MA 121-002

Thursday, November 8

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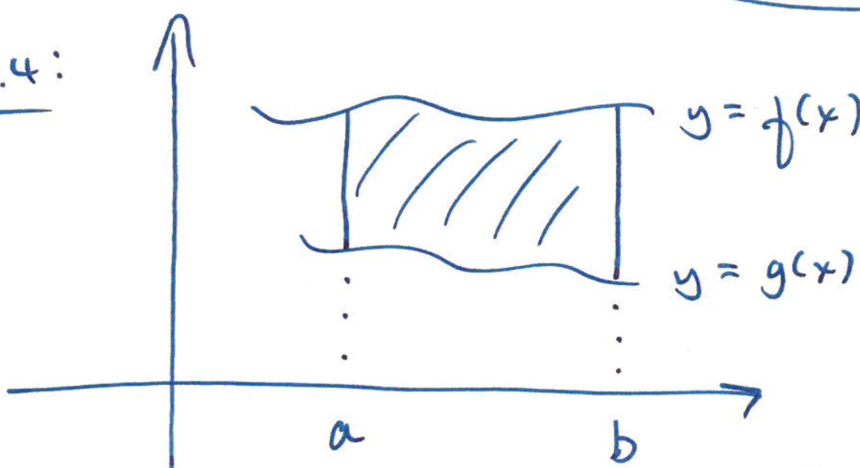
4.4: • area between 2 curves

• average value of a function

4.5: • integration using SUBSTITUTION

4.4:



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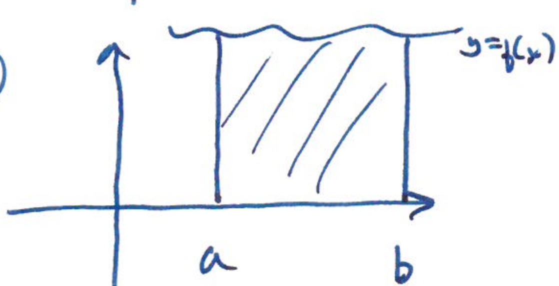

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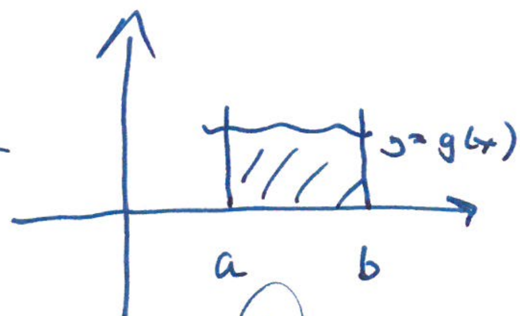
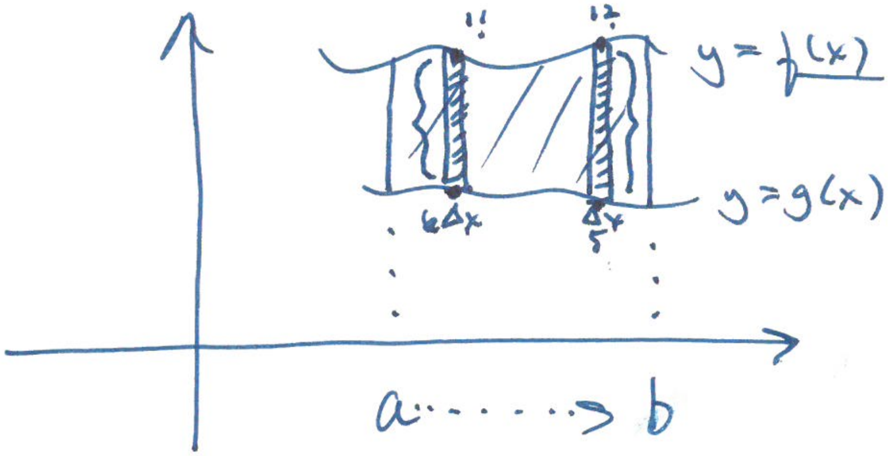


$$\int_a^b f(x)\,dx \quad - \quad \int_a^b g(x)\,dx$$

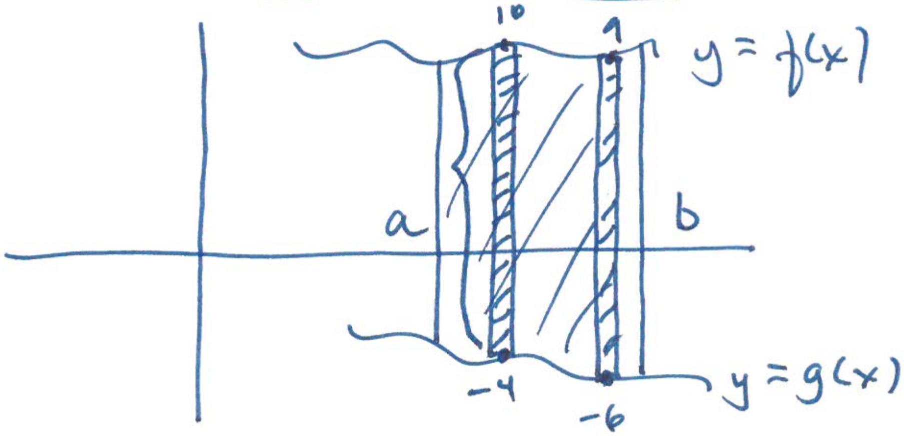
$$A = \int_a^b [f(x) - g(x)] \cdot dx \quad *$$

2

3



$$A = \int_a^b \Big[\underbrace{\big[f(x) - g(x)\big]}_{\text{HEIGHT}} \cdot \underbrace{dx}_{\text{WIDTH}}\Big]$$



$10 - (-4)$ $\quad 9 - (-6)$

$4 - 7$

$7 - 4$

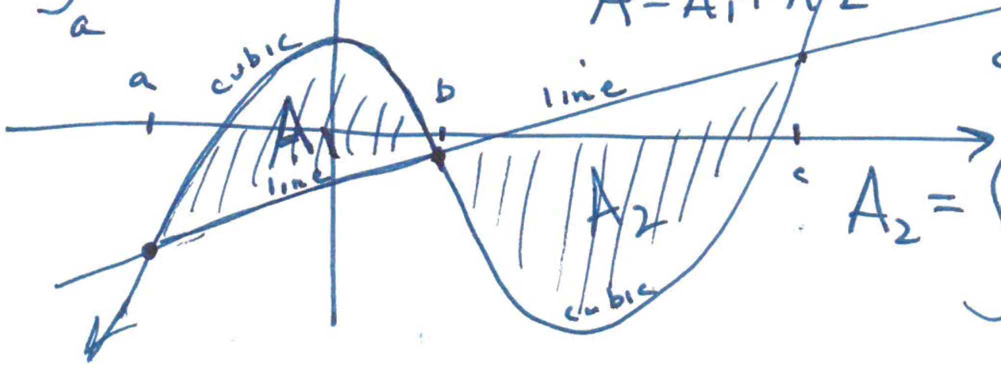
$$A_1 = \int_a^b \big[(\text{cubic}) - (\text{line})\big]\,dx$$

cubic polynomial

$A = A_1 + A_2$

ALGEBRA:

cubic = line



$$A_2 = \int_b^c \big([\text{line}] - [\text{cubic}]\big)\,dx$$

ex: $f(x) = 6x - x^2$ $\quad y = x(6-x)$ $\quad (0,0)$ $\quad (6,0)$

$g(x) = x$

parabola; opens down

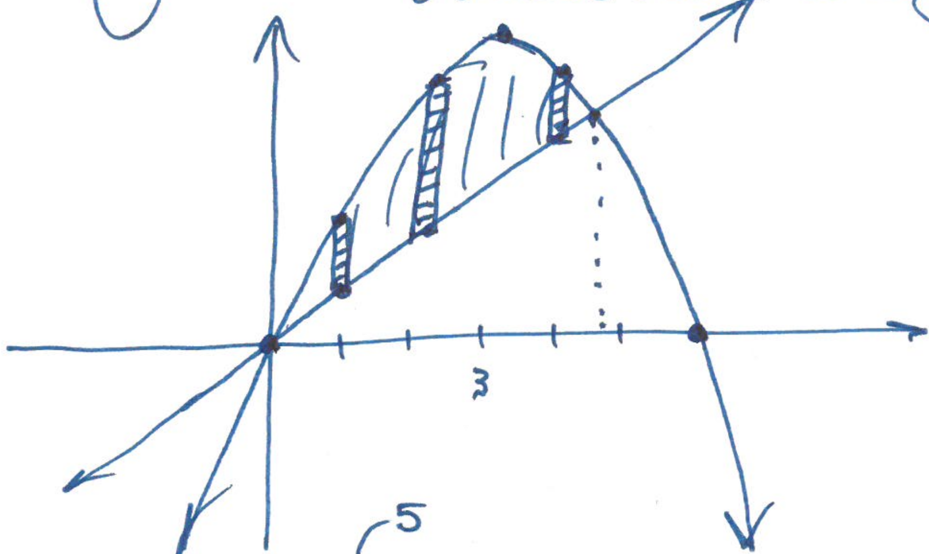
find the area of the bounded region

$V(3, 9)$

$y = 6x - x^2$

$y = 6 \cdot 3 - 3^2$

$18 - 9$



$6x - x^2 = x$

$0 = x + x^2 - 6x$

$0 = x^2 - 5x$

$0 = x(x-5)$

$x = 0 \quad x = 5$

$$A = \int_0^5 \Big[\underbrace{(6x - x^2)}_{\text{upper curve}} - \underbrace{x}_{\text{lower curve}} \Big] \, dx$$

$$= \int_0^5 (5x - x^2)\, dx$$

$$\left[5 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^5$$

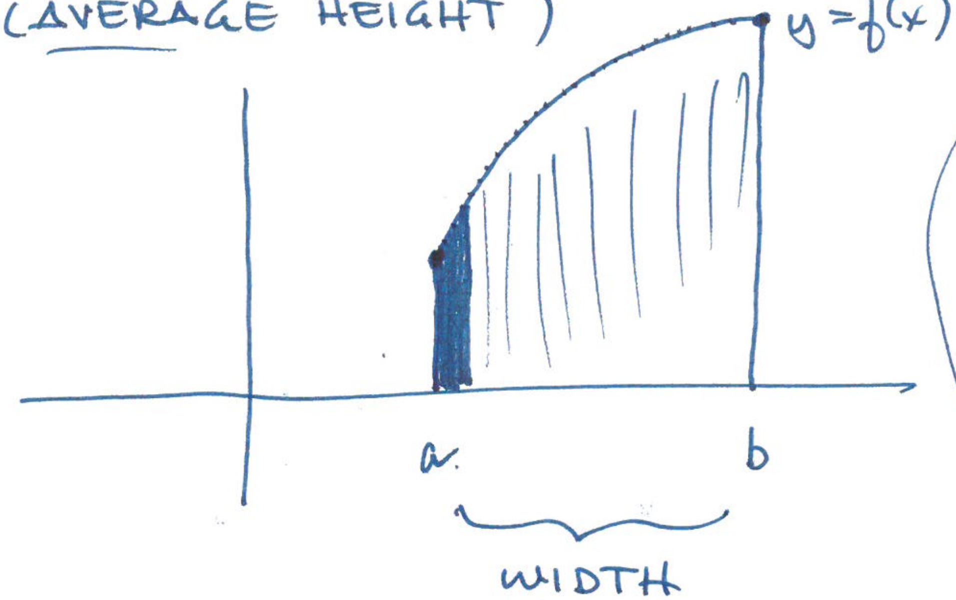
$$\left[\frac{5}{2}(x^2) - \frac{x^3}{3} \right]_0^5$$

$$= \left(\frac{5}{2} \cdot 5^2 - \frac{5^3}{3} \right) - \cancel{\left(\frac{5}{2}(0)^2 - \frac{0^3}{3} \right)}$$

$$= 3 \cdot \frac{125}{3 \cdot 2} - \frac{125 \cdot 2}{3 \cdot 2} = \frac{375}{6} - \frac{250}{6} = \left(\frac{125}{6} \right) = A$$

AVERAGE VALUE OF A FUNCTION

(AVERAGE Y-VALUE)
(AVERAGE HEIGHT)



AREA

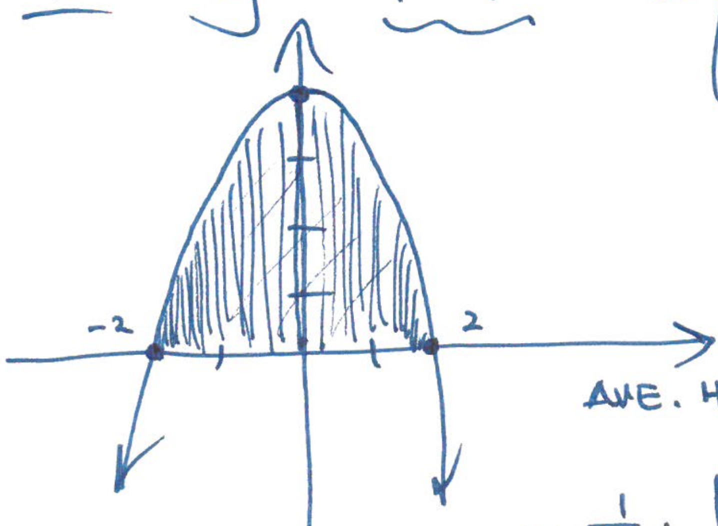
$$\frac{\int_a^b f(x)\,dx}{b-a}$$

WIDTH

$$\frac{\text{AREA}}{\text{WIDTH}} = \text{AVE HT}$$

$$\text{AVE HT} = \frac{1}{b-a}\boxed{\int_a^b f(x)\,dx}$$

ex: $y = 4 - x^2$ from $x = -2$ $= a$ to $x = 2$ $= b$



AVE. HT: 2

$$\text{AVE. HT} = \frac{1}{2-(-2)}\int_{-2}^{2} (4 - x^2)\,dx$$

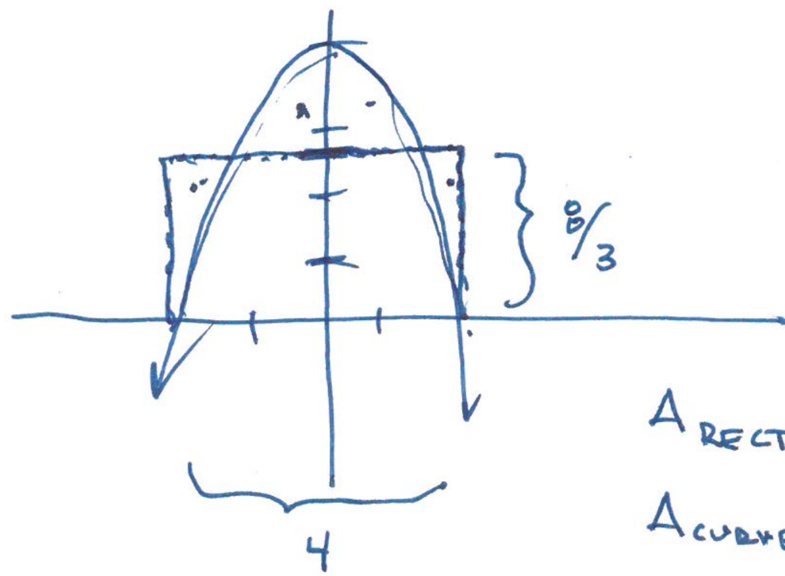
$$= \frac{1}{4}\cdot\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$\text{AVE HT} = \frac{1}{4}\cdot\left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$= \frac{1}{4}\left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right]$$

$$= \frac{1}{4}\left[16 - \frac{16}{3}\right] = \frac{1}{4}\left[\frac{48}{3} - \frac{16}{3}\right] = \frac{1}{4}\left[\frac{32}{3}\right]$$

$$= \frac{8}{3} = 2\tfrac{2}{3}$$



$$A_{RECT} = \frac{8}{3}\cdot 4 = \frac{32}{3}$$

$$A_{CURVE} = \frac{32}{3}$$

4.5: INTEGRATION USING SUBSTITUTION

(change-of-variable method)
(u - subst)

① $\int a \cdot x^n \cdot dx = a \cdot \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)

$\int a \cdot u^n \cdot du = a \cdot \frac{u^{n+1}}{n+1} + C$

"EASIER" LOOKING"

② $\int a \cdot x^{-1} \cdot dx = a \cdot \ln|x| + C$

$\int a \cdot u^{-1} \, du = a \cdot \ln|u| + C$

③ $\int a \cdot e^x \cdot dx = a \cdot e^x + C$

$\int a \cdot e^u \cdot du = a \cdot e^u + C$

ex: $\frac{1}{12}\int (3t^4+2)^5 \cdot t^3 \cdot dt \cdot 12$

let $\boxed{u = 3t^4 + 2}$

$dt \cdot \frac{du}{dt} = 12t^3 \cdot dt$

$\boxed{du = 12t^3 \cdot dt}$

$\frac{1}{12}\int u^5 \cdot du$

$\frac{1}{12} \frac{u^6}{6} + C = \frac{1}{72}(3t^4+2)^6 + C$

$$\int \boxed{x \cdot \sqrt[5]{1-x^2}} \cdot dx$$

$$-\frac{1}{2} \int (1-x^2)^{1/5} \cdot \boxed{x \cdot dx\,(-2)}$$

let $\boxed{u = 1-x^2}$

$\cancel{dx} \cdot \frac{du}{\cancel{dx}} = -2x \cdot dx$

$\boxed{du = -2x\,dx}$

$$-\frac{1}{2} \int u^{1/5} \cdot du = -\frac{1}{2} \cdot \frac{u^{6/5}}{(6/5)} + C$$

$$= -\frac{1}{2} \cdot \frac{5}{6} (1-x^2)^{6/5} + C$$

$$= \frac{-5}{12} (1-x^2)^{6/5} + C$$