MA121 - 002

Thursday, December 6

- 6.3 today
- final exam review
- FINAL EXAM: Thursday, December 13
  1:00 - 4:00 pm SAS 2203
- do class evaluation, please
  (before 12/10 at 8:00 am)

MAX / MIN / SADDLE POINT FOR $f(x,y)$:

final exam

* 1.) find f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}

2.) set $f_x = 0$ & $f_y = 0$ $\Rightarrow$ (a,b) $\uparrow z$ $(a,b,f(a,b))$

3.) D TEST (2nd DERIV TEST)

$$D = f_{xx}(a,b) \cdot f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

a.) if $D>0$ & $f_{xx}(a,b)<0$
$\longrightarrow$ $(a,b,f(a,b))$ is a MAX

b.) if $D>0$ & $f_{xx}(a,b)>0$
$\longrightarrow$ $(a,b,f(a,b))$ is a MIN

c.) if $D<0$ $\longrightarrow$ $(a,b,f(a,b))$ is a SADDLE POINT

d.) if $D=0$ $\longrightarrow$ TEST FAILS

$f_x = 2x + y(1) + 0 - 7$

$f_x = \underline{2x + y - 7}$

① $f_{xx} = 2 + 0 - 0 = 2$

② $f_{xy} = 0 + 1 - 0 = 1$ ✓

$f_y = 0 + x + 4y - 0 = x + 4y$

$f_y = \underline{x + 4y}$

① $f_{yy} = 0 + 4 = 4$

② $f_{yx} = 1 + 0 = 1$

example:

$$f(x,y) = x^2 + xy + 2y^2 - 7x$$

1.) find $f_x, f_y, f_{xx}, f_{xy}, f_{yy}, f_{yx}$:

$$f_x = 2x + y - 7$$

$$f_y = x + 4y$$

$$f_{xx} = 2 \qquad f_{yy} = 4 \qquad f_{xy} = f_{yx} = 1$$

2.) solve $f_x = 0$ and $f_y = 0$

$2x + y - 7 = 0$ $\qquad$ $x + 4y = 0$

$x = -4y$

SUBST.

$$2(-4y) + y - 7 = 0$$
$$-8y + y - 7 = 0$$
$$-7y = 7$$
$$y = -1$$
$$x = -4y = -4(-1) = 4$$

$(4, -1)$ actually $(4, -1, f(4,-1))$

is a possible max / min / saddle

2 ✓

$(4, -1, -14)$

3.) D-TEST:

$$D = f_{xx}(4,-1) \cdot f_{yy}(4,-1) - \left[f_{xy}(4,-1)\right]^2$$

$$D = (2) \cdot (4) - 1^2$$

$$D = 7 > 0$$

4.) $D = 7$ and $f_{xx}(4,-1) = 2$

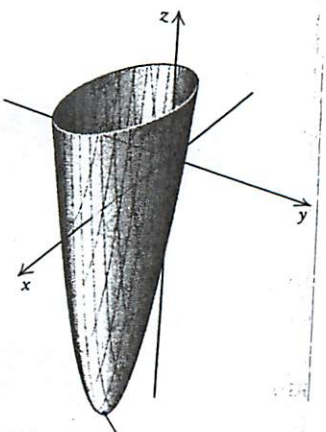
(since $D > 0$ and $f_{xx}(4,-1) > 0$, this is a relative MINIMUM.

$$f(+4,-1) = 4^2 + 4(-1) + 2(-1)^2 - 7 \cdot 4$$

$$= 16 - 4 + 2 - 28$$

$$= -14$$

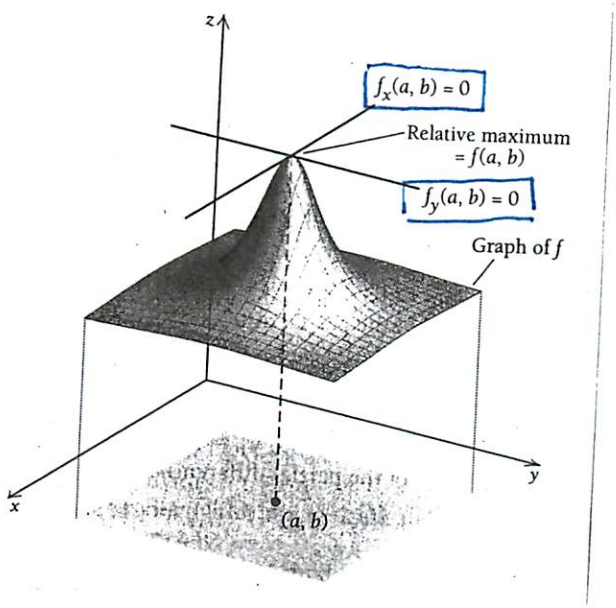
thus $(+4, -1, -14)$ is a relative MIN



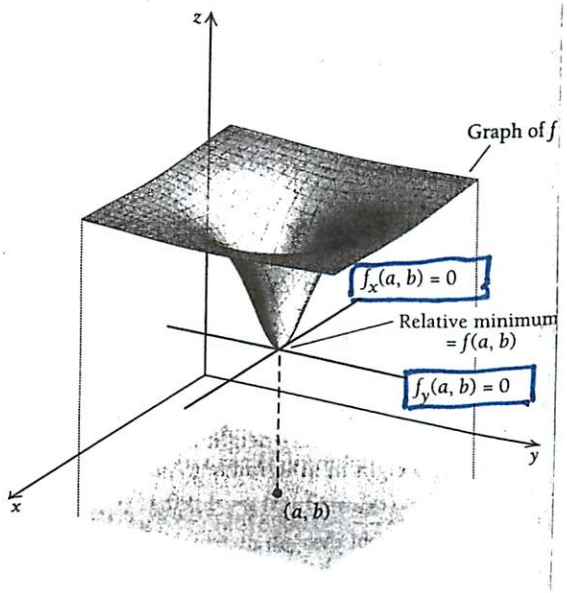
Relative minimum: $(4, -1, -14)$

$z = f(x, y) = x^2 + xy + 2y^2 - 7x$

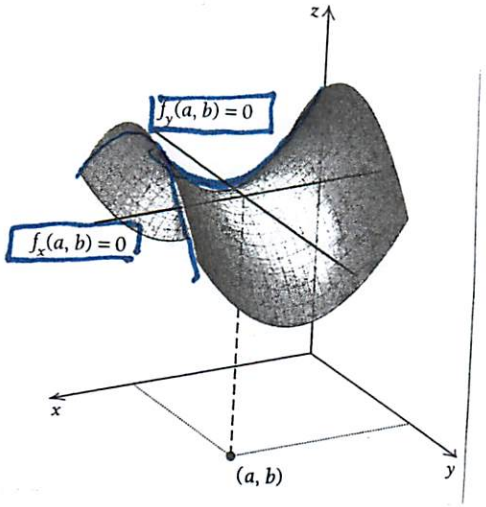
RELATIVE MAX:

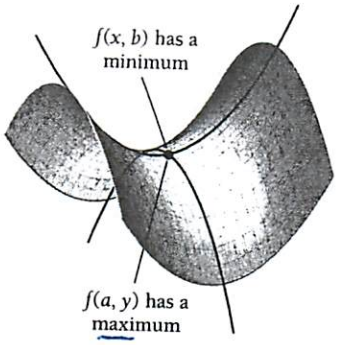


RELATIVE MIN:



SADDLE POINT:





MA 121 - 002

TEST #4 RESULTS

A's	71 (37.9%)	62.5%
B's	46 (24.6%)	
C's	32 (17.1%)	
D's	18 (9.6%)	20.3%
F's	20 (10.7%)	

AVE: 80.69