NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-001 Test #3 Form A Tuesday, November 6, 2018 Dr. J. Griggs

Please show all work and answers in the stamped blue book provided; one problem per page. Please put your **name**, **section** (121-001), and **form** (A or B) on the front of the blue book; put your **row letter** and **seat number** on the top right corner of your blue book. Do **not** use a graphing calculator, nor any other calculator that **does** calculus. Simplify completely. (8 questions; 12 points each; 4 points for following directions)

1.) The average cost of a prime-rib dinner was \$15.81 in 1986. In 2010, it was \$27.95. Assuming that the cost is growing exponentially, what will the cost of such a dinner be in 2020?

2.) Find the derivatives: a.) $y = \ln(x^4 + 5)$ b.) $y = e^{x^2} \ln(4x)$

3.) Find the derivatives: a.) $y = x^3 3^x$ b.) $y = 7\log_4(2x+1)$

4.) A metal plate that has been heated cools from 170 degrees to 140 degrees in 20 minutes when surrounded by air at a temperature of 60 degrees. Use Newton's Law of Cooling to approximate its temperature at the end of one hour of cooling. When will the temperature be 90 degrees? $(T = ae^{kt} + M)$

5.) Evaluate the indefinite integrals: a.) $\int (2x^3 - 4\sqrt{x} + \frac{5}{x^3})dx$ b.) $\int (3e^{5x} + \frac{4}{x})dx$

6.) Find the exact area under the curve $y = x^2 + 2x + 1$ from $x = 0$ *to* $x = 3$.

7.) An oilfield contains 8 wells that produce 1600 barrels of oil per day (200 barrels per well). For each additional well that is drilled, the average production per well decreases by 10 barrels per day. How many additional wells should be drilled to obtain the maximum amount of oil per day? What is the maximum amount of oil that could be drilled per day based on these facts? (let x = number of additional wells drilled; solution by calculus techniques only)

8.) Find $f(x)$ such that $f'(x) = 6x^2 - 8x + 3$ and $f(2) = 4$.

Bonus: (5 points) Evaluate $\int_{-2}^{4} f(x)dx$, *where* $f(x) = \begin{cases} x+2, & \text{for } x \le 0 \\ 2 - \frac{1}{2}\sqrt{x}, & \text{for } x > 0 \end{cases}$

MA 121 TEST #3 FORM A

(8 QUESTIONS − 12 PTS EACH)

1.) 1986 $(t=0)$ $y = 15.81$ $(y_0 = 15.81)$

2010 $(t=24)$ $y = 27.95$

2020 $(t=34)$ $y = ??$

$y = y_0 \cdot e^{kt}$

$y = (15.81)\, e^{kt}$

$27.95 = (15.81)\, e^{k(24)}$

$\dfrac{27.95}{15.81} = \dfrac{\cancel{15.81}\, e^{24k}}{\cancel{15.81}}$

$\dfrac{27.95}{15.81} = e^{24k}$

$\dfrac{\ln\left(\frac{27.95}{15.81}\right)}{24} = \dfrac{\cancel{24}k}{\cancel{24}} \approx .02374$

$y = (15.81)\, e^{(.02374)(34)}$

$y = \$35.44$

price of prime rib in 2020

2.) a.) $y = \ln(x^4+5)$ $y' = \dfrac{1}{x^4+5} \cdot (4x^3) = \dfrac{4x^3}{x^4+5}$

b.) $y = e^{x^2} \cdot \ln(4x)$ $y' = e^{x^2} \cdot \left(\frac{1}{4x} \cdot 4\right) + [\ln 4x] \cdot e^{x^2}(2x)$

$y' = \dfrac{e^{x^2}}{x} + 2x \cdot e^{x^2} \cdot \ln 4x$

3.) a.) $y = x^3 \cdot 3^x$ $y' = x^3(3^x \cdot \ln 3) + 3^x \cdot (3x^2)$

$y' = x^2 \cdot 3^x\, [x \cdot \ln 3 + 3]$

b.) $y = 7 \log_4(2x+1)$ $y' = 7 \cdot \dfrac{1}{(2x+1)\cdot \ln 4} \cdot (2)$

$y' = \dfrac{14}{(2x+1)\cdot \ln 4}$

4.) $T = a \cdot e^{kt} + m$ $\quad m = 60$

2 PTS

① $t = 0$ $T = 170$

② $t = 20$ $T = 140$

③ $t = 60$ $T = ??$

④ $t = ??$ $T = 90$

$t = 0$ $T = 170$

$170 = a \cdot e^{k(0)} + 60$ (solve for a)

$170 = a + 60$

$170 - 60 = a = 110$

$T = 110e^{kt} + 60$

$t = 20$ $T = 140$

$140 = 110e^{k(20)} + 60$ (solve for k)

$\frac{140 - 60}{110} = e^{20k}$

$\ln\left(\frac{140-60}{110}\right) = 20k$

$\frac{\ln\left(\frac{140-60}{110}\right)}{20} = k \approx -.01592$

$T = 110e^{-.01592t} + 60$

$t = 60$ $T = ??$

$T = 110e^{-.01592(60)} + 60$

$T = 102.32°$

$t = ??$ $\quad T = 90$

$$90 = 110 \cdot e^{-.01592t} + 60$$

$$\frac{90 - 60}{110} = e^{-.01592t}$$

$$\frac{\ln\left(\frac{90-60}{110}\right)}{-.01592} = \frac{\cancel{-.01592}\,t}{\cancel{-.01592}} \approx 81.6 \text{ min}$$

5.) a.) $\int \left(2x^3 - 4\sqrt{x} + \frac{5}{x^3}\right) dx$

$$= \int \left(2x^3 - 4x^{1/2} + 5x^{-3}\right) dx$$

$$= 2\left(\frac{x^4}{4}\right) - 4\left(\frac{x^{3/2}}{3/2}\right) + 5\left(\frac{x^{-2}}{-2}\right) + C$$

$$= \frac{1}{2}x^4 - \frac{8}{3}x^{3/2} - \frac{5}{2}x^{-2} + C$$

b.) $\int \left(3 \cdot e^{5x} + \frac{4}{x}\right) dx = 3\int e^{5x} dx + 4\int \frac{1}{x} dx$

$$= 3 \cdot \left(\frac{1}{5}e^{5x}\right) + 4 \cdot \ln|x| + C$$

$$= \frac{3}{5}e^{5x} + 4 \cdot \ln|x| + C$$

6.) $\int_0^3 (x^2 + 2x + 1) dx = \left. \frac{x^3}{3} + \cancel{2}\left(\frac{x^2}{\cancel{2}}\right) + x \right]_0^3$

$$= \left[\frac{(3)^3}{3} + (3)^2 + 3\right] - \left[\frac{(0)^3}{3} + (0)^2 + (0)\right] = \frac{27}{3} + 9 + 3 - 0$$

7.) let x = number of additional wells drilled

$$OIL = \underbrace{(8 + 1 \cdot x)}_{\text{\# of wells}}\underbrace{(200 - 10 \cdot x)}_{\text{barrels of oil per well}}$$

$$O(x) = (8+x)(200-10x) = 1600 + 200x - 80x - 10x^2$$

$$O(x) = 1600 + 120x - 10x^2$$

$$O'(x) = 120 - 20x = 0$$

$$120 = 20x$$

$$6 = x \quad \text{(additional wells)}$$

$O''(x) = -20$ ∴ concave down

∴ max

wells $(8+6) = \boxed{14 \text{ wells}}$

total barrels of oil =

$$(8+6)\ (200 - 10 \cdot 6)$$

$$(14)\ (140) = \boxed{1960 \text{ barrels}}$$

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8.) $f'(x) = 6x^2 - 8x + 3$ $\quad f(2) = 4$

$$f(x) = \int (6x^2 - 8x + 3)\,dx$$

$$f(x) = 6\cdot\frac{x^3}{3} - 8\cdot\frac{x^2}{2} + 3x + C$$

$$f(x) = 2x^3 - 4x^2 + 3x + C \quad \text{(find C)}$$

$x = 2$
$y = 4$

$$4 = 2(2)^3 - 4(2)^2 + 3(2) + C$$

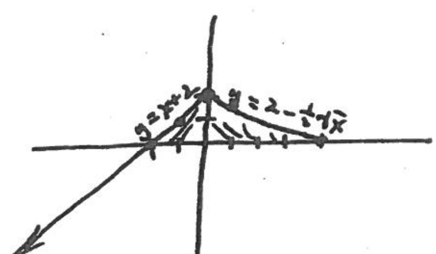
$$4 = 16 - 16 + 6 + C$$

$$4 = 6 + C \qquad C = -2$$

$$f(x) = 2x^3 - 4x^2 + 3x - 2$$

BONUS: (5 pts)

$$f(x) = \begin{cases} x+2, & x \le 0 \\ 2 - \frac{1}{2}\sqrt{x}, & x > 0 \end{cases}$$



$y = x+2$

x	y
0	2
-1	1
-2	0

$y = 2 - \frac{1}{2}\sqrt{x}$

x	y
0	2
1	3/2
4	0

$$\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$= \int_{-2}^{0} (x+2)\,dx + \int_{0}^{4} \left(2 - \tfrac{1}{2}\sqrt{x}\right)dx$$

$$\left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[2x - \frac{1}{2}\,\frac{x^{3/2}}{3/2}\right]_{0}^{4} = \left[\left(\cancel{\frac{0^2}{2} + 2(0)}\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)\right]$$

$$+ \left[\left(2(4) - \tfrac{1}{3}(4)^{3/2}\right) - \left(\cancel{2(0) - \tfrac{1}{3}(0)^{3/2}}\right)\right] = -(2-4) + \left(8 - \tfrac{8}{3}\right) = 10 - \tfrac{8}{3} = \frac{22}{3}$$