MA 121-002

Thursday, August 30

$f(x) = 3x^2 - 5x + 11$ domain: $\mathbb{R}$

$f(1) = 3(1)^2 - 5(1) + 11$

$f(1) = 3 - 5 + 11 = 9$

$(1, 9)$

$f(a) = 3 \cdot a^2 - 5 \cdot a + 11$

$(a, 3a^2 - 5a + 11)$

$f(x+h) = 3(x+h)^2 - 5(x+h) + 11$

$= 3(x^2 + 2xh + h^2) - 5x - 5h + 11$

$= 3x^2 + 6xh + 3h^2 - 5x - 5h + 11$

$(x+h, 3x^2 + 6xh + 3h^2 - 5x - 5h + 11)$

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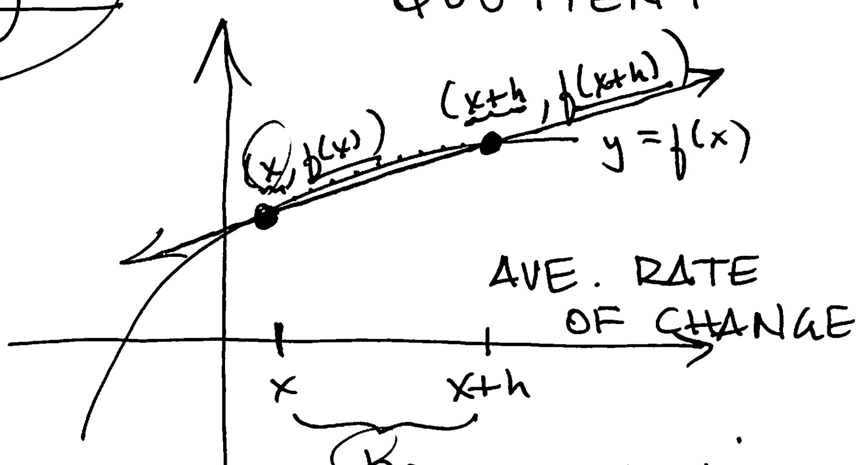
$$m = \frac{f(x+h) - f(x)}{h}$$

DIFFERENCE QUOTIENT

$$m = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$




AVE. RATE OF CHANGE

slope of secant line

$$\frac{f(x+h) - f(x)}{h}$$

$f(x) = 3x^2 - 5x + 11$

$$\frac{(3x^2 + 6xh + 3h^2 - 5x - 5h + 11) - (3x^2 - 5x + 11)}{h}$$

$$\frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 5x} - 5h \cancel{+ 11} \cancel{- 3x^2} \cancel{+ 5x} \cancel{- 11}}{h}$$

$$= \frac{\cancel{h}(6x + 3h - 5)}{\cancel{h}} \quad (h \neq 0)$$

$$= \boxed{6x + 3h - 5} = m$$

initial x-value

change in x

"split-domain" function:

(piecewise)

$$f(x) = \begin{cases} 2x-1, & x \leq -1 \\ x^2+4, & -1 < x \leq 2 \\ 3, & x > 2 \end{cases}$$

$y = 2x-1$
$(x \leq -1)$

| $x$ | $y$ |
|---|---|
| $-1$ | $-3$ |
| $-2$ | $-5$ |
| $-3$ | $-7$ |
| $\vdots$ | $\vdots$ |

$y = x^2+4$
$(-1 < x \leq 2)$

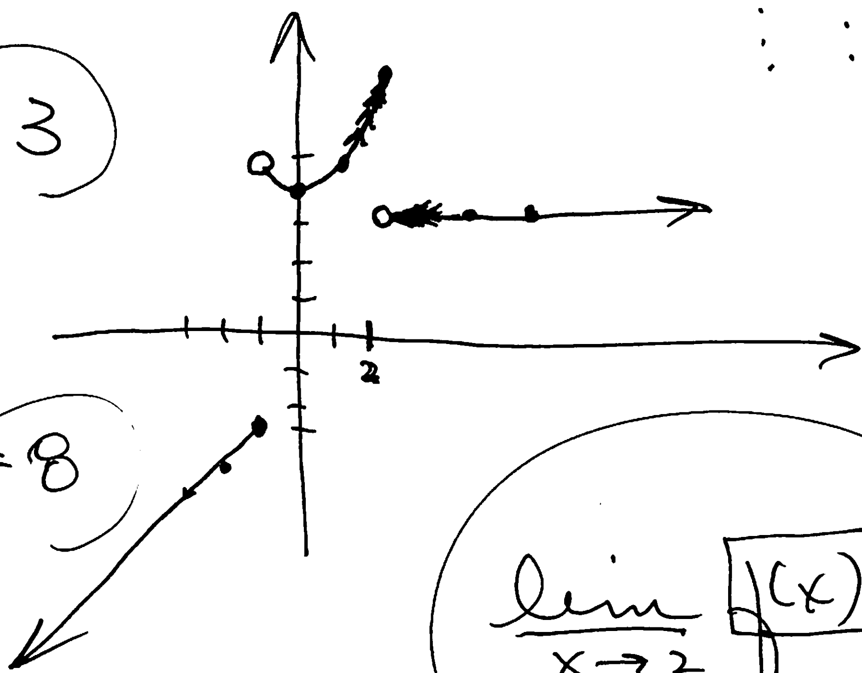
| $x$ | $y$ |
|---|---|
| $-1$ | $5$ DELETE |
| $0$ | $4$ |
| $1$ | $5$ |
| $2$ | $8$ |

$y = 3$
$(x > 2)$

| $x$ | $y$ |
|---|---|
| $2$ | $3$ DELETE |
| $3$ | $3$ |
| $4$ | $3$ |
| $\vdots$ | $\vdots$ |

$\lim\limits_{\boxed{x \to 2^+}} f(x) = 3$

"from the right"




$\lim\limits_{x \to 2^-} f(x) = 8$

"from the left"

$\lim\limits_{x \to 2} \boxed{f(x)} =$ D.N.E.
no limit

$x$ "approaches" 2

$$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{4}$$

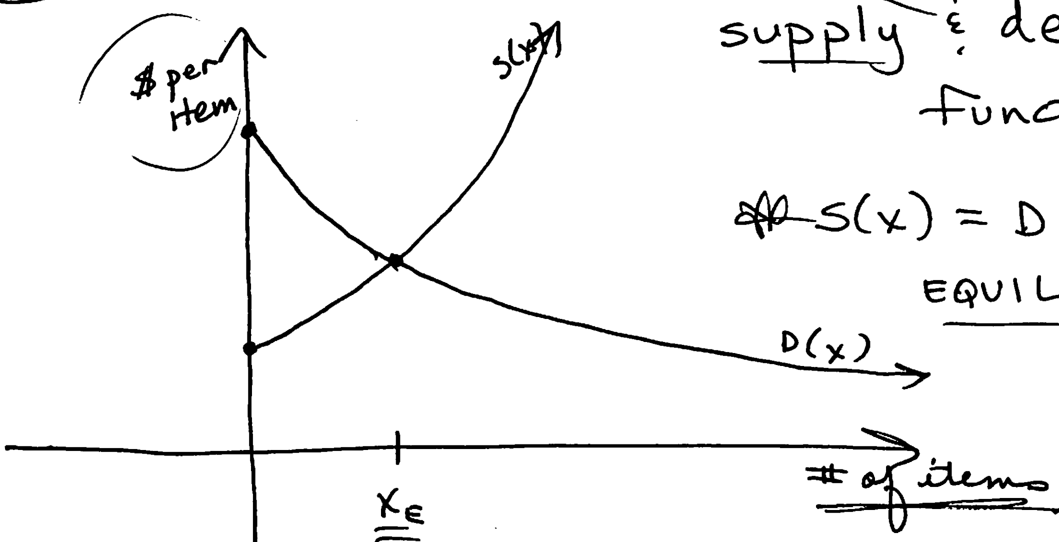
square

cube root

?

$$\frac{1}{\sqrt[3]{[8^2]}}$$

supply & demand functions




$\cancel{\phantom{a}}\ S(x) = D(x)$

EQUIL. PT.

$(x_E,\ D(x_E) \text{ or } S(x_E))$

restricted domains:

$f(x) = \sqrt{x-5}$

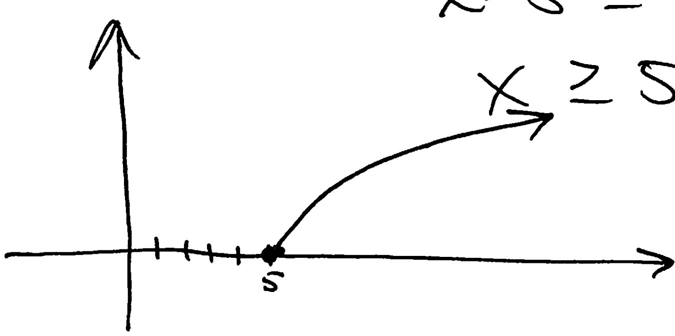
$\sqrt{-1} = i$

imagin.

$x - 5 \geq 0$

$x \geq 5$

$[5, \infty)$




range: $[0, \infty)$

$$f(x) = \frac{4}{x-3} \begin{array}{l} \leftarrow \text{polynomial} \\ \leftarrow \text{polynomial} \end{array}$$

domain: $x \neq 3$
$(x \in \mathbb{R})$

$(-\infty, 3) \cup (3, \infty)$

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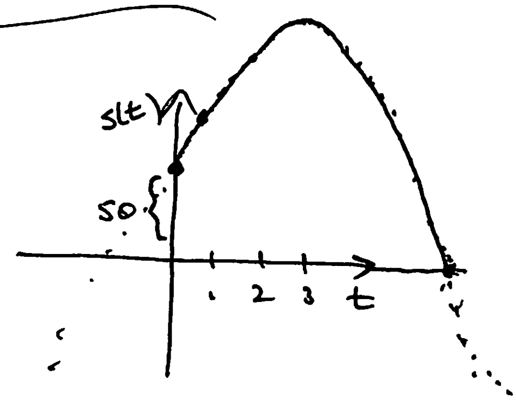
real world prob:

1.) $t \geq 0$

2.) $v(t): +, 0, -$

3.) $a(t): +, -, 0$

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$f(x) = 3x^2 - 4x - 2$ parabola:

vertex $\left(\frac{-b}{2a}, \underline{\underline{f\left(\frac{-b}{2a}\right)}}\right)$

$f(x) = \underline{a}x^2 + \underline{b}x + c$
$(a \neq 0)$

$V\left(2/3, -10/3\right)$

$\frac{-b}{2a} = \frac{-(-4)}{2(3)} = \frac{4}{6}$

$= \frac{2}{3}$

$f(2/3) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 2$

$= \cancel{3} \cdot \frac{4}{\cancel{9}\,3} - \frac{8}{3} - \frac{6}{3}$

$= \frac{4}{3} - \frac{8}{3} - \frac{6}{3} = \frac{-10}{3}$

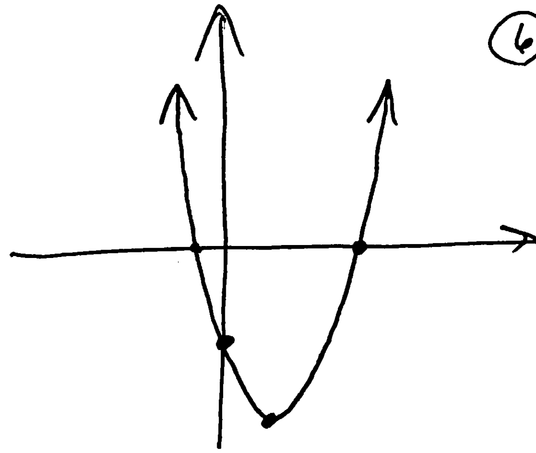
find vertex:

find ALL intercepts:

GRAPH:

⑥

$f(x) = 3x^2 - 4x - 2$



INTERCEPTS:

① x-int: (set $y=0$)

$0 = 3x^2 - 4x - 2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(1.72, 0)$

$(-.39, 0)$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$

$x = \dfrac{4 \pm \sqrt{16 + 24}}{6} = \dfrac{4 \pm \sqrt{40}}{6}$

$x = \dfrac{4 \pm \sqrt{4 \cdot 10}}{6} = \dfrac{4 \pm 2\sqrt{10}}{6}$

$x = \dfrac{2 \pm \sqrt{10}}{3}$

$x_1 = \dfrac{2 + \sqrt{10}}{3} \approx \underline{1.72}$

$x_2 = \dfrac{2 - \sqrt{10}}{3} \approx \underline{-.39}$

② y-int: (set $x = 0$)

$f(0) = 3(\cancel{0})^2 - 4(\cancel{0}) - 2 = -2$

$(0, -2)$