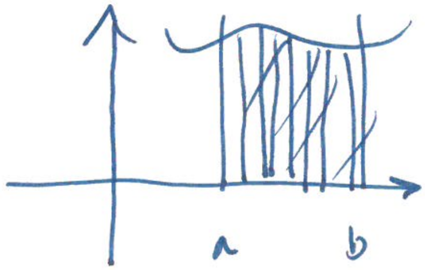
MA121-002

Thursday, November 15

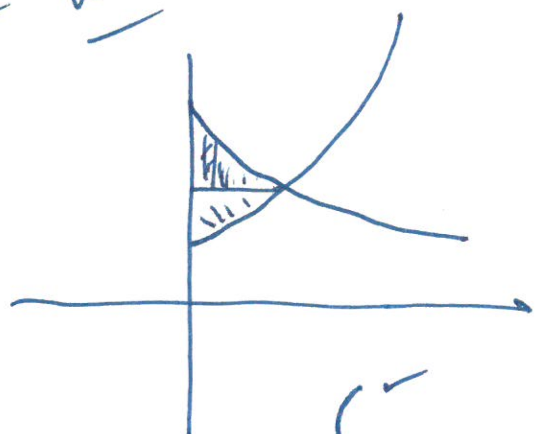
5.2: APPLICATION OF INTEGR.



$$\int f(x)\cdot dx$$

HT WIDTH

AREA



ACCUMULATION MODEL

$\int$??

① one-time contribution (investment)

y_0: initial amt.

y: future amt.

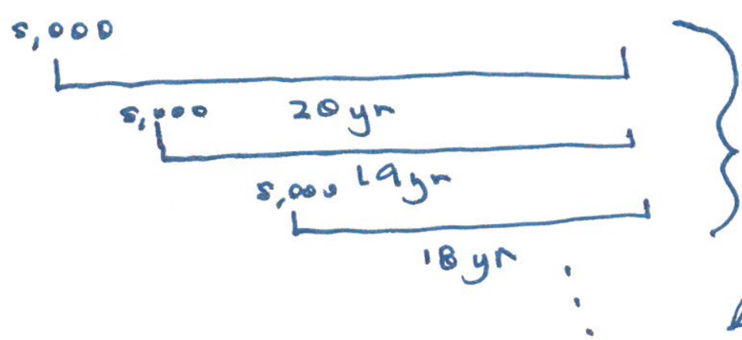
k: rate (yr.)

t: time (yr.)

$$y = y_0 \cdot e^{kt}$$

$(A = P \cdot e^{rt})$

② multiple contributions (investments)



sum

$$\int_{T1}^{T2} y_0 \cdot e^{kt}\, dt$$

ACCUM. MODEL

5.2:

25. **Future value of an inheritance.** Upon the death of his uncle, David receives an inheritance of \$50,000, which he invests for 16 yr at 4.3%, compounded continuously. What is the future value of the inheritance? \$99,486.60

$y = y_0 \cdot e^{kt}$ $(A = P \cdot e^{rt})$

$y = (50{,}000)\, e^{(.043)(16)}$

$y = \$99{,}486.^{60}$

31. Trust fund. Bob and Ann MacKenzie have a new grandchild, Brenda, and want to create a trust fund for her that will yield \$250,000 on her 24th birthday.

a) What lump sum should they deposit now at 5.8%, compounded continuously, to achieve \$250,000? \$62,144.41

b) The amount in part (a) is more than they can afford, so they decide to invest a constant amount, $R(t)$ dollars per year. Find $R(t)$ such that the accumulated future value of the continuous money stream is \$250,000, assuming an interest rate of 5.8%, compounded continuously. \$4796.74

a.)
$$\frac{250,000}{e^{(.058)(24)}} = \frac{y_0 \cdot \cancel{e^{(.058)(24)}}}{\cancel{e^{(.058)(24)}}}$$

$$\int a \cdot e^{bt}\,dt = a \cdot \frac{1}{b} e^{bt} + C$$

$$\$62,144.^{41} = y_0$$

b.)
$$250,000 = \int_0^{24} R \cdot e^{(.058)t}\,dt$$

$$250,000 = R \int_0^{24} e^{.058t}\,dt$$

$$250,000 = R \cdot \left[\frac{1}{.058} e^{.058t}\right]_0^{24}$$

$$250,000 = \frac{R}{.058}\left[e^{.058(24)} - e^{.058(0)}\right]$$

$$250,000 = \frac{R}{.058}\left[e^{.058(24)} - 1\right]$$

$$\frac{(.058)(250{,}000)}{\left[e^{.058(24)}-1\right]} = \frac{\frac{R}{\cancel{.058}}\cancel{\left[e^{.058(24)}-1\right]}(\cancel{.058})}{\cancel{\left[e^{.058(24)}-1\right]}}$$

$$\$4796.^{74} = R$$

$$(4796.^{74})(24) = 115{,}121 + \text{INT}$$

42 **Demand for aluminum ore (bauxite).** In 2013 ($t = 0$), bauxite production was approximately 232 million metric tons, and the demand was growing exponentially at a rate of 2.6% per year. (*Source:* minerals.usgs.gov.) If the demand continues to grow at this rate, how many metric tons of bauxite will the world use from 2015 to 2030?
4483.3 million metric tons

$t=2 \quad t=17$

$$\int y_0 e^{kt}\, dt$$

$$= \int_2^{17} 232\, e^{(.026)t}\, dt$$

$$= 232 \int_2^{17} e^{.026t}\, dt$$

$$= 232 \left[\frac{1}{.026} e^{.026t}\right]_2^{17}$$

$$= \frac{232}{.026}\left[e^{.026t}\right]_2^{17}$$

$$4483.3 \text{ millions} = \frac{232}{.026}\left[e^{.026(17)} - e^{.026(2)}\right]$$

metric tons

44 **Depletion of aluminum ore (bauxite).** In 2013, the world reserves of bauxite were about 65 billion metric tons. (*Source:* U.S. Geological Survey summaries, Jan. 2014.) Assuming that the growth described in Exercise 42 continues and that no new reserves are discovered, when will the world reserves of bauxite be depleted?

.026

In 81.32 yr, or about 2094

65 billion

$$65{,}000 \text{ million metric tons} = \int_0^T 232 \cdot e^{.026t}\, dt$$

$$65{,}000 = 232 \int_0^T e^{.026t}\, dt$$

$$65{,}000 = 232 \left[\frac{1}{.026} e^{.026t} \right]_0^T$$

$$\frac{(.026)\, 65{,}000}{232} = \frac{\cancel{232}}{\cancel{.026}} \left[e^{.026T} - e^{.026(0)} \right] \frac{\cancel{.026}}{\cancel{232}}$$

$$\frac{(.026)(65{,}000)}{232} + 1 = e^{.026T} - \cancel{1} + \cancel{1}$$

$$\frac{\ln\left[\frac{(.026)(65{,}000)}{232} + 1\right]}{.026} = \frac{\cancel{.026}\, T}{\cancel{.026}}$$

$$T = 81.32 \text{ yr.}$$

46. **Radioactive buildup.** Plutonium-239 has a decay rate of approximately 0.003% per year. Suppose plutonium-239 is released into the atmosphere for 20 yr at a constant rate of 1 lb per year. How much plutonium-239 will be present in the atmosphere after 20 yr? Approximately 19.994 lb

$k = -.00003$

$$\int_0^{20} 1 \cdot e^{-.00003t}\, dt$$

$$\int_0^{20} e^{-.00003t}\, dt$$

$$= \frac{1}{-.00003}\left[e^{-.00003t}\right]_0^{20}$$

$$= \frac{1}{-.00003}\left[e^{-.00003(20)} - e^{-.00003(0)}\right]$$

$$= \frac{1}{-.00003}\left[e^{-.00003(20)} - 1\right]$$

$= 19.94$ lb.