Tuesday, November 27

- today: 5.7; QUIZ #3 (collected/graded)
- TEST #4: Thursday, November 29

5.7: SEPARABLE DIFFERENTIAL EQUATIONS (D.E.'s)

(D.E. — has a derivative in it)

ex: $y' = 11x$ (1st order DE) $y(2) = 4$ ($x=2$, $y=4$)

find y:

$$y = \int 11x\,dx$$

$$y = 11 \cdot \frac{x^2}{2} + C$$

$$4 = \frac{11}{2}(2)^2 + C$$

$$4 = \frac{11}{2}(4) + C$$

$$4 = 22 + C$$

$$C = -18$$

$$y = \frac{11}{2}x^2 - 18$$

ex:

$$\boxed{\frac{dy}{dx} = \frac{2x}{y}}$$

$$dx \cdot \frac{dy}{dx} = \frac{2x}{y} \cdot dx$$

y's & dy's ← → x's & dx's

$$y \cdot dy = \frac{2x}{y} \cdot dx \cdot y$$

(separate the x's & dx's from the y's & dy's)

$$y\,dy = 2x\,dx$$

$$\int y\,dy = \int 2x\,dx$$

$$\frac{y^2}{2} + C_1 = 2 \cdot \frac{x^2}{2} + C_2$$

$$\frac{y^2}{2} = x^2 + C_2 - C_1$$

$$\frac{y^2}{2} = x^2 + C$$

(solve for y ....)

$$y^2 = 2x^2 + K$$

$$y = \pm\sqrt{2x^2 + K}$$

$y^2 = 4$

$y = \pm 2$

ex: $\frac{dy}{dx} = 5x^4 \cdot y$

① SEP.

② INTEGR.

$\frac{1}{y}\, dx\, \frac{dy}{dx} = 5x^4 \cdot y \cdot dx \cdot \frac{1}{y}$

$\frac{1}{y}\, dy = 5x^4\, dx$

$\int \frac{1}{y}\, dy = \int 5x^4\, dx$

$\ln y = 5 \cdot \frac{x^5}{5} + C$

$\ln y = x^5 + C$

(solve for y . . .)

exponentiate both sides

$e^{\ln y} = e^{x^5 + C}$

$y = e^{x^5} \cdot e^{C}$ $e^{C} = A$

$$\boxed{y = A \cdot e^{x^5}}$$

$$\frac{dP}{dt} = K \cdot P$$

rate of change of population is directly proport. to population

P = population

SEP. DE.

$$dt\,\frac{dP}{dt} = K \cdot P \cdot dt$$

$$\frac{1}{P}\,dP = K \cdot P \cdot dt \cdot \frac{1}{P}$$

$$\frac{1}{P}\,dP = K \cdot dt$$

$$\int \frac{1}{P}\,dP = \int K \cdot dt$$

$$\ln P = Kt + C$$

$$e^{\ln P} = e^{Kt+C}$$

$$P = e^{Kt} \cdot e^{C} \qquad e^{C} = B$$

$$P = B \cdot e^{Kt} \qquad \begin{cases} t = 0 \\ P = P_0 \end{cases}$$

$$P_0 = B \cdot e^{K(0)}$$

$$P_0 = B$$

$$P = P_0 \cdot e^{Kt}$$

$$y' = 2x + xy$$

sep. x & y

$$dx \frac{dy}{dx} = x(2+y) \cdot dx$$

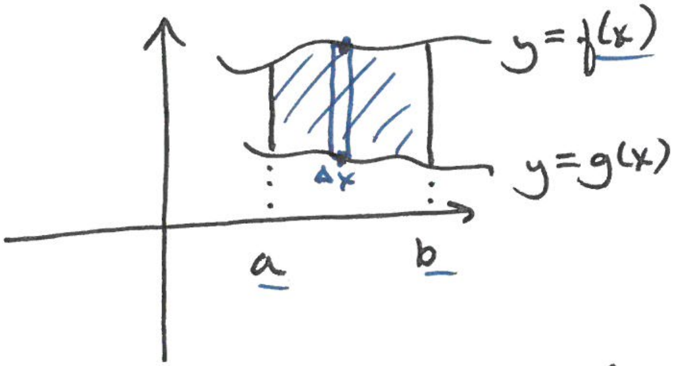
$$\frac{1}{2+y}\, dy = x(2+y) \cdot dx \, \frac{1}{2+y}$$

$$\int \frac{1}{2+y}\, dy = \int x \cdot dx$$

$$\vdots$$

MA121 — TEST #4:

4.4: • area between 2 curves:



$$A = \int_a^b [f(x) - g(x)]\,dx$$

• average value of a function

$$\text{AVE HT.} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

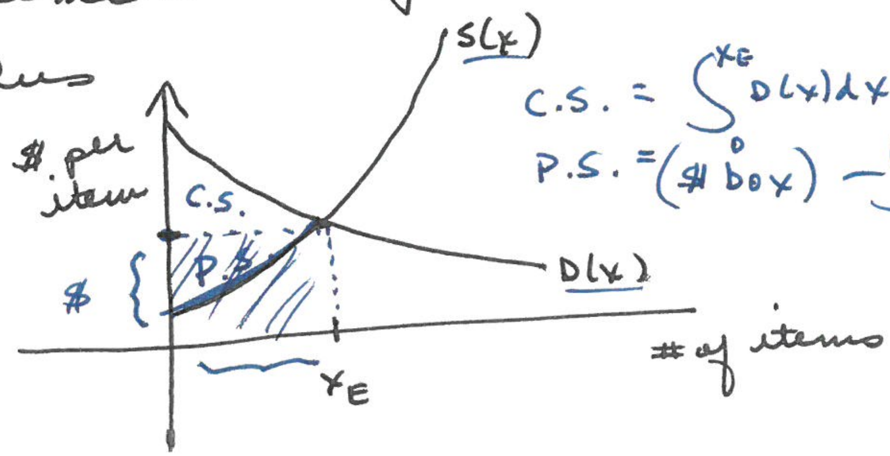
4.5: • integration using SUBSTITUTION ✓

let $u =$ ____

$du = 8t^4 \cdot dt$ $\qquad \frac{1}{8}\int \qquad t^4 \cdot dt\, 8$

$\frac{1}{8}\int u^4\,du$

5.1: • consumer surplus and producer surplus



$$C.S. = \int_0^{x_E} D(x)\,dx - \$\text{ box}$$

$$P.S. = (\$\text{ box}) - \int_0^{x_E} S(x)\,dx$$

EQ. PT: $D(x) = S(x)$

⋮

$x_E =$ ____ → $D(x_E)$ or $S(x_E)$

$ per item

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5.2: • accumulation models

① one-time contribution:

$$y = y_0 \, e^{kt}$$

② multiple (yearly) contributions:

$$\int_{T_0}^{T_1} y_0 \, e^{kt} \, dt \quad \overset{\text{ACCUM F.V.}}{=} \int_0^{24} 5000 \cdot e^{.031t} \, dt$$

(accumulated future value of a continuous income stream)

5.3: • improper integrals

$$\int_3^{\infty} f(x)\,dx \rightarrow \lim_{A \to \infty} \int_3^{A} f(x)\,dx$$

→ ① delay ∞ 'til end of prob. (lim ...)

② integrate

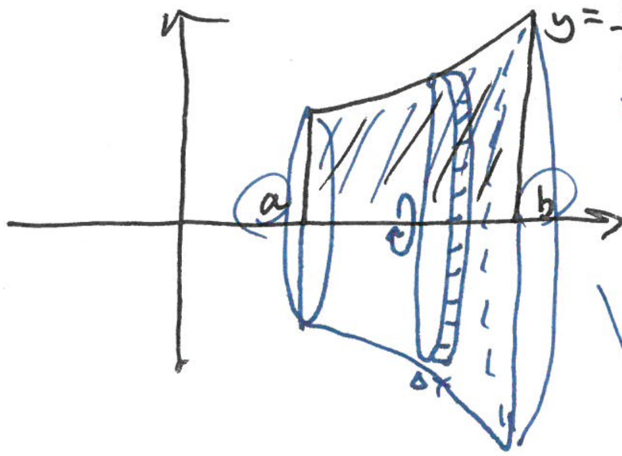
③ evaluate

④ simplify

→ ⑤ take limit

⑥ converge or diverge

5.6: • volumes of solids of revolution



$$V = \int \pi \cdot r^2 \cdot h$$

$r = f(x)$; $h = \Delta x \to dx$

$$V = \int_a^b \pi \left[f(x)\right]^2 \cdot dx$$

5.7: • separable differential equations

1. separate x's & dx's from y's & dy's.
2. integrate both sides ✓
3. if possible, solve for y ✓

QUIZ #3

NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA121 — J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

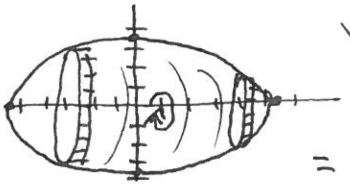
1.) Evaluate the improper integral $\int_2^{\infty} 7x^{-2}\,dx$. Does this integral converge or diverge?

$$\lim_{A\to\infty} 7\int_2^A x^{-2}\,dx = 7\cdot\lim_{A\to\infty}\left[\frac{x^{-1}}{-1}\right]_2^A = 7\cdot\lim_{A\to\infty}\left[\frac{-1}{x}\right]_2^A$$

$$= 7\cdot\lim_{A\to\infty}\left[\left(\frac{-1}{A}\right)-\left(\frac{-1}{2}\right)\right] = 7\cdot\lim_{A\to\infty}\left[\frac{1}{2}-\frac{1}{A}\right] = 7\cdot\frac{1}{2} = \frac{7}{2}$$

$\therefore$ integral converges

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5$ *for* $-5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.



$$\text{VOL} = \int_{-5.5}^{5.5} \pi (f(x))^2\,dx = \pi\int_{-5.5}^{5.5}(-.116x^2+3.5)^2\,dx$$

$$= \pi\int_{-5.5}^{5.5}(.013456x^4 - .812x^2 + 12.25)\,dx$$

$$= \pi\left[.013456\frac{x^5}{5} - .812\frac{x^3}{3} + 12.25x\right]_{-5.5}^{5.5} = \pi\left[\left(\frac{.013456}{5}(5.5)^5 - \frac{.812}{3}(5.5)^3 + (12.25)(5.5)\right) - \left(\frac{.013456}{5}(-5.5)^5 - \frac{.812}{3}(-5.5)^3 + (12.25)(-5.5)\right)\right]$$

$$\approx 71.774\pi \approx 225.48 \text{ in}^3$$

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of $200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?

$$\int_0^{34} 200{,}000\cdot e^{.029t}\,dt = 200{,}000\left[\frac{1}{.029}e^{.029t}\right]_0^{34}$$

$$= \frac{200{,}000}{.029}\left[e^{.029(34)} - e^{.029(0)}\right] = \frac{200{,}000}{.029}\left[2.68049 - 1\right]$$

$$\approx \$11{,}589{,}593.35$$