MA121 – 002

Thursday, September 20

1.7: chain rule

1.8: higher order deriv.

1.7: CHAIN RULE

why do we need this rule?

power rule:

$f(x) = 5x^3 + \ldots$

$f'(x) = 5(3 \cdot x^2) + \ldots$

$g(x) = (3x+2)^5$

$\cancel{g(x) = (3x+2)(3x+2)(3x+2)(3x+2) \cdot (3x+2)}$

$\cancel{g'(x) =}$

① EXTENDED POWER RULE:

$y = [f(x)]^n$

$y' = n \cdot [f(x)]^{n-1} \cdot f'(x)$

ex: $g(x) = (3x+2)^5$

$g'(x) = 5 \cdot (3x+2)^4 \cdot 3$

$g'(x) = 15(3x+2)^4$ $\qquad g'(1) =$

(2) COMPOSITION OF FUNCTIONS:

$$y = (f \circ g)(x) = f(g(x))$$

$$y = (3x+2)^5$$

$$y' = \underbrace{f'(g(x))}_{\text{DERIV OF "OUTSIDE" FUNCTION}} \cdot \underbrace{g'(x)}_{\text{DERIV OF "INSIDE" FUNCTION}}$$

$$g(x) = 3x+2$$

$$f(x) = x^5$$

$$y = (f \circ g)(x) = f(g(x)) = f(3x+2) = (3x+2)^5$$

$$f'(x) = y' = \frac{dy}{dx} \quad y' = 5 \cdot (3x+2)^4 \cdot 3 = 15(3x+2)^4$$

(3) y in terms of u

$$y = 8 + u^3 \qquad \boxed{\frac{dy}{du} = 0 + 3u^2}$$

u in terms of x

$$u = x^2 + 5 \qquad \boxed{\frac{du}{dx} = 2x + 0}$$

$$\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$$

$$(3u^2)(2x) = \frac{dy}{dx}$$

$$3(x^2+5)^2 \cdot 2x = \frac{dy}{dx} = 6x \cdot (x^2+5)^2$$

$$\left.\begin{aligned} y &= 8 + u^3 \\ u &= \boxed{x^2+5} \end{aligned}\right\} \quad \frac{dy}{du}\cdot\frac{du}{dx} =$$

y in terms of x (SUBST....)

$$y = 8 + (x^2+5)^3$$

$$y' = \frac{dy}{dx} = \cancel{0} + 3(x^2+5)^2 \cdot 2x$$

$$\frac{dy}{dx} = 6x(x^2+5)^2$$

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chain....?

$$\boxed{\frac{dy}{du}\cdot\frac{du}{dr}\cdot\frac{dr}{dw}\cdot\frac{dw}{dx} \stackrel{?}{=} \frac{dy}{dx}}$$

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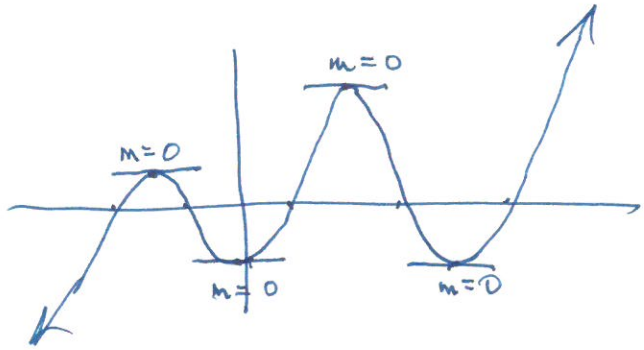
ex:

$$y = \boxed{(2x-1)^4} \cdot \boxed{(x^2+x+1)^7}$$

~~product~~ rule ....

$$y' = (2x-1)^4 \cdot \boxed{7(x^2+x+1)^6 \cdot (2x+1)}$$
$$+ (x^2+x+1)^7 \cdot \boxed{4(2x-1)^3 \cdot 2}$$

$$y' = (2x-1)^3(x^2+x+1)^6\Big[\overbrace{(2x-1)\cdot 7(2x+1)}^{(4x^2-1)}$$
$$+ 8(x^2+x+1)\Big]$$



$$y' = (2x-1)^3(x^2+x+1)^6\left[28x^2 - 7 + 8x^2 + 8x + 8\right]$$

$$y' = (2x-1)^3(x^2+x+1)^6\left[36x^2+8x+1\right]$$

$$0 = (2x-1)^3\,(x^2+x+1)^6\,(36x^2+8x+1)$$

$(2x-1)^3 = 0$ $\quad x^2+x+1=0$ $\quad 36x^2+8x+1=0$

$(2x-1) = 0$

$$h(x) = \sqrt[3]{\frac{x-3}{x+4}} = \left(\frac{x-3}{x+4}\right)^{1/3} \qquad \left(\quad\right)^{1/3}$$

$$h'(x) = \frac{1}{3}\left(\frac{x-3}{x+4}\right)^{-2/3}\left[\underbrace{\frac{(x+4)(1)-(x-3)(1)}{(x+4)^2}}_{\text{DERIV OF "INSIDE" FUNCTION}}\right]$$

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find the equation of the tangent line to $f(x)=\sqrt{x^2+3x}$ at $(1, 2)$

$$y - y_1 = m(x - x_1) \qquad f'(x) = m_{TAN}$$

$$y - 2 = \boxed{}(x-1)$$

$$f(x) = (x^2 + 3x)^{1/2}$$

$$f'(x) = m_{TAN} = \frac{1}{2}(x^2+3x)^{-1/2} \cdot (2x+3)$$

$$f'(1) = \frac{(2x+3)}{2 \cdot \sqrt{x^2+3x}} = \frac{2(1)+3}{2 \cdot \sqrt{1^2+3(1)}}$$

$$f'(1) = \frac{5}{2 \cdot 2} = \frac{5}{4}$$

$$y - 2 = \boxed{}(x-1)$$

$$\boxed{y - 2 = \frac{5}{4}(x-1)}$$

1.8: HIGHER ORDER DERIVATIVES

$$f(x) = 3x^2 - 8x + 5$$

$$f'(x) = 3(2x) - 8(1) + 0$$

$$f'(x) = 6x - 8$$

$$f''(x) = 6$$

$$f'''(x) = 0$$

$f^{(4)}(x)$

4^{th} DERIV

$f^{(7)}(x)$ 7^{th} DER.

$$\frac{dy}{dx}\ ;\ \frac{d^2y}{dx^2}\ ;\ \frac{d^3y}{dx^3}\ ;\ \ldots$$

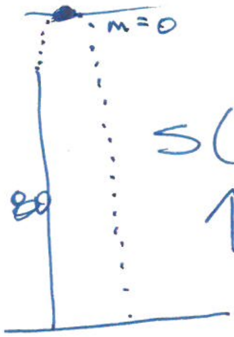
$$y',\ y'',\ y''',\ \ldots$$

$$f(x) = \overbrace{(10x^2 - x + 5)}^{7}$$

$$f'(x) = 7(10x^2 - x + 5)^6 \cdot (20x - 1)$$

$$f'(x) = \underbrace{7(20x-1)} \cdot \underbrace{(10x^2 - x + 5)^6}$$

$f''(x) =$ prod. rule . . .



$$s(t) = -16t^2 + 64t + 80$$ ✓

free falling object

$s(t)$: height (ft.)

(dist; position)

t : time (sec.)

$t=0$:

$$s(0) = -16(\cancel{0})^2 + 64(\cancel{0}) + 80$$

pos. (HT) at $t=0$:

$$s(0) = 80 \text{ ft}$$

$$s(1) = -16(1)^2 + 64(1) + 80$$

$$s(1) = 128 \text{ ft}.$$

$s(2)$: . . .

$$s'(t) = \boxed{-32t + 64 = v(t)}$$

$$v(0) = -32(0) + 64 = 64 \frac{\text{ft}}{\text{sec}}$$

$$v(1) = -32(1) + 64 = 32 \text{ ft/sec}$$

$$v(2) = -32(2) + 64 = 0 \text{ ft/sec}$$

$$s''(t) = v'(t) = a(t)$$

$$v'(t) = a(t) = -32 \frac{\text{ft/sec}}{\text{sec}}$$