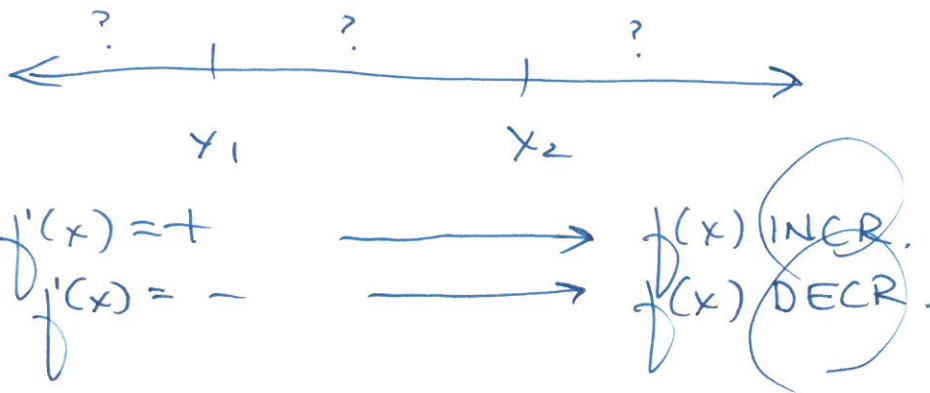## 2.2: USING $f''(x)$:

$f'(x)$:

① find $f'(x)$

② (a) $f'(x) = 0$ or $f'(x)$ undef.   (critical values)

"FLAT"
(horizontal tangent line)

"STEEP"
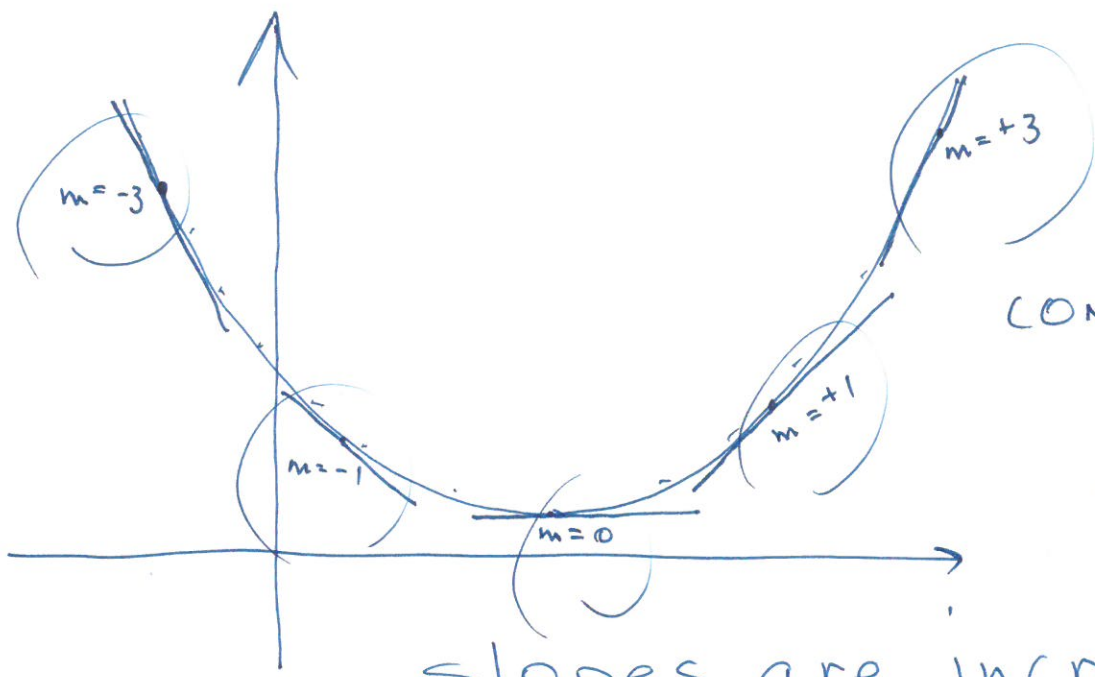(vertical tangent line)

③ $f'(x)$ chart:

?   ?   ?
⟵————|————————|—————⟶
      x_1 x_2

$f'(x) = +$ ⟶ $f(x)$ INCR.
$f'(x) = -$ ⟶ $f(x)$ DECR.

$f''(x)$:

① $f''(x)$; find it

② @ $f''(x) = 0$ $f''(x)$ undef.

③ $f''(x)$ chart:

$m = -3$

$m = +3$

$m = +1$

$m = -1$

$m = 0$

(C. UP)

CONCAVE
UP

slopes are increasing
(from left to right)

$f'(x) = m_{TAN}$

$\begin{cases} f''(x) = + \\ f''(x) = - \end{cases}$

CONCAVE UP

$f'(x)$ INCR

$f'(x)$ DECR
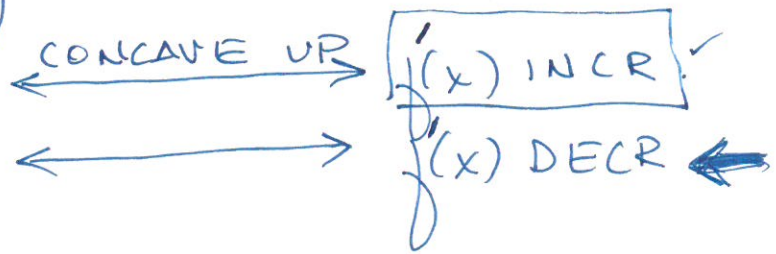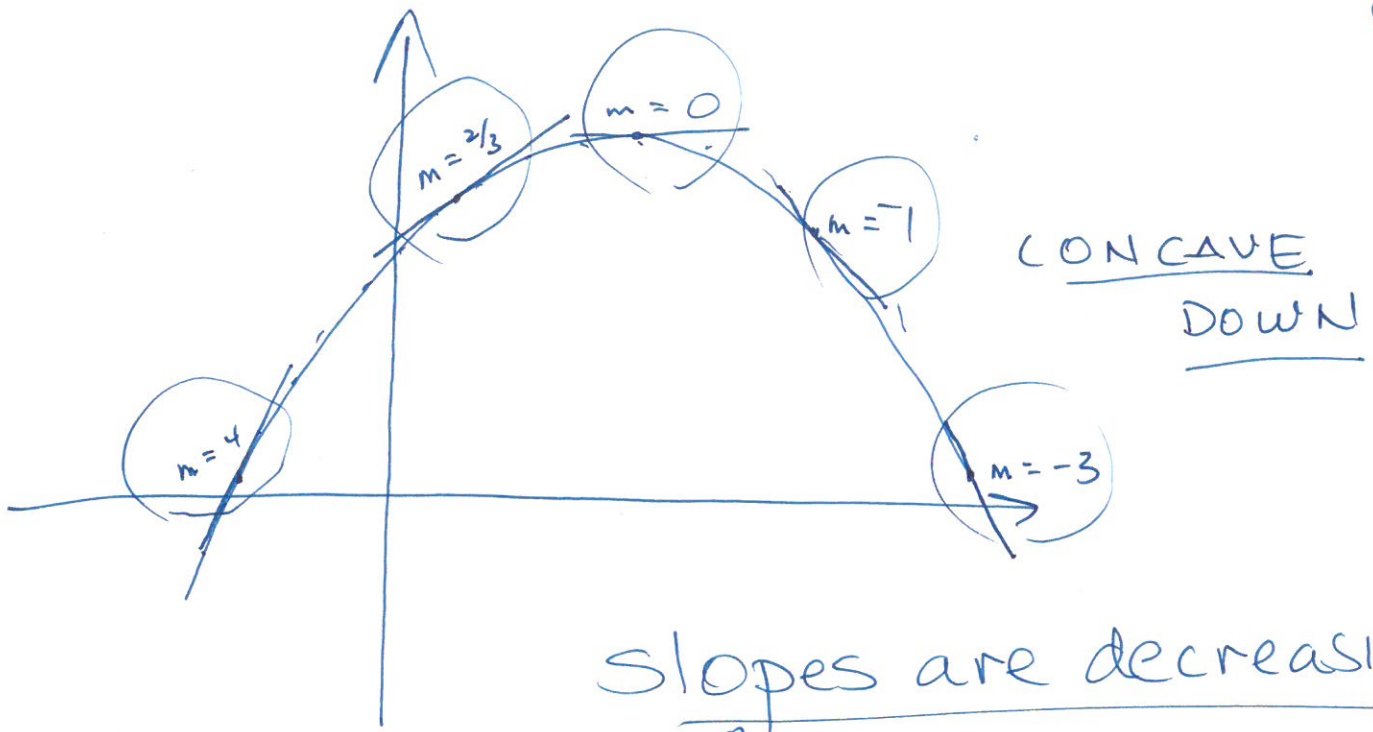
CONCAVE
DOWN

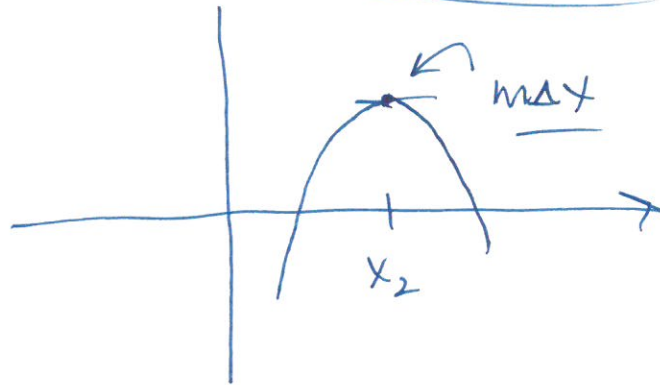slopes are decreasing
$f'(x)$

$f''(x) = +$ $\longleftrightarrow$ $f'(x)$ INCR

$f''(x) = -$ $\longleftrightarrow$ $f'(x)$ DECR

then $f(x)$ is CONCAVE DOWN

$$\left\{ \begin{array}{l} f'(x_1) = 0 \\[4pt] \underline{f''(x_1) = +} \end{array} \right\}$$

$\underbrace{}$ concave up

MIN

x_1

$$\left\{ \begin{array}{l} f'(x_2) = 0 \\[4pt] f''(x_2) = - \end{array} \right.$$

$\underbrace{}$ concave down

MAX

x_2
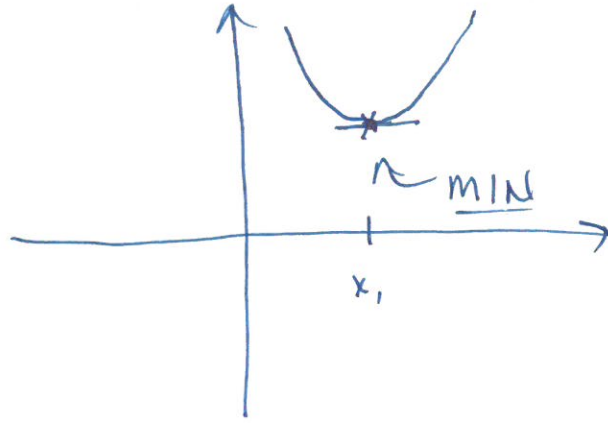
$$f(x) = 2x^3 - 3x^2 - 36x + 28$$

$$f'(x) = 6x^2 - 6x - 36$$

ⓐ $f'(x) = 0$ ⓑ $f'(x)$ ~~undef.~~

$$6x^2 - 6x - 36 = 0$$

$$6(x^2 - x - 6) = 0$$

$$\boxed{6(x-3)(x+2) = 0}$$

$x - 3 = 0$ $x + 2 = 0$

$x = 3$ $x = -2$ $(-2, 72)$

$(3, -53)$

$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 28$ $f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 28$

$f(3) = 54 - 27 - 108 + 28$ $f(-2) = -16 - 12 + 72 + 28 = 72$

$f(3) = -53$

$f'(x):$

$$f'(x) = 6(x-3)(x+2)$$

$f'(-3) = +\cdot - \cdot - \quad\quad f'(0) = +\cdot - \cdot + \quad\quad f'(4) = +++ = +$

$f(x)$ INCR $\quad\quad f(x)$ DECR $\quad\quad f(x)$ INCR

$\boxed{x=-2}$ $\quad\quad$ $\boxed{x=3}$

$(\frac{1}{2}, 9\frac{1}{2})$

−2 $\quad$ 3
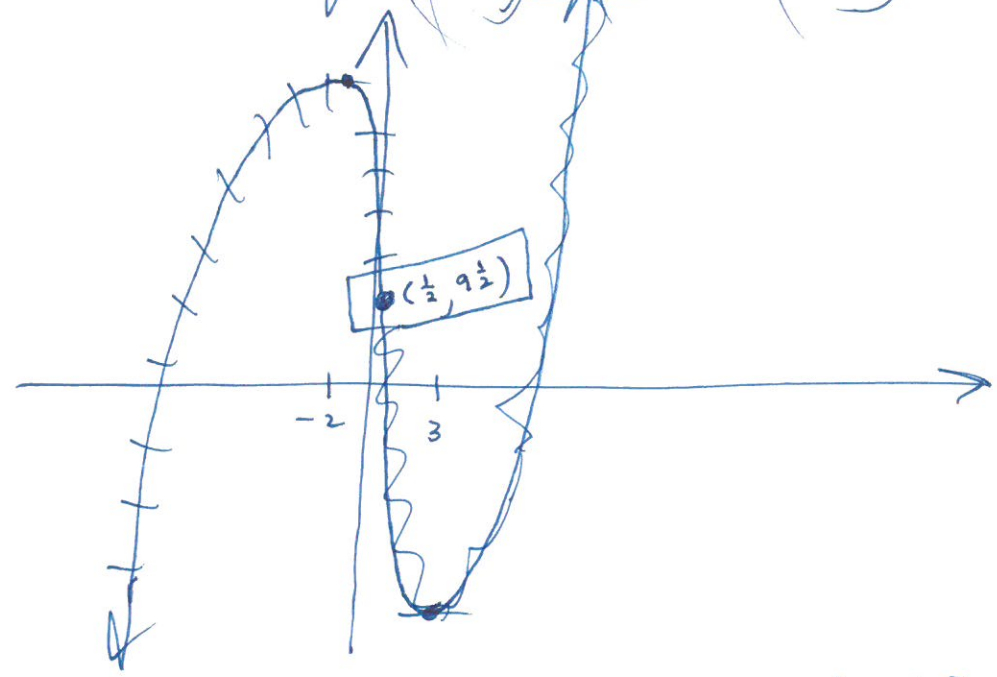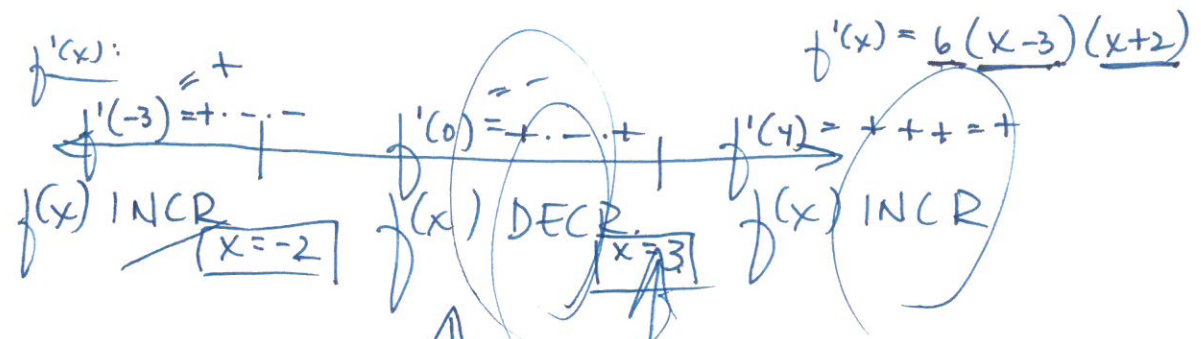
$$f'(x) = 6x^2 - 6x - 36$$

$$\boxed{f''(x) = 12x - 6}$$

① $f''(x) = 0$ $\quad\quad$ $f''(x)$ ~~undef.~~

$12x - 6 = 0$

$12x = 6$

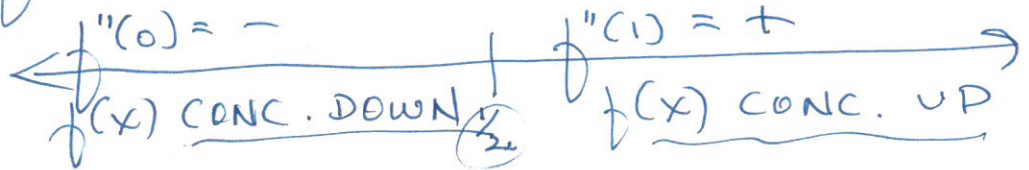$x = \frac{1}{2}$ $\quad\longrightarrow\quad$ $\left(\frac{1}{2}, 9\frac{1}{2}\right)$ $\quad$ changed CONCAV.
point of inflection

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 36(\frac{1}{2}) + 28$

$f(\frac{1}{2}) = \frac{1}{4} - \frac{3}{4} - 18 + 28 = 9\frac{1}{2}$

$f''(x):$

$f''(0) = -$ $\quad\quad\quad\quad$ $f''(1) = +$

$f(x)$ CONC. DOWN $\frac{1}{2}$ $\quad$ $f(x)$ CONC. UP

non - polynomial

$$f(x) = (1 + x)^{2/3}$$

$f'(x):$

$$f'(x) = \frac{2}{3}(1+x)^{-1/3} \cdot (1) = \boxed{\frac{2 \cdot}{3 \sqrt[3]{1+x}}} = f'(x)$$

ⓐ $f'(x) = 0$ ~~crossed out~~
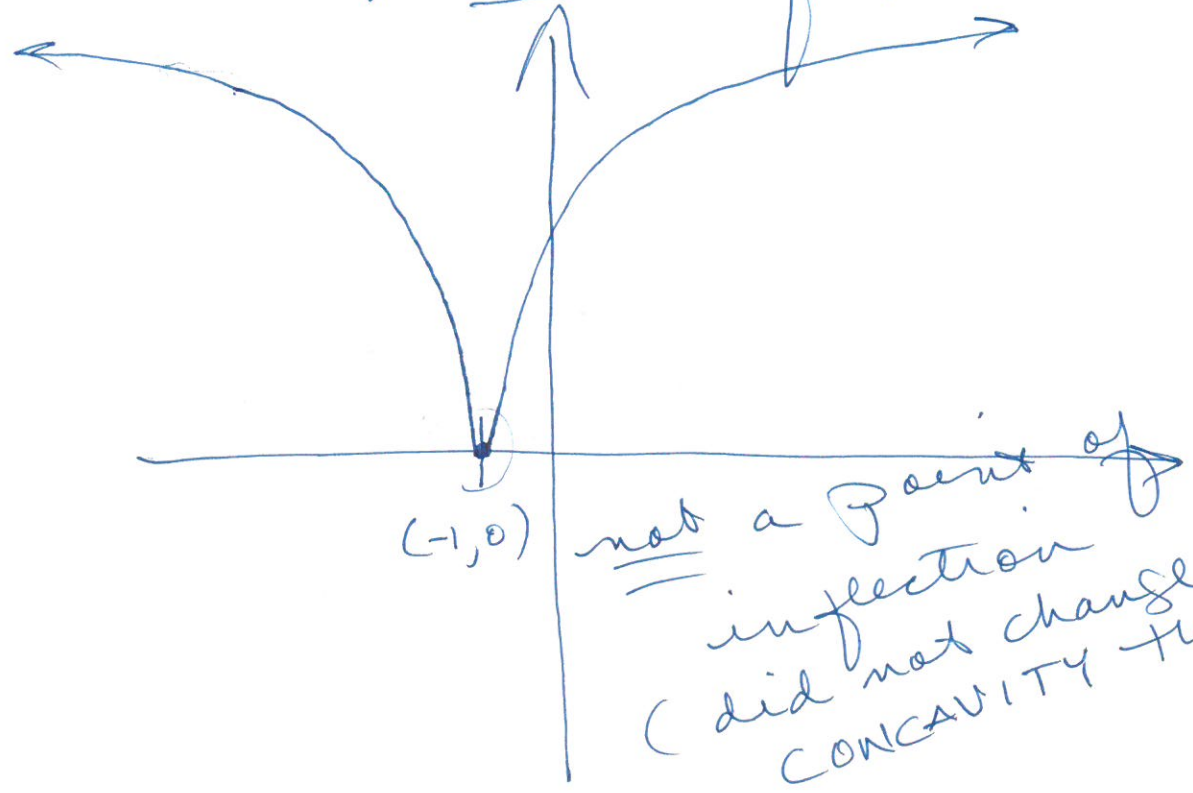
no flat places

ⓑ $f'(x)$ undef.

$$\frac{2}{3 \cdot \sqrt[3]{1+x}} \quad \text{undef}$$

when $x = -1$ VERT TANG.

$$(-1, f(-1)) = (-1, 0)$$

$$f(-1) = (1 + -1)^{2/3}$$

$$f(-1) = 0$$

$f'(x):$

$$f'(-2) = \frac{+}{+ \cdot -} = -$$

$f(x)$ DECR $x = -1$

$$f'(0) = \frac{+}{+ \cdot +} = +$$

$f(x)$ INCR

$(-1, 0)$ not a point of inflection (did not change CONCAVITY there)

$$f(x) = (1+x)^{2/3}$$

$$f'(x) = \frac{2}{3}(1+x)^{-1/3}$$

$$f''(x) = \frac{2}{3} \cdot \left[-\frac{1}{3} \cdot (1+x)^{-4/3} \cdot (1) \right]$$

$$f''(x) = \boxed{\frac{-2}{9 \cdot \left[\sqrt[3]{1+x} \right]^4}}$$
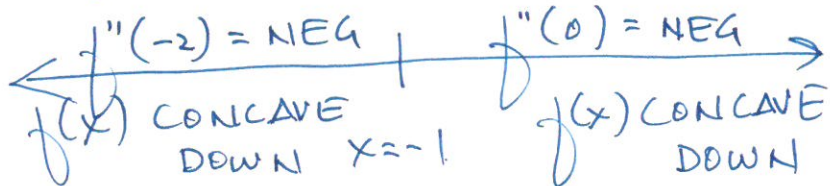
$$\frac{-}{+ \cdot +}$$

$$\cancel{f''(x) = 0}$$

$$f''(x) \text{ undef.}$$

$$\frac{-2}{9 \cdot \left[\sqrt[3]{1+x} \right]^4}$$

when $x = -1$

$(-1, f(-1))$

$= (-1, 0)$

$f''(x)$:

$f''(-2) = \text{NEG}$ $f''(0) = \text{NEG}$

$f(x)$ CONCAVE DOWN $x = -1$ $f(x)$ CONCAVE DOWN

Quiz #1: Interview; documented with a **1 -2 page paper**

1.) paper should be typed (word processed), not handwritten
2.) Q & A format is OK; paragraph format is OK
3.) Interview a person (possibly a parent; not a student) who is currently doing for a living what you would eventually like to do for a living (based on your current major – or desired major)
4.) Face-to-face OK; Phone or skype OK; email OK
5.) One question you **MUST** ask: How is math used in this career?
6.) Other suggested questions you will probably want to ask: Average salary? Stress level? Is it seasonal (lots of work at certain times of the year) or fairly steady? Work/Life balance? Lifestyle/perks? Do you find your job rewarding? Is there continuing education associated with this job? Travel associated with this job? Do you have any summer jobs or internships for college students? Did your degree adequately prepare you for this job?   And many, many other questions specific to your interviewee....
7.) DUE DATE: Tuesday, October 9, at the beginning of class

TA Office Hours: (121-002)

A. Mendez

    Fridays 10:30 – 11:30

    SAS 3201

P. Misra

    Wednesdays 3:00-4:00

    SAS 4125

J. Wang

    Mondays 3:00 – 4:00

    SAS 4117