MA121-002

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Tuesday, October 23

$\begin{cases} 10/23: & \underline{3.3}; \underline{3.4} \\ 10/25: & 3.5; \underline{4.1} \end{cases}$

$\begin{cases} 10/30: & 4.2; \underline{4.3} \\ 11/1: & \underline{4.3}; \text{review} \end{cases}$

11/6: TEST #3

check:

$$e^{.08t} = 143 \qquad\qquad e^{.08(62.04)} \approx 143$$

(take the ln of both sides)

$$\ln_e \boxed{e^{.08t}} = \ln 143$$

$$\frac{\cancel{.08}t}{\cancel{.08}} = \frac{\ln 143}{.08} \approx \underline{62.04}$$

$$y = \frac{\ln x}{x^2} \quad \text{find } y':$$

$$y' = \frac{(x^2)\cdot\left(\frac{1}{x}\right) - (\ln x)(2x)}{[x^2]^2}$$

$$y' = \frac{x - 2x\ln x}{x^4} = \frac{\cancel{x}(1-2\ln x)}{\cancel{x}\cdot x^3}$$

$$y = \ln\left(\frac{x^2+2x}{x-3}\right)$$

rewrite this

$$\ln\left(\frac{a}{b}\right) = \underline{\ln a} - \underline{\ln b}$$

$$\boxed{y = \ln(x^2+2x) - \ln(x-3)} \; *$$

$$y' = \frac{1}{x^2+2x} \cdot (2x+2) - \frac{1}{x-3} \cdot (1)$$

$$y' = \frac{2x+2}{x^2+2x} - \frac{1}{x-3}$$

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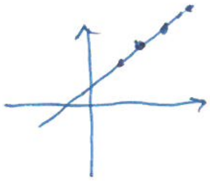
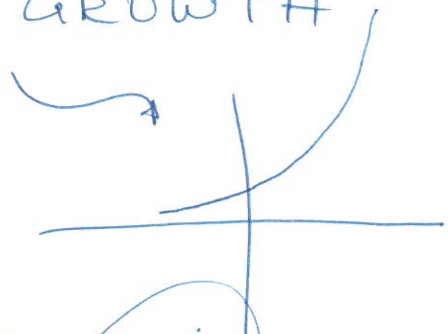
if not . . .

$$y' = \frac{1}{\frac{x^2+2x}{x-3}} \left[\frac{(x-3)(2x+2) - (x^2+2x)(1)}{(x-3)^2} \right]$$

3.3: EXPONENTIAL GROWTH





rate of growth is directly proportional to the amount (of y) at any time "t".

$$\frac{dy}{dt} = k \cdot y$$

ch5

$$\boxed{y = C \cdot e^{kt}} \longrightarrow \frac{t = 0}{y = y_0}$$

y_0 ← initial amt.

$$y_0 = C \cdot e^{k(0)}$$

$$y_0 = C \cdot 1$$

check:

$$\frac{dy}{dt} = C \cdot e^{kt} \cdot k$$

$$\frac{dy}{dt} = k \cdot \left[C \cdot e^{kt}\right]$$

$$\frac{dy}{dt} = k \cdot y$$

$$\boxed{y = y_0 \cdot e^{kt}}$$

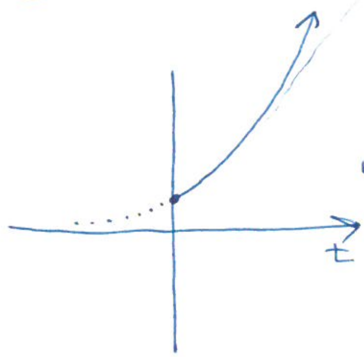
← time

future amt — initial amt — growth rate ($k = +$ growth)

$$\underline{\underline{A}} = P \cdot e^{rt}$$

contin. comp. int.

Pop. prob:



uninhibited growth

"standard" exp. growth

SPEEDWAY, INDIANA

① 2000: 10,882 ← y_0 ($t=0$)

② 2010: 12,461 ($t=10$)

pred. pop. 2025 ($t=25$)

① $y = \underline{y_0} \cdot e^{kt}$

$y = (10,882) \cdot e^{kt}$

② $t = 10$, $y = 12,461$

$$\frac{12,461}{10,882} = \frac{\cancel{(10,882)} \cdot e^{k(\underline{10})}}{\cancel{10,882}}$$

$$\frac{12,461}{10,882} = e^{10k}$$

$$\frac{\ln\left(\frac{12,461}{10,882}\right)}{10} = \frac{\cancel{10}k}{\cancel{10}} \approx \underline{+.0135}$$

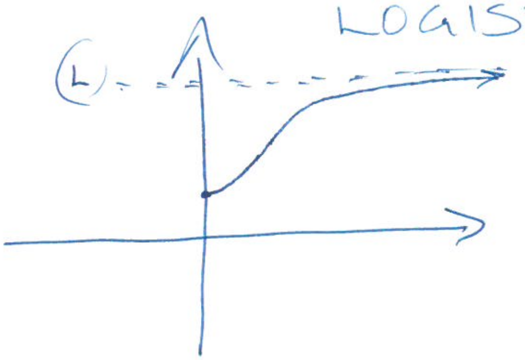
$$y = (10,882) \cdot e^{(.0135)t}$$

Pop in 2025 $(t = 25)$

$$y = (10,882) \cdot e^{(.0135)(25)}$$

$$y \approx \underline{15,250}$$

LOGISTIC GROWTH:



(long-term model)

$$y = \frac{L}{1 + b \cdot e^{kt}}$$

L ← limiting value

k is NEG

b ↑ solved by using initial pop

RALEIGH, NC:

① 2010 $(t=0)$: 403,892 (find b)

② 2018 $(t=8)$: 464,758 (find k)

L: 700,000

predict when $(t = ??)$ pop. would be 550,000 ??

① $y = \dfrac{700,000}{1 + b \cdot e^{kt}}$

$t = 0 \quad y = 403,892$

$$403,892 = \frac{700,000}{1 + b \cdot e^{k \cdot 0}}$$

$e^{k \cdot 0} = 1$

$$\frac{403,892}{1} = \frac{700,000}{1 + b}$$

$$\frac{\cancel{403{,}892}(1+b)}{\cancel{403{,}892}} = \frac{700{,}000}{403{,}892}$$

$$\cancel{-1}\ \cancel{1} + b = \frac{700{,}000}{403{,}892} - 1$$

$$b = \frac{700{,}000}{403{,}892} - 1 \approx \underline{.7331}$$

$$y = \frac{700{,}000}{1 + (.7331)\cdot e^{kt}}$$

2 $t = 8$ $y = 464{,}758$

$$\underline{464{,}758} = \frac{700{,}000}{1 + (.7331)\cdot e^{k(8)}}$$

$$\frac{\cancel{464{,}758}\left[1 + (.7331)\, e^{8k}\right]}{\cancel{464{,}758}} = \frac{700{,}000}{464{,}758}$$

$$\cancel{-1}\ \cancel{1} + \frac{(\cancel{.7331})\, e^{8k}}{\cancel{.7331}} = \frac{\frac{700{,}000}{464{,}758} - 1}{.7331}$$

$$e^{8k} = \frac{\frac{700{,}000}{464{,}758} - 1}{.7331}$$

$$\underline{-.0463} \approx \frac{\cancel{8}k}{\cancel{8}} = \frac{\ln\left[\frac{\frac{700{,}000}{464{,}758} - 1}{.7331}\right]}{8}$$

$$y = \frac{700,000}{1 + (.7331)e^{-.0463t}}$$

$t = ?$

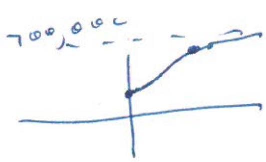
pop: 550,000

$$\frac{550,000}{1} = \frac{700,000}{1 + (.7331)e^{-.0463t}}$$

$$\frac{\cancel{550,000}\left[1 + (.7331)e^{-.0463t}\right]}{\cancel{550,000}} = \frac{700,000}{550,000} - 1$$

$$e^{-.0463t} = \frac{\frac{700,000}{550,000} - 1}{.7331}$$

$$\frac{\cancel{-.0463}\, t}{\cancel{-.0463}} = \frac{\ln\left[\frac{\frac{700,000}{550,000} - 1}{.7331}\right]}{-.0463}$$



$t \approx \underline{21.36}$

$t = 0 \rightarrow 2010$

550,000 in 2032

3.4: EXPONENTIAL DECAY

$$y = y_0 \cdot e^{kt}$$ (k is NEG.)

100 lbs. radioactive material
(half-life: 2 yrs.) → find k
when will there be 5 lbs left?
($t = ??$)

$t = 0 \qquad y_0 = 100$

$$y = 100 \cdot e^{kt}$$

$$\frac{50}{100} = \frac{\cancel{100} \cdot e^{k(2)}}{\cancel{100}}$$

$$\frac{1}{2} = e^{2k} \qquad \frac{\ln\left(\frac{1}{2}\right)}{2} = \frac{2k}{2}$$

$$k \approx -.3466$$

100
(2)
50
(2)
25
(2)
12.5
(2)
6.25

$$y = 100 \cdot e^{-.3466t} \qquad t = ?$$

$$5 = 100 \cdot e^{-.3466t} \qquad y = 5$$

$$\frac{5}{100} = e^{-.3466t}$$

$$\frac{\ln\left(\frac{5}{100}\right)}{-.3466} = \frac{\cancel{-.3466}\, t}{\cancel{-.3466}} \approx 8.64 \text{ yrs}$$

NEWTON'S LAW OF COOLING:

rate at which an object cools is directly proportional to the difference in temp. between the object and the surrounding medium.

$$\frac{dT}{dt} = k \cdot (T - m)$$

T = Temp. of object
t = time
m = surrounding medium

$$T = \boxed{a}\, e^{kt} + m$$

$T_0 = 400^\circ$ $t = 0$
$m = 70^\circ$
$T = 320^\circ$ $t = 10\text{ min}$
$T = 125^\circ$ $t = ??$