ground (in feet) at time *t* (in seconds) is given by $s(t)=-16t^2+130t+55$. Find the height of the stone at t=2 sec, the velocity of the stone at t=2 sec, and the acceleration of the stone at t=2 sec. (appropriate units are important)

4.) Find the critical point(s); determine where the function is increasing and decreasing; and graph: $y=1+(x-4)^{2/3}$

5.) Find the absolute maximum and absolute minimum values of the function on the given closed interval using calculus techniques: $f(x)=x^3-x^2-x+5 \text{ on } [0,3]$

6.) Find the equation of the tangent line to $y=\sqrt{3x^2+6x}$ at the point (1, 3).

7.) a.) Find y' using the product rule: $y=(2x-3)(5x^2-4x+8)$

b.) Find y' and y": $y=(3x-4)^7$

Bonus: (5 points) For the function $f(x)=\dfrac{x^3-6x^2+5x+12}{x^2-2x-8}=\dfrac{(x-3)(x^2-3x-4)}{x^2-2x-8}$ find all intercepts and all asymptotes (including oblique asymptotes). Does this function have a "hole" in it? If so, where is it? Use all of the above information to sketch a graph of this function. ($f'(x)$ and $f''(x)$ are not necessary for this sketch)

MA 121 TEST #2 FORM B

(14 points each)

1.) $f(x) = \dfrac{4x+1}{x-3}$

$$f'(x) = \frac{(x-3)(4) - (4x+1)(1)}{(x-3)^2}$$

$$f'(x) = \frac{\cancel{4x} - 12 - \cancel{4x} - 1}{(x-3)^2}$$

$$f'(x) = \frac{-13}{(x-3)^2}$$

① $f'(x) \neq 0$

② $f'(x)$ undef at $x=3$

$f(3) = ?$ no! VERT. ASYMP.

intercepts:

① y-int: $(x=0)$

$$y = \frac{4(0)+1}{0-3} = \frac{-1}{3}$$

$(0, -1/3)$

② x-int: $(y=0)$

$$0 = \frac{4x+1}{x-3}$$

$4x+1=0$

$x = -1/4$

$(-1/4, 0)$

$f'(x)$:

$f'(0) = -$ | $f'(4) = -$

$f(x)$ DECR ③ $f(x)$ DECR

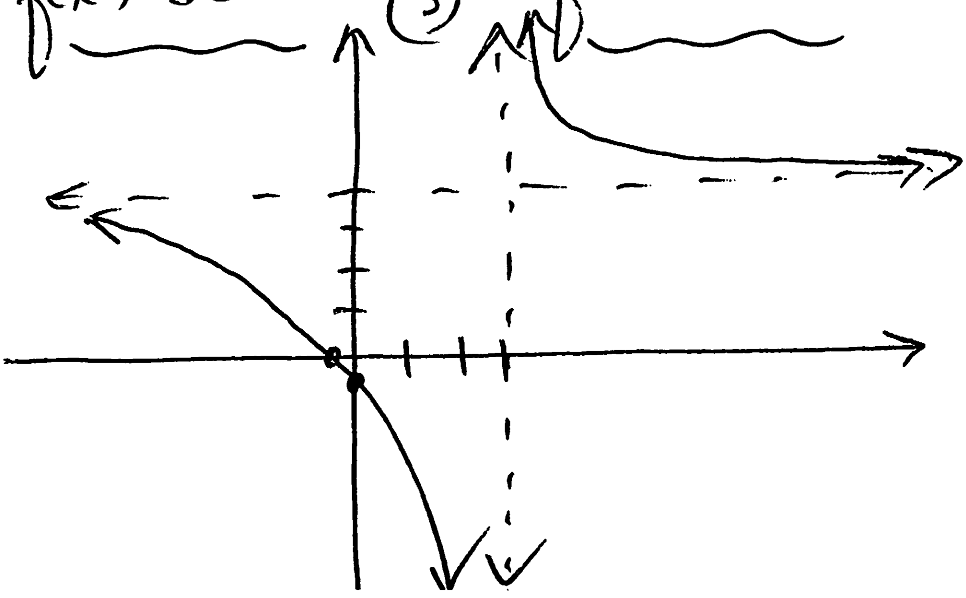
ASYMP:

① VERT: $\boxed{x=3}$

② HORIZ.

$$\lim_{x \to \infty} \frac{4x+1}{x-3} = 4$$

$\boxed{y=4}$



(page 2)

2.) $f(x) = x^3 - 3x + 6$

$f'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1)$

① $f'(x) = 3(x-1)(x+1) = 0$

$x = 1$ & $x = -1$

$f(1) = 1^3 - 3(1) + 6$

$f(1) = 1 - 3 + 6$

$f(1) = 4$

$(1, 4)$ CRIT PT

$f(-1) = (-1)^3 + 3(-1) + 4$

$f(-1) = -1 + 3 + 6$

$f(-1) = 8$

$(-1, 8)$ CRIT. PT

(horizontal tangent lines there)

$\underline{f'(x):}$

$f'(-2) = +$ | $f'(0) = -$ | $f'(2) = +$

$f(x)$ INCR | -1 | $f(x)$ DECR | 1 | $f(x)$ INCR

$f''(x) = 6x$

① $f''(x) = 6x = 0$

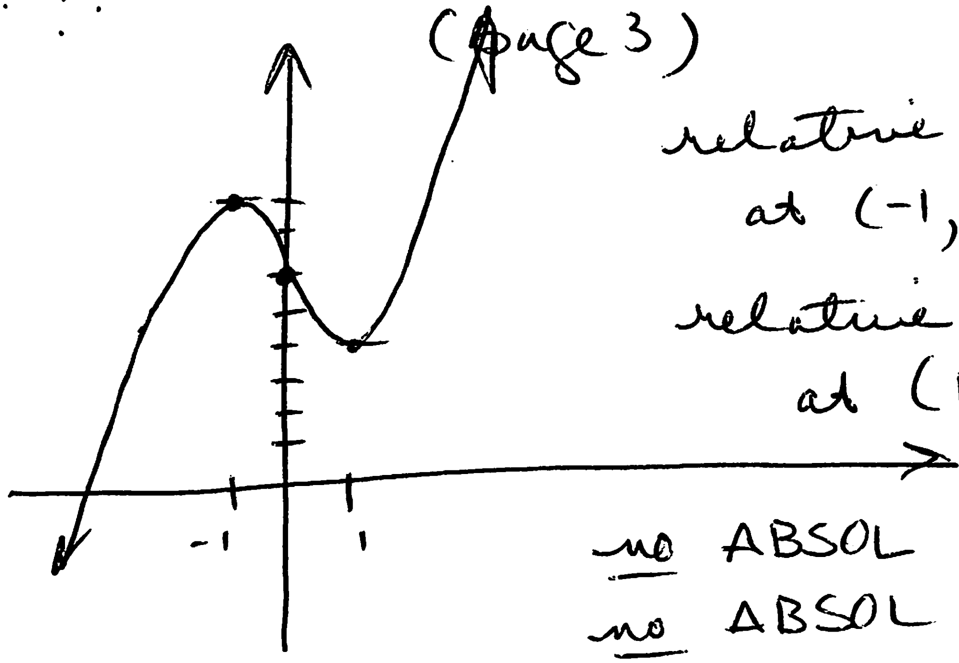
$x = 0$

$f(0) = 6$

$\Rightarrow$ $(0, 6)$ point of inflection

$\underline{f''(x):}$

$f''(-1) = -$ | $f''(1) = +$



relative max at $(-1, 8)$

relative min at $(1, 4)$

no ABSOL MAX

no ABSOL MIN

3.) $s(t) = -16t^2 + 130t + 55$

$s(2) = -16(2)^2 + 130(2) + 55$

$s(2) = -64 + 260 + 55 = 251$ feet

$v(t) = s'(t) = -32t + 130$

$v(2) = -32(2) + 130$

$v(2) = -64 + 130 = 66$ feet/sec

$a(t) = v'(t) = s''(t) = -32$

$a(2) = -32 \dfrac{\text{ft/sec}}{\text{sec}}$ or -32 ft/sec^2

4.) $y = 1 + (x-4)^{2/3}$

$y' = \frac{2}{3}(x-4)^{-1/3}(1) = \dfrac{2}{3\sqrt[3]{x-4}} = y'$

① $y' = 0$ no!

② y' undef when $x = 4$

$f(4) = 1 + (4-4)^{2/3} = 1 + 0 = 1$

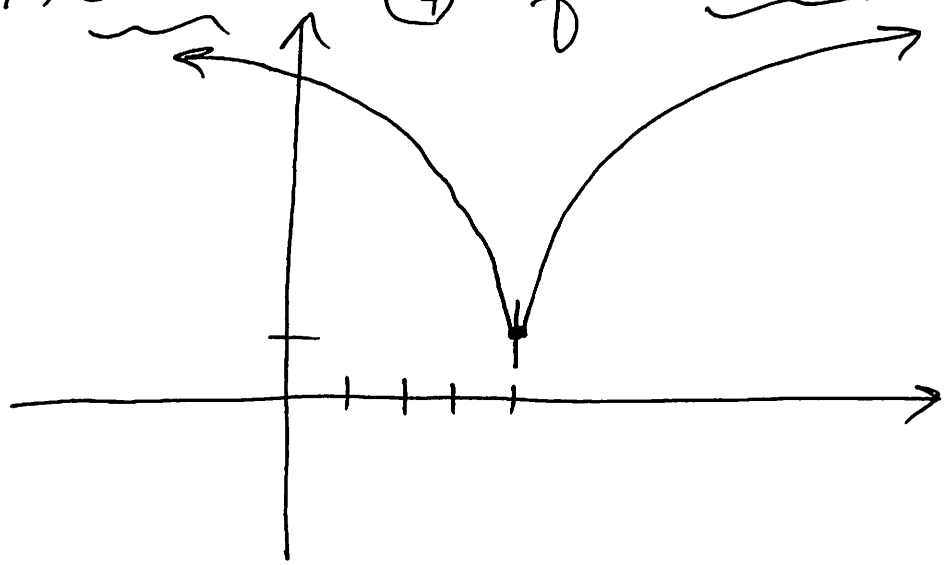
(page 4)

(4, 1) vertical tangent line there

$f'(x)$:

$f'(0) = -$ | 4 | $f'(5) = +$

$f(x)$ DECR | | $f(x)$ INCR



5.) $f(x) = x^3 - x^2 - x + 5$ on $[0, 3]$

endpoints:

$x = 0$: $f(0) = 0^3 - 0^2 - 0 + 5 \Rightarrow (0, 5)$

$x = 3$: $f(3) = 3^3 - 3^2 - 3 + 5 = 27 - 9 - 3 + 5 = 20$

$\Rightarrow (3, 20)$

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x^2 - 2x - 1 = (3x+1)(x-1)$

$\cancel{x = -1/3}$ outside of interval

$x = 1$

$(1, f(1)) \Rightarrow (1, 4)$

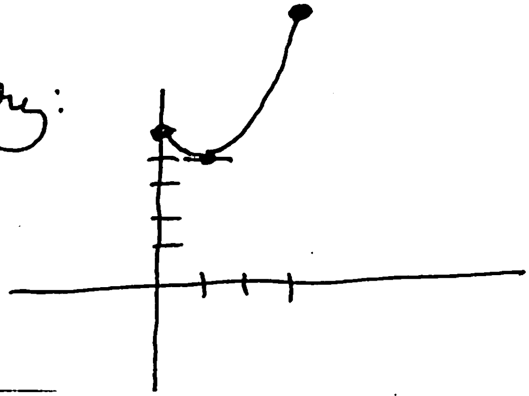
$f(1) = 1^3 - 1^2 - 1 + 5$

$f(1) = 4$

absolute max: 20 at (3, 20)

absolute min: 4 at (1, 4)

graph not necessary:



6.) $y = (3x^2+6x)^{1/2}$ $\quad (1, 3)$

$y' = \frac{1}{2}(3x^2+6x)^{-1/2}\cdot(6x+6)$

$y' = \dfrac{6x+6}{2\cdot\left(\sqrt{3x^2+6x}\right)}$

y' when $x=1$: $\dfrac{6(1)+6}{2\cdot\sqrt{3\cdot1^2+6(1)}} = \dfrac{12}{2\cdot 3} = 2$

eq: $y - 3 = 2(x-1)$

7.) a.) $y = (2x-3)(5x^2-4x+8)$

$y' = (2x-3)(10x-4) + (5x^2-4x+8)(2)$

$y' = 20x^2 - 30x - 8x + 12 + 10x^2 - 8x + 16$

$y' = 30x^2 - 46x + 28$

b.) $y = (3x-4)^7$

$y' = 7(3x-4)^6\cdot(3) = 21(3x-4)^6$

$y'' = 21\left[6(3x-4)^5\cdot 3\right]$

$y'' = 378(3x-4)^5$

BONUS: (5 PTS)

$$f(x) = \frac{(x-3)(x^2-3x-4)}{x^2-2x-8} = \frac{(x-3)\cancel{(x-4)}(x+1)}{\cancel{(x-4)}(x+2)}$$

$$f(x) = \frac{(x-3)(x+1)}{(x+2)} \quad (x \neq 4)$$

"hole" in the graph at $x=4$

$(4, \frac{5}{6})$

$$\frac{(x-3)(x+1)}{(x+2)} = \frac{(4-3)(4+1)}{(4+2)} = \frac{(1)(5)}{(6)} = \frac{5}{6}$$

$$f(x) = \frac{x^2-2x-3}{x+2}$$

1) VERT ASYMP:

$x = -2$

2) HORIZ ASYMP:

$$\lim_{x \to \infty} \frac{x^2-2x-3}{x+2} = \text{D.N.E.}$$

no horiz asymp

3) OBLIQUE (SLANT) ASYMP:

$$\begin{array}{r} x-4+\frac{5}{x+2} \\ x+2 \overline{\big) x^2-2x-3} \\ -(x^2+2x) \\ \hline -4x-3 \\ -(-4x-8) \\ \hline 5 \end{array}$$

$y = x-4$

4) int:

set $x=0$

$(0, -\frac{3}{2})$

$$\frac{0^2-2(0)-3}{0+2}$$

set $y=0$

$$0 = \frac{(x-3)(x+1)}{x+2}$$

$x=3$ or $x=-1$

$(-1, 0)$ & $(3, 0)$