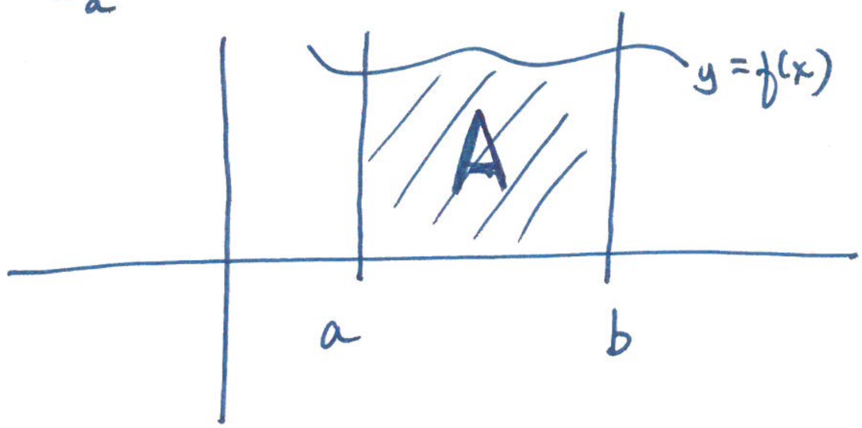
MA 121-002

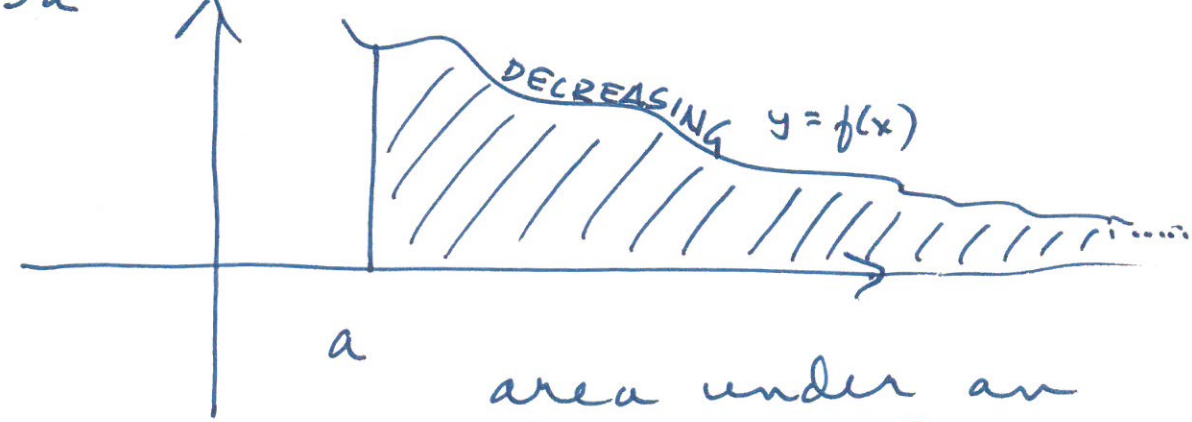
Tuesday, November 20

5.3: IMPROPER INTEGRALS

proper: $A = \int_a^b \underline{f(x) \cdot dx} = F(x) \Big]_a^b = F(b) - F(a)$

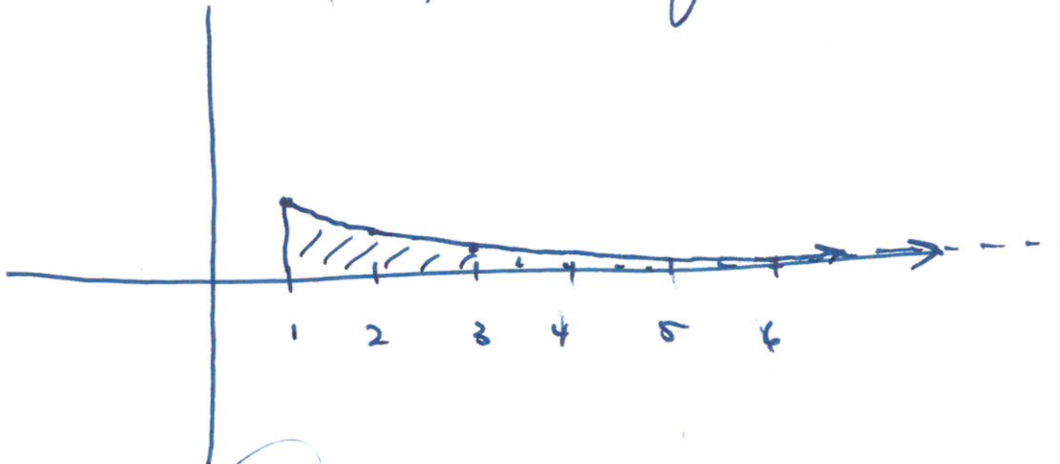


improper: $\int_a^\infty \underline{f(x)\,dx} = F(x) \Big]_a^\infty$???



area under an unbounded region

ex: $y = \frac{1}{x}$ from $x = 1$ to $x = \infty$



$$\int_1^{\infty} \frac{1}{x}\, dx = \lim_{A \to \infty} \int_1^A \left(\frac{1}{x}\right) dx$$

$$= \lim_{A \to \infty} \left[\ln|x| \right]_1^A = \lim_{A \to \infty} \left[\ln A - \overset{0}{\ln 1} \right]$$

$e^? = 1$

$$= \lim_{A \to \infty} \left[\ln A \right]^{\infty} = \text{no limit}$$

$A = 100$; $\ln 100$; $A = 10{,}000$; $\ln 10{,}000$; $A = 100{,}000{,}000$; $\ln 100{,}000{,}000$

∴ integral DIVERGES

Ex: $y = \frac{1}{x^2}$ $x = 1$ to $x = \infty$

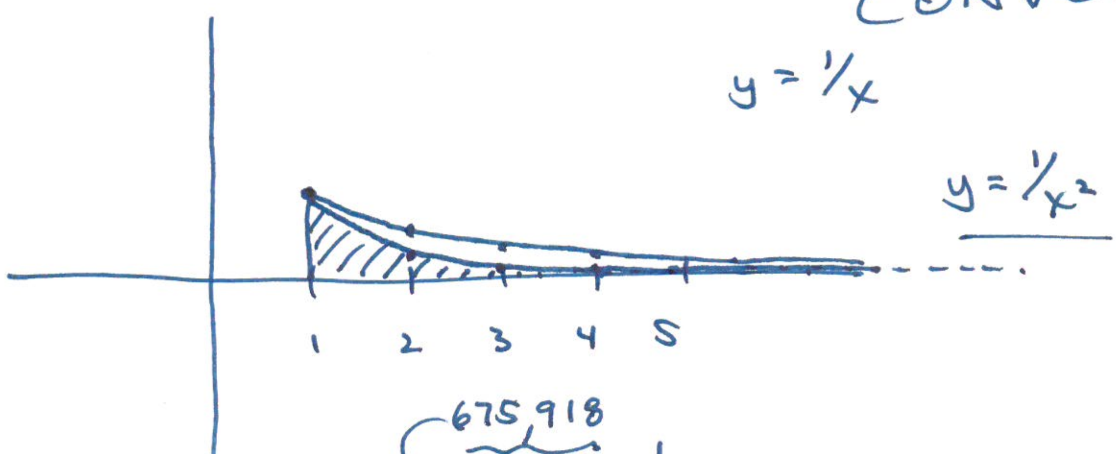
$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{B\to\infty} \int_1^B x^{-2}\,dx$$

$$= \lim_{B\to\infty} \left[\frac{x^{-1}}{-1}\right]_1^B = \lim_{B\to\infty} \left[\frac{-1}{x}\right]_1^B$$

$$= \lim_{B\to\infty} \left[\left(\frac{-1}{B}\right) - \left(\frac{-1}{1}\right)\right]$$

$$= \lim_{B\to\infty} \left[1 - \frac{1}{B}\right] = 1 \qquad \frac{1}{B} \to 0$$

∴ integral CONVERGES



$$\int_1^{675{,}918} \frac{1}{x^2}\,dx = .999998143$$

(close to 1)

(< one)

$$\int_0^\infty 8\cdot e^{-3x}\,dx$$

lim?

converge / diverge?

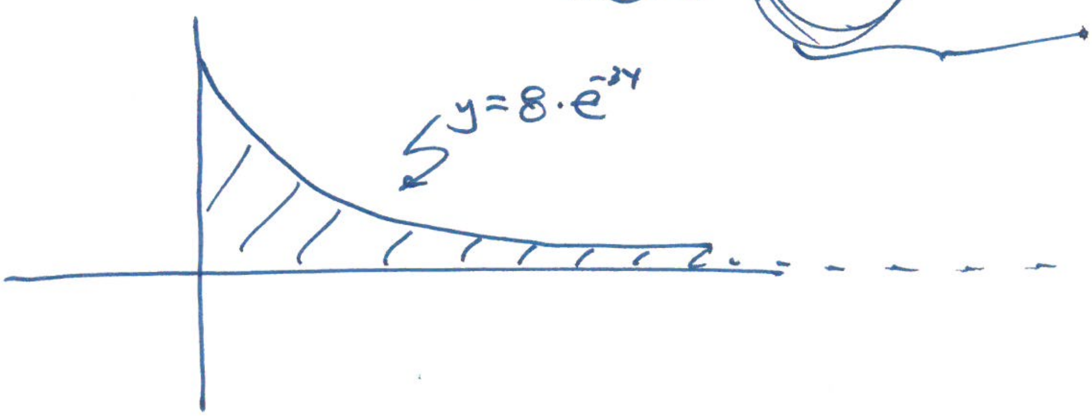
$$\int e^{bx}\,dx = \frac{1}{b}e^{bx} + C$$

$$\lim_{A\to\infty} 8\int_0^A e^{-3x}\,dx = \lim_{A\to\infty} 8\cdot\left[\frac{1}{-3}\cdot e^{-3x}\right]_0^A$$

$$= \frac{8}{-3}\cdot\lim_{A\to\infty}\left[\frac{1}{e^{3x}}\right]_0^A = \frac{-8}{3}\lim_{A\to\infty}\left[\frac{1}{e^{3\cdot A}} - \frac{1}{e^{3\cdot 0}}\right]$$

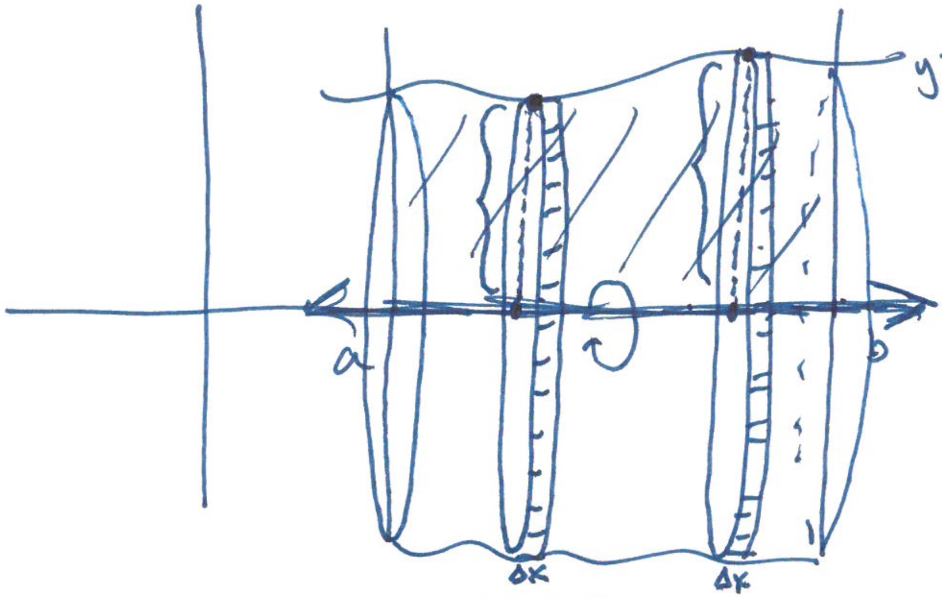
$$= \frac{-8}{3}\lim_{A\to\infty}\left[\underbrace{\frac{1}{e^{3A}}}_{\to 0} - 1\right] = \frac{-8}{3}\cdot(-1)$$

$$= \frac{8}{3}$$



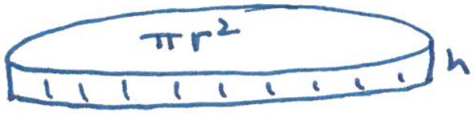
∴ integral CONVERGES

5.6: VOLUMES OF SOLIDS OF REVOLUTION



revolve this bounded region about (around) the x-axis

DISKS



CYLINDER

VOL = $\pi r^2 \cdot h$

$$V = \int_a^b \underbrace{\pi \cdot r^2 \cdot h}_{\text{VOL. OF ONE DISK}}$$

$r = ? = y = f(x)$

$h = ?$

$h = \Delta x \to dx$

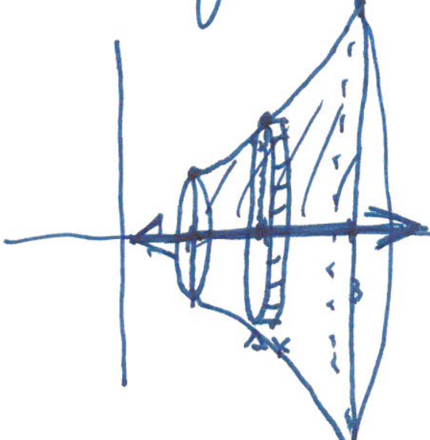
$$V = \int_a^b \pi \cdot \underbrace{(f(x))^2}_{r^2} \cdot \underbrace{dx}_{h}$$

ex: $f(x) = x^2$

bounded region:

$y = x^2$; $x = 1$; $x = 3$; x-axis

revolve about the x-axis



$r = y = f(x)$

$h = \Delta x \to dx$

$$V = \int_1^3 \pi \cdot (x^2)^2 \cdot dx$$

$$V = \pi \int_1^3 x^4 \cdot dx$$

$$V = \pi \left[\frac{x^5}{5}\right]_1^3 = \frac{\pi}{5}\left[x^5\right]_1^3$$

$$V = \frac{\pi}{5}\left[3^5 - 1^5\right] = \frac{\pi}{5}\left[243 - 1\right]$$

$$V = \frac{242\pi}{5}$$

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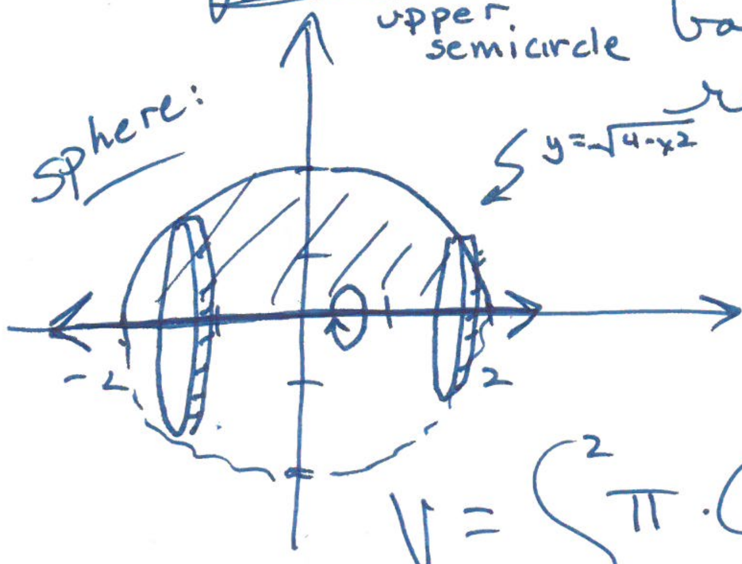
ex: $f(x) = \sqrt{4 - x^2}$ $x = -2$ to $x = 2$

upper semicircle bounded region revolved about the x-axis

sphere:



$y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

$\underbrace{x^2 + y^2 = 4}$

circle; $C(0,0)$; $r = 2$

$$V = \int_{-2}^{2} \pi \cdot (f(x))^2 \cdot dx$$

$$V = \pi \int_{-2}^{2} \left(\sqrt{4 - x^2} \right)^2 \cdot dx$$

$$V = \pi \int_{-2}^{2} \left(\underline{4} - x^2 \right) \cdot dx$$

$$V = \pi \left[4x - \frac{x^3}{3} \right]_{-2}^{2}$$

$$V = \pi \left[\left(4 \cdot 2 - \frac{2^3}{3} \right) - \left(4(-2) - \frac{(-2)^3}{3} \right) \right]$$

$$V = \pi \left[\underline{8} - \frac{8}{3} + \underline{8} - \frac{8}{3} \right] = \pi \left[\underline{16} - \frac{16}{3} \right]$$

$$V = \pi \left[\frac{48}{3} - \frac{16}{3} \right] = \pi \left(\frac{32}{3} \right) = \frac{32\pi}{3}$$

<u>check:</u> <u>sphere</u>

$$V = \frac{4}{3} \pi \cdot r^3 \qquad (r = 2)$$

$$V = \frac{4}{3} \pi (2)^3 = \frac{32\pi}{3}$$

$$V = \pi \int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$V = \frac{4}{3} \pi r^3$$

NORTH CAROLINA STATE UNIVERSITY | Department of Mathematics

MA121 | Quiz #2 | J. Griggs

Three points per question; you are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet.

3 PTS

1.) Find the area bounded by the two curves: $f(x)=x^2-x-5$ *and* $g(x)=x+10$.

pts of int: $x^2-x-5=x+10$

$x^2-x-5-x-10=0$

$x^2-2x-15=0$

$(x+3)(x-5)=0$

$x=-3$ and $x=5$

$A=\int_{-3}^{5}\left[(x+10)-(x^2-x-5)\right]dx$

$=\int_{-3}^{5}(-x^2+2x+15)\,dx$

$=\left[\frac{-x^3}{3}+x^2+15x\right]_{-3}^{5}$

$=\left[\frac{-5^3}{3}+5^2+15(5)\right]-\left[\frac{-(-3)^3}{3}+(-3)^2+15(-\right.$

$=\left(\frac{-125}{3}+25+75\right)-\left(\frac{27}{3}+9-45\right)$

$=145-18-\frac{125}{3}$

$=\frac{256}{3}=A\approx 85\frac{1}{3}$

3 PTS

2.) Find the average value of the function $f(x)=x^2-x+1$ *on* $[0,2]$.

$\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{2-0}\int_0^2(x^2-x+1)\,dx=\frac{1}{2}\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^2$

$=\frac{1}{2}\left[\left(\frac{2^3}{3}-\frac{2^2}{2}+2\right)-(0)\right]=\frac{1}{2}\left[\frac{8}{3}\cancel{-2}\cancel{+2}\right]=\frac{4}{3}=f_{AVE}$

3 PTS

3.) Integrate using substitution: $\int(2t^5-3)^2\,t^4\,dt$

let $u=2t^5-3$

$du=\left[2(5t^4)-0\right]dt$

$du=10t^4\,dt$

$\frac{1}{10}\int\underbrace{(2t^5-3)^2}_{u^2}\cdot\underbrace{t^4\cdot dt\cdot 10}_{du}$

$\frac{1}{10}\int u^2\cdot du=\frac{1}{10}\cdot\frac{u^3}{3}+C$

$=\frac{1}{30}(2t^5-3)^3+C$

NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Quiz #3 Due Tuesday, November 27, 2018 (at the beginning of class) J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

1.) Evaluate the improper integral $\int_{2}^{\infty} 7x^{-2}\,dx$. Doe this integral converge or diverge?

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5 \;\; for -5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of $200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?