Tuesday, October 30

TUES, NOVEMBER 6: TEST #3

2.5; 3.1 → 3.5; 4.1 → $\underline{\underline{4.3}}$

4.1: ANTIDERIVATIVES (INTEGRALS)

$$\int f(x)\cdot dx = F(x) + C$$

indef. integral: family of curves

(where $F'(x) = f(x)$)

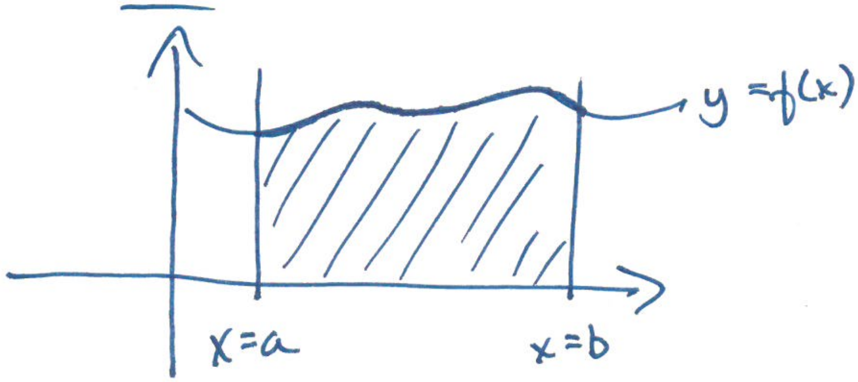
① $$\int a\cdot x^n \cdot dx = a\cdot \frac{x^{n+1}}{n+1} + C$$

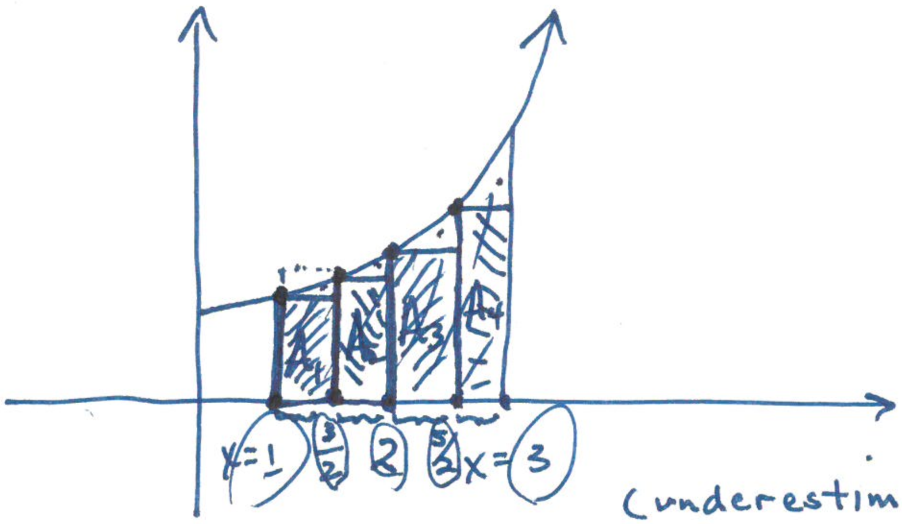
(for $n \neq -1$)

② $$\int a\cdot x^{-1}\cdot dx = \int a\cdot \frac{1}{x}\cdot dx = a\cdot \ln|x| + C$$

③ $$\int a\cdot e^{bx}\, dx = a\cdot\frac{1}{b}\cdot e^{bx} + C$$

4.2: AREAS (UNDER CURVES)





$f(x) = x^2 + 1$

find the area under $f(x) = x^2+1$ from $x = 1$ to $x = 3$

APPROX THE AREA: (using rectangles)

$$A \approx A_1 + A_2 + A_3 + A_4$$

$$\Delta x = \text{WIDTH} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$A_1 = (1^2 + 1)\left(\frac{1}{2}\right) = (2)\left(\frac{1}{2}\right) = 1$$

$$A_2 = \left(\left(\frac{3}{2}\right)^2 + 1\right)\left(\frac{1}{2}\right) = \left(\frac{13}{4}\right)\left(\frac{1}{2}\right) = \frac{13}{8}$$

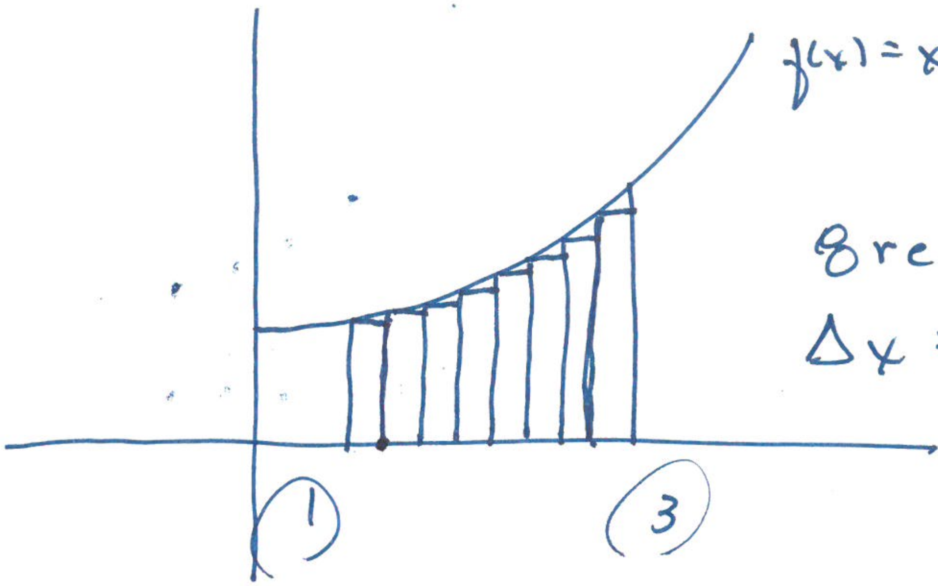
$$A_3 = (2^2 + 1)\left(\frac{1}{2}\right) = (5)\left(\frac{1}{2}\right) = \frac{5}{2}$$

$$A_4 = \left(\left(\frac{5}{2}\right)^2 + 1\right)\left(\frac{1}{2}\right) = \left(\frac{29}{4}\right)\left(\frac{1}{2}\right) = \frac{29}{8}$$

$$A \approx \underset{\uparrow \; 8/8}{1} + \frac{13}{8} + \underset{\uparrow \; 20/8}{\frac{5}{2}} + \frac{29}{8} = \frac{70}{8} = \frac{35}{4}$$

$$\boxed{A \approx 8\tfrac{3}{4}}$$

more rectangles $\longrightarrow$ better accuracy



8 rectangles:

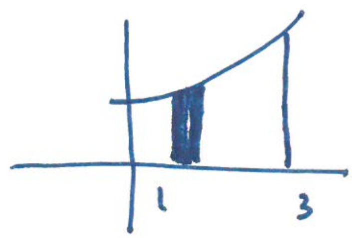
$$\Delta x = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$A \approx \left[f(1)\right]\cdot\left[\tfrac{1}{4}\right] + \left[f\left(\tfrac{5}{4}\right)\right]\cdot\left(\tfrac{1}{4}\right) + \dots + \left[f\left(2\tfrac{3}{4}\right)\right]\cdot\left[\tfrac{1}{4}\right]$$

$$A \approx \underbrace{f(x_1)\cdot\Delta x}_{A_1} + \underbrace{f(x_2)\cdot\Delta x}_{A_2} + \dots + \underbrace{f(x_8)\cdot\Delta x}_{A_8}$$

$$A \approx \boxed{\sum_{i=1}^{8} f(x_i)\cdot\Delta x}$$

summation



better

$$A \approx \sum_{i=1}^{700,000,000} f(x_i)\cdot\Delta x$$

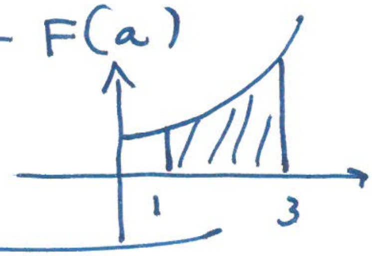
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

DEF:

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x = \int_{x=a}^{x=b} f(x) \cdot dx$$

$$= F(x) \Big]_a^b = F(b) - F(a)$$



$f(x) = x^2 + 1$ find the area under $f(x) = x^2 + 1$ from $x = 1$ to $x = 3$

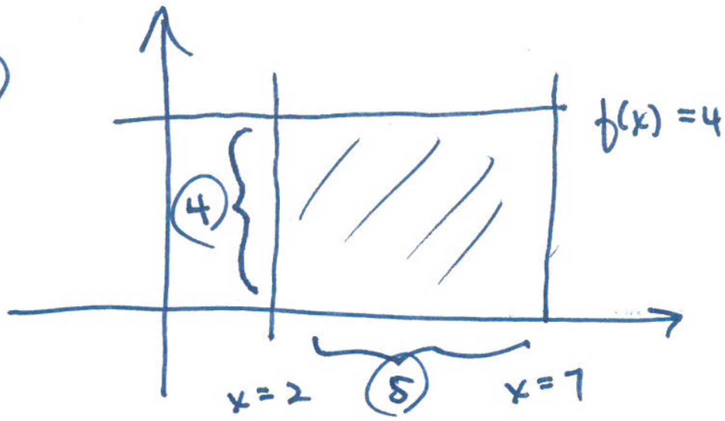
$\boxed{A \approx 8\tfrac{3}{4}}$ using 4 rectangles

$$A = \int_1^3 (x^2 + 1)\, dx = \frac{x^3}{3} + x \Big]_1^3$$

$$= \left[\frac{3^3}{3} + 3\right] - \left[\frac{1^3}{3} + 1\right]$$

$$= 9 + 3 - \tfrac{1}{3} - 1 = \boxed{10\tfrac{2}{3}} \text{ or } \tfrac{32}{3}$$

1



$$A = \int_a^b f(x) \cdot dx$$

$$A = \int_2^7 4 \cdot dx = 4x\Big]_2^7 = 4(7) - 4(2)$$

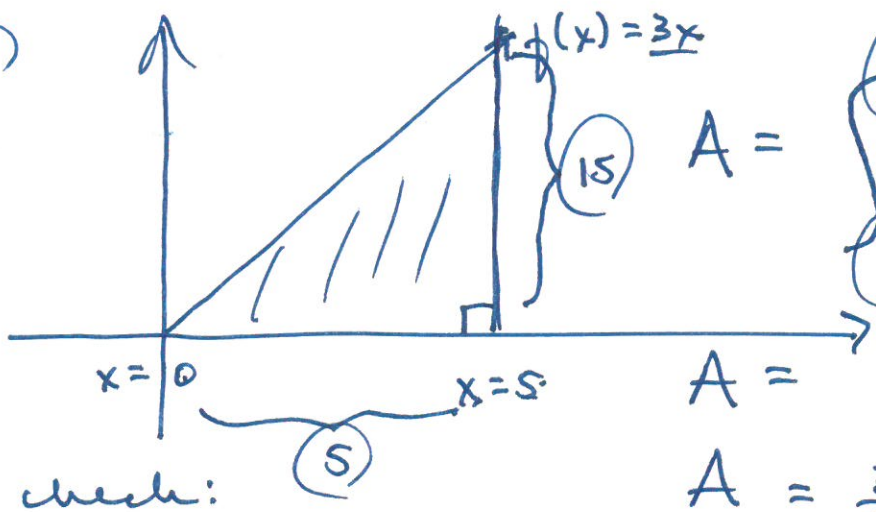
$$= 28 - 8$$

$$= \underline{20}$$

check:

$$A = b \cdot h = 5 \cdot 4 = 20$$

2



DEFINITE INTEGRAL

$$A = \int_0^5 3x \cdot dx$$

$$A = 3\frac{x^2}{2}\Big]_0^5$$

$$A = \frac{3}{2}x^2\Big]_0^5$$

$$A = \frac{3}{2}(5)^2 - \cancel{\frac{3}{2}(0)^2}$$

$$A = \frac{75}{2}$$

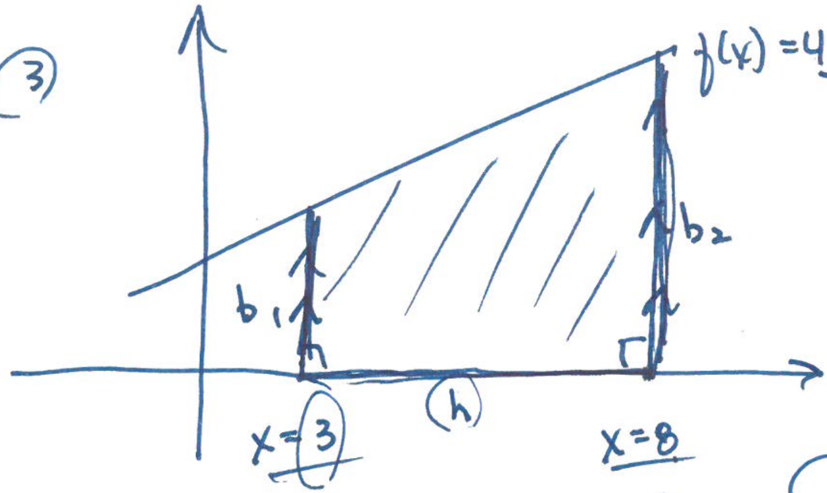
check:

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot 5 \cdot 15$$

$$A = \frac{75}{2}$$

3



TRAPEZOID

$A = \underbrace{\frac{1}{2}(b_1 + b_2)} \cdot h$

$A = \frac{1}{2}(13 + 33) \cdot 5$

$A = (23)(5) = 115$

$$A = \int_3^8 (4x + 1)\, dx$$

$$A = 4 \cdot \frac{x^2}{2} + x \Big]_3^8$$

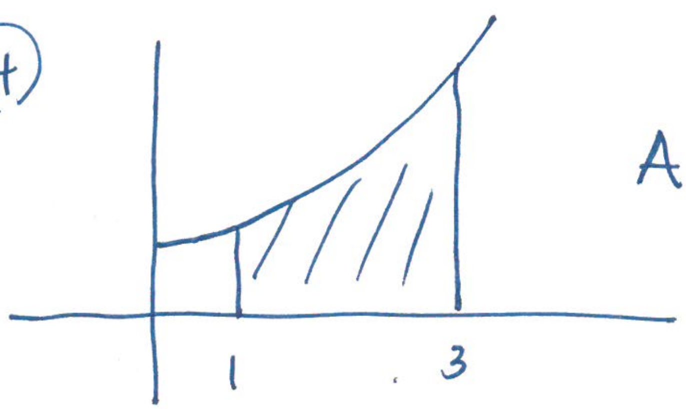
$$A = \underline{2x^2} + \underline{x} \Big]_3^8$$

$$A = [2(8)^2 + 8] - [2 \cdot 3^2 + 3]$$

$$A = 128 + 8 - 18 - 3 = \underline{115}$$

$136 - 21$

4



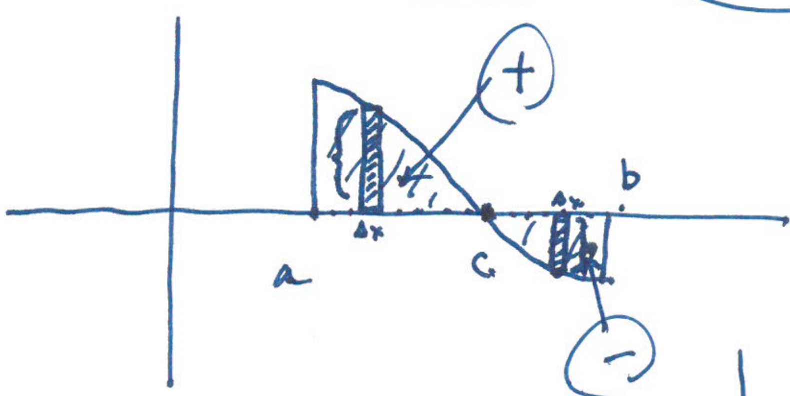
$$A = \int_1^3 (x^2 + 1)\, dx = 10\tfrac{2}{3}$$

indef. integral:

$$\int f(x)\cdot dx = \underbrace{F(x)+C}_{\text{family of curves}}$$

def. integral:

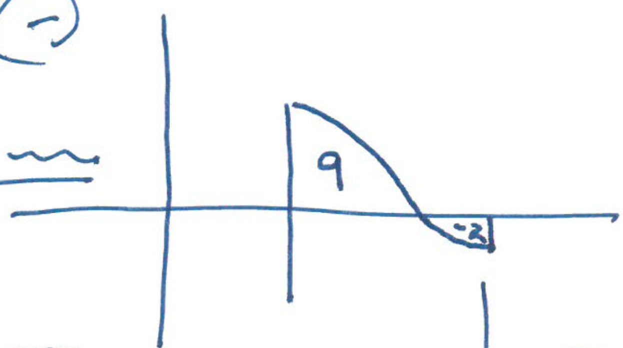
$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = \underbrace{F(b)-F(a)}_{\#\ (\text{area})}$$

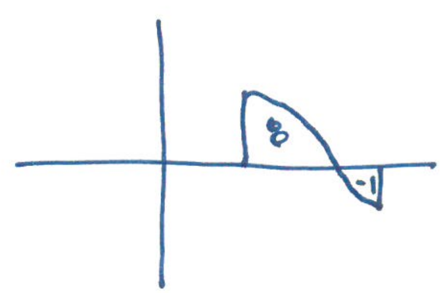


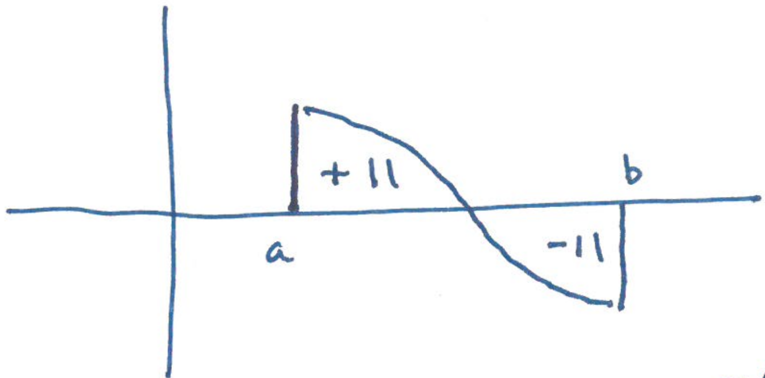
$$\int_a^b f(x)\,dx = \underline{7}$$

$$A_1 = \int_a^c f(x)\,dx = \underline{+\sim}$$

$$A_2 = \int_c^b f(x)\,dx = \underline{-\sim}$$







$$\int_a^b f(x)\,dx = 0$$

$$\int_a^a f(x)\,dx = 0$$

$f'(x) = 3x+2$ $\boxed{f(1) = 5}$?? $x=1$, $y=5$

find $f(x)$:

$$\int (3x+2)\,dx = f(x)$$

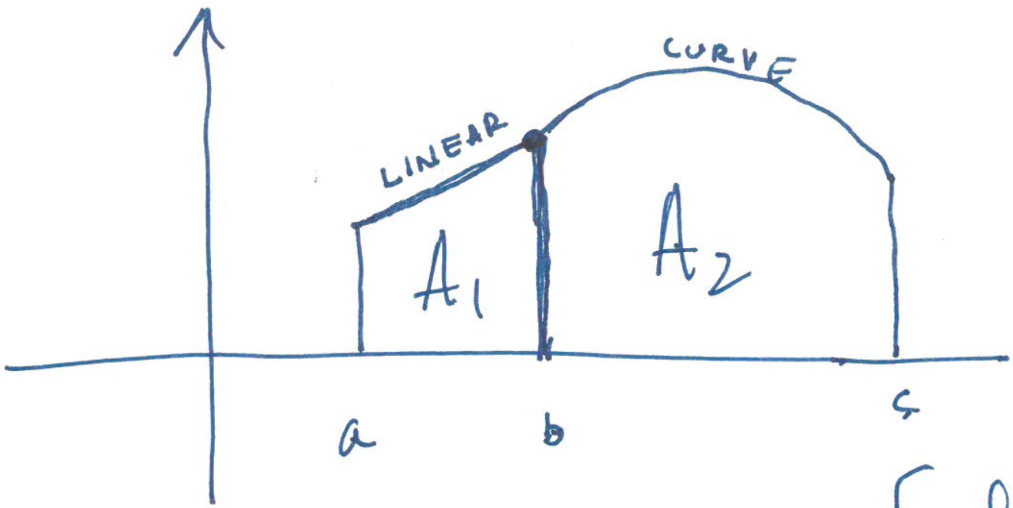
$$\boxed{3\frac{x^2}{2} + 2x + C = f(x)}$$

$$\frac{3}{2}(1)^2 + 2(1) + C = 5$$

$$\frac{3}{\cancel{2}} + \cancel{2} + C = 5 - 3\tfrac{1}{2}$$

$$-3\tfrac{1}{2} \quad C = 1\tfrac{1}{2}$$

$$\boxed{f(x) = \frac{3}{2}x^2 + 2x + 1\tfrac{1}{2}}$$



$$f(x) = \begin{cases} \text{line} \\ \text{curve} \end{cases}$$

$$A_1 = \int_a^b \underline{\text{line}} \cdot dx$$

$$A = A_1 + A_2$$

$+$

$$A_2 = \int_b^c \underline{\text{curve}} \cdot dx$$