Wednesday, September 26

today: $\dfrac{1.7 \text{ (chain rule)}}{1.8 \text{ (higher order deriv.)}}$

(T2: prod & quotient rules)

1.7: CHAIN RULE

① EXTENDED POWER RULE:

$$f(x) = 8x^3 - 5x^2 + 11x - 7$$
$$f'(x) = 8(3 \cdot x^2) - 5(2 \cdot x) + 11(1) - 0$$

$$g(x) = (3x+1)^7$$

"inside" function

$$y = [f(x)]^n$$
$$y' = n \cdot [f(x)]^{n-1} \cdot f'(x)$$

DERIV. OF THE "INSIDE" FUNCTION

$$g(x) = (3x+1)^7$$
$$g'(x) = 7 \cdot (3x+1)^6 \cdot d(3x+1)$$
$$g'(x) = 7 \cdot (3x+1)^6 \cdot 3 = 21(3x+1)^6$$

M TAN   INST. RATE OF CHANGE

$$y = (3x+1)^7$$

$$y = (3x+1)(3x+1)(3x+1)(3x+1)(3x+1)(3x+1)(3x+1)$$

chain rule:

$$y' = 7(3x+1)^6 \cdot 3$$

② COMPOSITION OF FUNCTIONS:

$$y = (f \circ g)(x) = f(g(x)) \qquad g(x) \text{ "INSIDE" FUNCTION}$$

$$y = f(g(x))$$

$$y' = \boxed{f'(g(x))} \cdot g'(x)$$

$$f(x) = \bigcirc^7 \qquad g(x) = 3x+1$$

$$y = (f \circ g)(x) = f(g(x))$$

$$y = f(3x+1) = (3x+1)^7$$

$$y' = 7(3x+1)^6 \cdot 3$$

③ y in terms of u:

$$y = 8 + u^3 \qquad \frac{dy}{du} = 0 + 3u^2$$

u in terms of x

$$u = x^2 + 5 \qquad \frac{du}{dx} = 2x + 0$$

find y': (find f'(x))

$$\left(\text{find } \frac{dy}{dx} \right) \quad \begin{array}{c} \text{DERIV OF Y} \\ \text{WITH RESPECT} \\ \text{TO X} \end{array}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (3u^2) \cdot (2x)$$

$$\frac{dy}{dx} = 3 \cdot (x^2+5)^2 \cdot (2x)$$

$$\frac{dy}{dx} = 6x(x^2+5)^2$$

$$y = 8 + u^3 \qquad u = \boxed{x^2+5}$$

$$y = 8 + (x^2+5)^3$$

now find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = 3(x^2+5)^2 \cdot (2x)$$

$$\frac{dy}{dx} = (6x)(x^2+5)^2$$

find the equation of the
tangent line to the
curve $\boxed{y = \sqrt{x^2 + 3x}}$ at the
point $(\underline{1}, 2)$ $\searrow$ $\underline{y = (x^2 + 3x)^{\frac{1}{2}}}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 2 = \frac{5}{4}(x - 1)}$$

$M_{TAN} = y' = ?$

$y' = \frac{1}{2}(x^2 + 3x)^{-\frac{1}{2}} \cdot (2x + 3)$

y' at $x = 1$:

$$y' = \frac{2x + 3}{2\sqrt{x^2 + 3x}}$$

$$y' = \frac{2(1) + 3}{2\sqrt{1^2 + 3(1)}} = \frac{5}{4}$$

V. Bo:

$$g(x) = \boxed{(2x - 1)^4} \cdot \boxed{(x^2 + x + 1)^7}$$

prod. rule ..... then use
the chain rule

$$g'(x) = (2x - 1)^4 \cdot \left[7 \cdot (x^2 + x + 1)^6 \cdot (2x + 1) \right]$$
$$+ (x^2 + x + 1)^7 \cdot \left[4 \cdot (2x - 1)^3 \cdot 2 \right]$$

$$h(x) = \sqrt[3]{\frac{x-3}{x+4}} = \left(\frac{x-3}{x+4}\right)^{1/3}$$

$$\left(\underset{\cdots\cdots\cdot}{\underbrace{\text{\textbardbl\textbardbl\textbardbl}}}\right)^{1/3}$$

$$\text{deriv: } \frac{1}{3}\left(\text{\textbardbl\textbardbl\textbardbl}\right)^{-2/3} \cdot d(\text{\textbardbl})$$

begin with chain rule . . . . .

then use quat. rule

$$h(x) = \left(\frac{x-3}{x+4}\right)^{1/3}$$

deriv of $\frac{x-3}{x+4}$

$$h'(x) = \frac{1}{3}\left(\frac{x-3}{x+4}\right)^{-2/3} \cdot \left[\frac{\overset{x+4}{(x+4)}(1) - \overset{-x+3}{(x-3)(1)}}{(x+4)^2}\right]$$

$$h'(x) = \frac{1}{3}\left(\frac{x-3}{x+4}\right)^{-2/3} \cdot \frac{7}{(x+4)^2}$$

1.8: HIGHER ORDER DERIV.

$$f(x) = 3x^5 - 8x^3 + 14x$$

$$f'(x) = 15x^4 - 24x^2 + 14$$

$$f''(x) = 60x^3 - 48x$$

$$f'''(x) = 180x^2 - 48$$

$$f^{(7)}(x) = 7^{th} \text{ DERIV.}$$

$$\frac{dy}{dx} \; ; \; \frac{d^2y}{dx^2} \; ; \; \frac{d^3y}{dx^3} \; ; \quad \cdots$$

$$S(t) = -16t^2 + 84t + 50 \checkmark$$
$$\text{(free falling object)} \qquad t: \text{TIME (sec)}$$

$$S(0) = -16(0)^2 + 84(0) + 50 \qquad S(t): \text{DIST; HT; POS}$$
$$\text{initial pos (at } t=0) \qquad\qquad \text{(FT.)}$$
$$S(0) = 50 \text{ FT.}$$
$$S(1) = -16(1)^2 + 84(1) + 50 = -16 + 84 + 50 = 118 \text{ FT.}$$

$$S'(t) = V(t) = -32t + 84$$
$$V(0) = -32(0) + 84 = 84 \text{ }^{FT}/_{SEC}$$
$$\text{initial vel.}$$
$$V(1) = -32(1) + 84 = 52 \text{ }^{FT}/_{SEC}$$

$$v(t) = -32t + 84$$

$$s''(t) = v'(t) = a(t) = -32 \frac{FT/SEC}{SEC} = -32 \left(\frac{FT}{SEC^2} \right)$$

GRAVITY

$$a(0) = -32 \frac{FT/SEC}{SEC}$$

$$a(1) = -32 \frac{FT/SEC}{SEC}$$

$$s(t) \ \cdots \overset{DERIV}{\cdots} \ v(t) \ \cdots \overset{DERIV}{\cdots} \ a(t)$$

Quiz #1: Interview; documented with a **1 -2 page paper**

1.) paper should be typed (word processed), not handwritten
2.) Q & A format is OK; paragraph format is OK
3.) Interview a person (possibly a parent; not a student) who is currently doing for a living what you would eventually like to do for a living (based on your current major – or desired major)
4.) Face-to-face OK; Phone or skype OK; email OK
5.) One question you **MUST** ask: How is math used in this career?
6.) Other suggested questions you will probably want to ask: Average salary? Stress level? Is it seasonal (lots of work at certain times of the year) or fairly steady? Work/Life balance? Lifestyle/perks? Do you find your job rewarding? Is there continuing education associated with this job? Travel associated with this job? Do you have any summer jobs or internships for college students? Did your degree adequately prepare you for this job?   And many, many other questions specific to your interviewee….
7.) DUE DATE: Monday, October 8, at the beginning of class