ma 121-002

Thursday, October 25

$$T = a \cdot e^{kt} + \underline{m}$$

$\underline{m = 70^\circ}$

$$\begin{cases} t = 0 & T = 400^\circ \\ t = 10\text{ min} & T = 320^\circ \end{cases}$$

$\underline{t = ??}$ $\quad T = 125^\circ$

$$T = a \cdot e^{kt} + 70$$

1 $t = 0$ $\quad T = 400^\circ$

$$400 = a \cdot e^{k(0)} + 70$$

$$400 = a + \cancel{70}$$

$$-70 \qquad \cancel{-70}$$

$$330 = a$$

$$T = 330 e^{kt} + 70$$

2 $t = 10$ min $\quad T = 320^\circ$

$$320 = 330 e^{k(10)} + \cancel{70}$$

$$-70 \qquad \cancel{-70}$$

$$\frac{250}{330} = \frac{\cancel{330} e^{10k}}{\cancel{330}}$$

$$\frac{250}{330} = e^{10k}$$

$$\ln\left(\frac{250}{330}\right) = 10k$$

$$\frac{\ln\left(\frac{250}{330}\right)}{10} = \frac{\cancel{10}k}{\cancel{10}} \approx \underline{-.0278}$$

$$T = 330 \cdot e^{-.0278t} + 70$$

$t = ??$ $\qquad T = 125°$

$$125 = 330 \cdot e^{-.0278t} + \cancel{70}$$

$$-70 \qquad\qquad -\cancel{70}$$

$$\frac{55}{330} = \frac{\cancel{330} \cdot e^{-.0278t}}{\cancel{330}}$$

$$\frac{55}{330} = e^{-.0278t}$$

$$\frac{\ln\left(\frac{55}{330}\right)}{-.0278} = \frac{\cancel{-.0278}\,t}{\cancel{-.0278}} \approx \underline{64.5} \text{ min}$$

$$y = y_0 \cdot e^{kt}$$

RADIOCARBON DATING

(CARBON DATING)

C_{14}: half-life is 5730 years

$$y = y_0 \cdot e^{kt}$$

$$\frac{\frac{1}{2}\cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} \cdot e^{k(5730)}}{\cancel{y_0}}$$

$$\frac{1}{2} = e^{5730k}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{5730} = \frac{\cancel{5730}k}{\cancel{5730}}$$

$$\boxed{k \approx \underline{-.00012097}}$$

how old is the artifact that has lost 42% of its C_{14}?

$$58\% = \underbrace{100\%} \cdot e^{-.00012097t}$$

$$.58 = 1 \cdot e^{-.00012097t}$$

3.5:

exponentials:

$$y = e^x$$
$$y' = e^x$$

chain rule: $y = e^{2x^2+x}$

$$y' = e^{2x^2+x} \cdot (4x+1)$$

$$y = 2^x \qquad y = 5^x \qquad y = 10^x$$

$$\boxed{y = a^x}$$

$$e^{\ln u} = u$$

$$\ln e^u = u$$

rewrite:

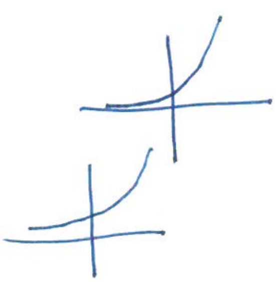
$$a^x = e^{\ln a^x}$$

$$\ln 2^3$$
$$= 3(\ln 2)$$

$$y = e^{\ln a^x}$$
$$y = e^{x \cdot \ln a}$$

$a = 5$

$\ln a = \ln 5$



take DERIV:

$$y' = e^{x \cdot \ln a} \cdot d(x \cdot \boxed{\ln a})$$
$$y' = e^{x \cdot \ln a} \cdot \ln a$$
$$y' = a^x \cdot \ln a$$

$$y = a^x$$
$$y' = a^x \cdot \ln a$$

ex: $y = 8^x$

$$y' = 8^x \cdot \ln 8$$

$$y = 5^{2x-1}$$
$$y' = 5^{2x-1} \cdot \ln 5 \cdot 2$$

$$y = e^x$$
$$y' = e^x \cdot \ln e \to 1$$
$$y' = e^x$$

LOGS:

$y = \ln(x)$

$y' = \frac{1}{x}$

chain rule: $y = \ln(5x^2 - 8x + 1)$

$y' = \frac{1}{5x^2-8x+1} \cdot (10x-8)$

$y' = \frac{10x-8}{5x^2-8x+1}$

$$\underbrace{y = \log_4 x \qquad y = \log_{10} x}$$

$y = \log_a x$

$\boxed{f(x) = \log_a(x)}$ find $f'(x)$:

rewrite in exponential form

$\boxed{a^{f(x)} = x}$

take DERIV:

$\frac{\cancel{a^{f(x)}} \cdot \cancel{\ln a} \cdot f'(x)}{\cancel{a^{f(x)}} \cdot \cancel{\ln a}} = \frac{1}{a^{f(x)} \cdot \ln a}$

$f'(x) = \frac{1}{x \cdot \ln a}$

$\left. \begin{array}{l} y = \ln x \\ y' = \frac{1}{x} \end{array} \right\}$

$\left. \begin{array}{l} y = \log_a x \\ y' = \frac{1}{x \cdot \ln a} \end{array} \right\}$

ex: $y = \log_7 x$

$y' = \frac{1}{x \cdot \ln 7}$

ex: $y = \log_4 (3x+5)$

$y' = \frac{1}{(3x+5) \cdot \ln 4} \cdot 3 = \frac{3}{(3x+5) \cdot \ln 4}$

4.1:

$$\begin{tikzcd} f(x) \arrow[r, "\text{DERIV.}"] & f'(x) \arrow[l, bend left, "\text{ANTIDERIVATIVE (INTEGRAL)}"] \end{tikzcd}$$

$$f(x) = 4x^2 - 8x + 3$$

$$f'(x) = 8x - 8 + 0$$

$$f'(x) = \boxed{8x} - 8 + 0$$

find $f(x)$:

$$f(x) = 4x^2 - 8x + C$$

indefinite integral

$y = x^2 + C$

$y = x^2$

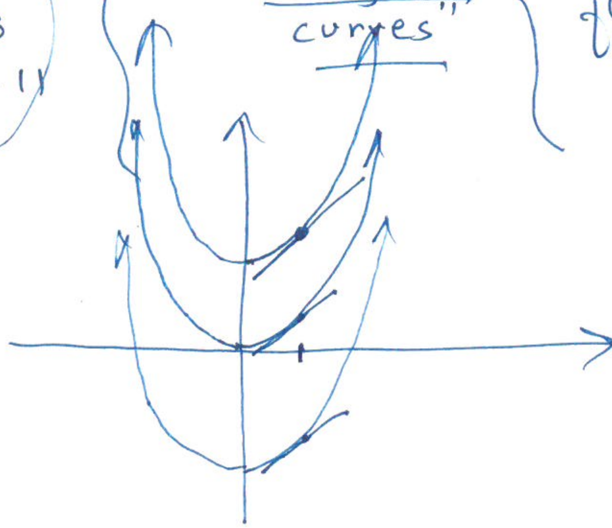
$y = x^2 + 2$

$y = x^2 - 3$

$y = x^2 + 11$

"family of curves"

$$\left\{ \begin{array}{l} f(x) = 4x^2 - 8x + 1 \\ f(x) = 4x^2 - 8x - 17 \\ f(x) = 4x^2 - 8x \\ \vdots \end{array} \right.$$



$$\int \underbrace{(8x - 8)}_{\text{integrand}}\, dx = 4x^2 - 8x + C$$

$4(2x)$

integral sign

antidiff. with respect to x

$$\int \boxed{4}\, dt = 4t + C$$

$$\int \boxed{4}\, dx = 4x + C$$

① power rule:

$$\boxed{\int a \cdot x^n\, dx = \frac{a \cdot x^{n+1}}{n+1} + C} \text{ for } n \neq -1$$

$$\int \boxed{4}\,\boxed{x^7}\, dx = 4x^8 + C$$

check: $d(4x^8 + C) \overset{?}{=} 4 \cdot 8x^7$??

$$\int 4 \cdot x^7\, dx = \frac{4 \cdot x^8}{8} + C$$

check: $d\left(\frac{1}{2}x^8 + C\right) = \frac{1}{2} 8x^7 = 4x^7$

$$\int \boxed{\frac{17}{9}\cdot x^{-3}}\, dx$$

$$= \frac{17}{9}\,\frac{x^{-2}}{-2} + C$$

$$= \frac{-17}{18}\, x^{-2} + C$$

check: $d\left(-\frac{17}{18}x^{-2} + C\right)$

$$\int 22\, x^{2/3}\, dx = 22 \cdot \frac{x^{5/3}}{5/3} + C$$

$$= \underline{22} \cdot \frac{3}{5}\, x^{5/3} + C$$

$$= \frac{66}{5}\, x^{5/3} + C$$

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(2) $\int a \cdot x^{-1}\, dx = \int \underline{a} \cdot \frac{1}{x} \cdot dx$

$$= a \cdot \ln|x| + C$$

(3) $\int e^{x}\, dx = e^{x} + C$

$$\int e^{ax}\, dx =$$

$$\int \boxed{e^{5x}}\, dx = e^{5x} + C$$

check: $d\left(e^{5x} + C\right) \stackrel{?}{=} e^{5x}$

$$= e^{5x} \cdot 5$$

$$\boxed{③ \quad \int e^{ax}\, dx = \frac{e^{ax}}{a} + C}$$

ex: $\int e^{8x}\, dx = \frac{e^{8x}}{8} + C$

check: $d\left(\frac{1}{8}\left(e^{8x}\right)\right)$

$$= \cancel{\frac{1}{8}}\left(e^{8x} \cdot \cancel{8}\right)$$