MA 121 - 003

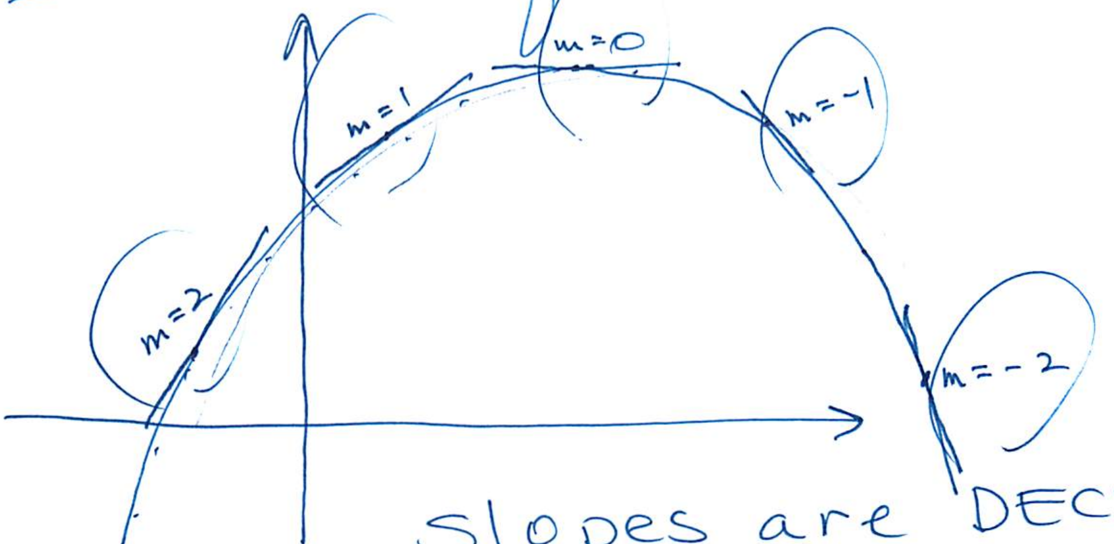
Wednesday, October 3

TEST #2

WED OCT 10^{th} (not 2.5)

2.1: y': (1) $y' = 0$ (FLAT)
(2) y' undef (STEEP)
} critical points
(3) $y' = +$; $y' = -$ (INCR ; DECR)

2.2: USING $f''(x)$:



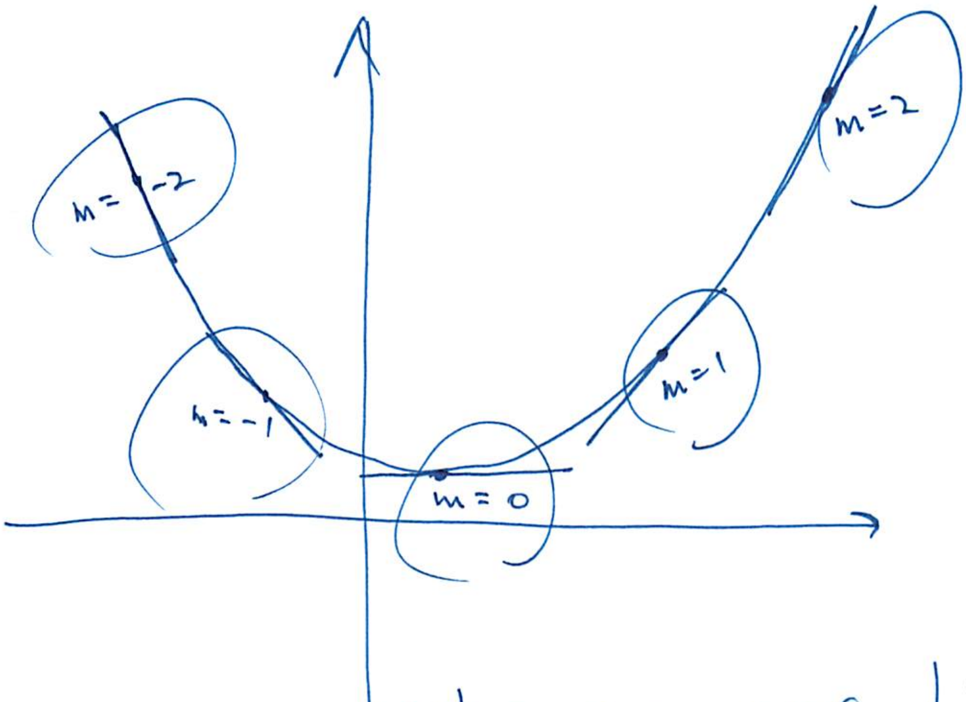
slopes are DECREASING

$m_{TAN} = f'(x)$

$f'(x) = +$ $\longleftrightarrow$ $f(x)$ INCREASING

* $\boxed{f''(x) = -}$ $\longleftrightarrow$ $f'(x)$ DECREASING

* if $f''(x) =$ NEG, then the curve is CONCAVE DOWN.



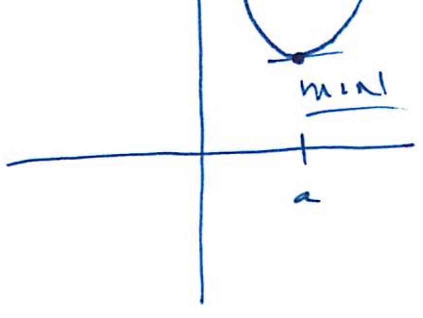
Slopes are INCREASING

$\uparrow$

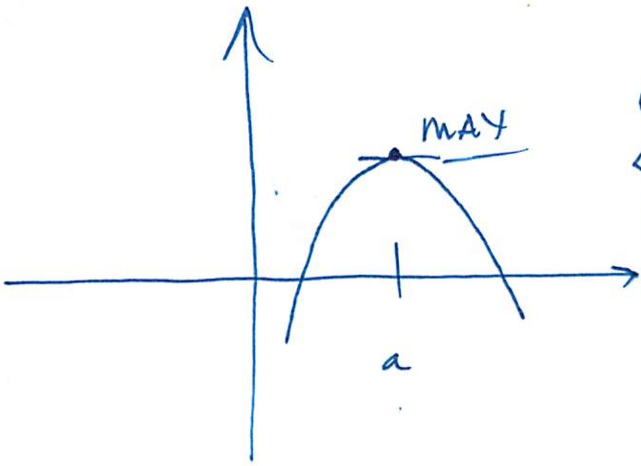
$m_{TAN} = f'(x)$

$f''(x) = +$ $\longleftrightarrow$ $f'(x)$ INCREASING

if $f''(x) = +$, then the function is CONCAVE UP (C. UP)



$$\begin{cases} f'(a) = 0 \\ f''(a) \text{ is } + \end{cases}$$



$$\begin{cases} f'(a) = 0 \\ f''(a) \text{ is NEG} \end{cases}$$

1. find $f''(x)$:
2. $f''(x) = 0$ or $f''(x)$ undef
3. chart: $f''(x)$: x_1 x_2

&
CONCAVITY

ex: $f(x) = 2x^3 - 3x^2 - 36x + 28$

$f'(x) = 6x^2 - 6x - 36$

$\boxed{f'(x) = 0} = 6(x^2 - x - 6) = \boxed{6(x-3)(x+2)} = 0$

(~~$f'(x)$ undef~~)

$x - 3 = 0$ $\qquad$ $x + 2 = 0$

$x = 3$ $\qquad$ $x = -2$

$(3, -53) = (3, ?)$ $\qquad$ $(-2, ?) = (-2, 72)$

82 - 135

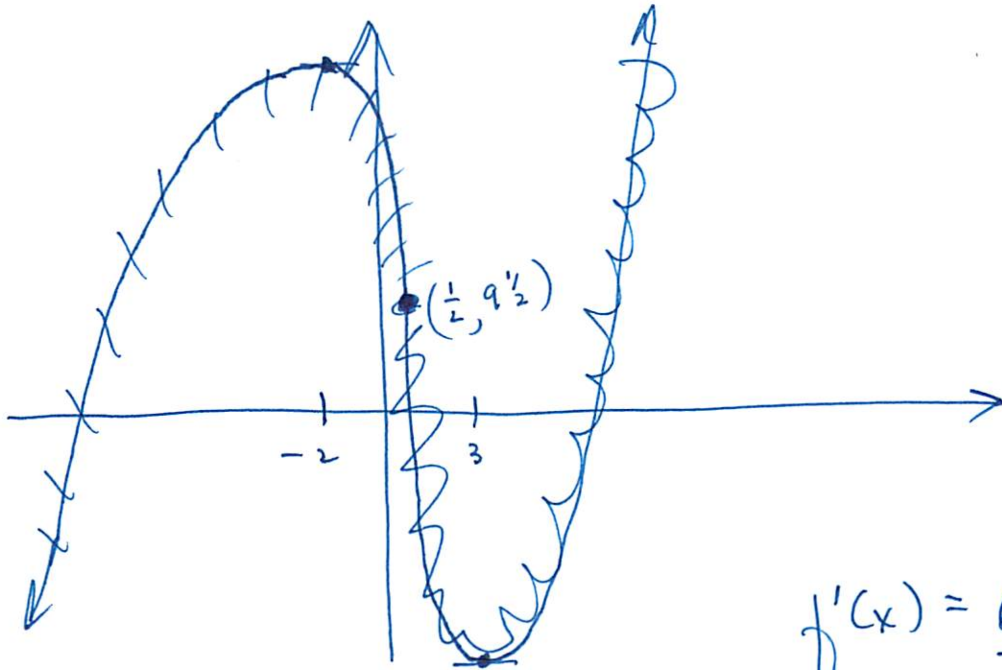
$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 28 = 54 - 27 - 108 + 28 = -53$

$f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 28 = -16 - 12 + 72 + 28 = 72$

$f'(x)$:

$f'(-3) = + \cdot - \cdot - = +$ $\qquad$ $f'(0) = + \cdot - \cdot + = -$ $\qquad$ $f'(4) = + \cdot + \cdot + = +$

$f(x)$ INCR -2 $\quad$ $f(x)$ DECR 3 $\quad$ $f(x)$ INCR



$f'(x) = 6x^2 - 6x - 36$

$\cancel{(f''(x) \text{ undef})}$

$f''(x) = \boxed{12x - 6} = 0$

$12x = 6$

$x = \frac{6}{12} = \frac{1}{2}$

$(\frac{1}{2}, ?) = (\frac{1}{2}, 9\frac{1}{2})$

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 36(\frac{1}{2}) + 28$

$= \frac{1}{4} - \frac{3}{4} - 18 + 28 = 10 - \frac{1}{2} = 9\frac{1}{2}$

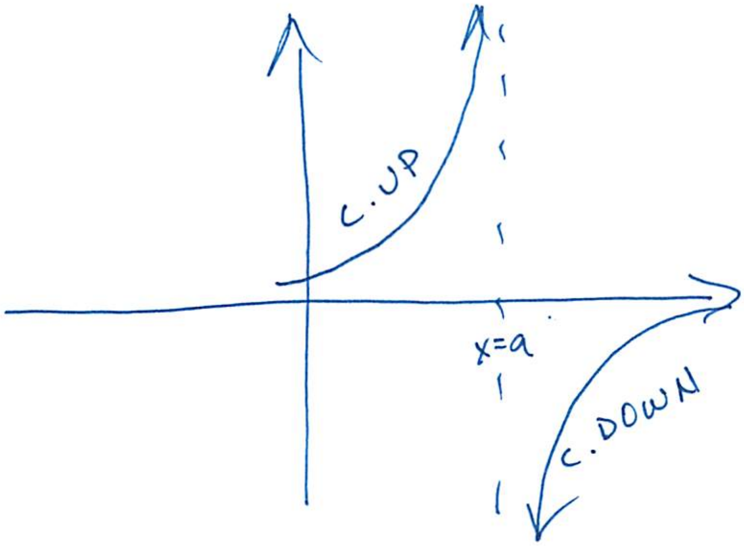
$f''(x)$:

$f''(0) = -$ | $\frac{1}{2}$ | $f''(1) = +$

$f(x)$ CONCAVE DOWN | | $f(x)$ CONCAVE UP

change in CONCAVITY at $(\frac{1}{2}, 9\frac{1}{2})$

→ point of inflection



no point of INFL.

non-polynom.

$f(x) = (x+1)^{2/3}$

$\boxed{f'(x) = \frac{2}{3}(x+1)^{-1/3}} \cdot (1) = \frac{2}{3 \cdot \sqrt[3]{x+1}}$

① $f'(x) = \boxed{0 \neq \frac{2}{3 \cdot \sqrt[3]{x+1}}}$

no flat places

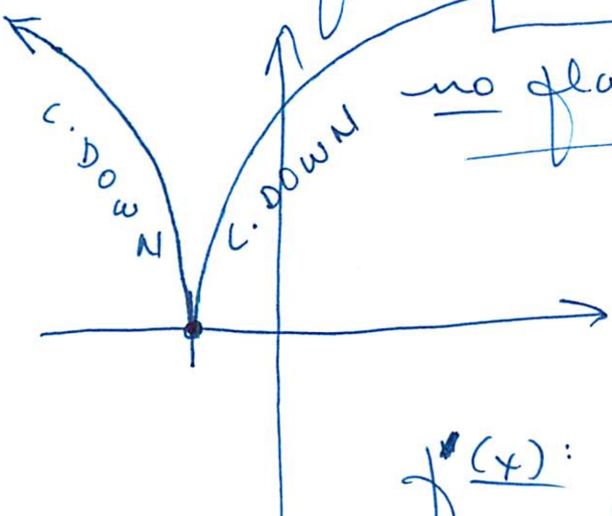
② $f'(x)$ undef

$\frac{2}{3 \cdot \sqrt[3]{x+1}}$ undef

when $x = -1$

$(-1, ?) = (-1, 0)$

$f(-1) = (-1+1)^{2/3} = 0$



$f'(x)$:

$f'(-2) = \frac{+}{+ \cdot -} = -$ | $f'(0) = \frac{+}{+ \cdot +} = +$

$f(x)$ DECR $\cdot\, -1$ | $f(x)$ INCR.

$f''(x)$:

$f''(x) = \frac{2}{3} \cdot -\frac{1}{3}(x+1)^{-4/3}(1) = \frac{-2}{9\left[\sqrt[3]{x+1}\right]^4}$

$f''(x) = 0 \neq \frac{-2}{9 \cdot \left[\sqrt[3]{x+1}\right]^4}$

$f''(x)$ undef

$\frac{-2}{9\left[\sqrt[3]{x+1}\right]^4}$ undef when $x = -1$

$(-1, 0)$

$$f''(x) = \frac{-2}{9 \cdot \left[\sqrt[3]{x+1}\,\right]^4} = \frac{-}{+ \cdot +} = -$$

$\underline{f''(x):}$

$f''(-2) = -$ | $f''(0) = -$

$f(x)$ C. DOWN -1 $f(x)$ C. DOWN

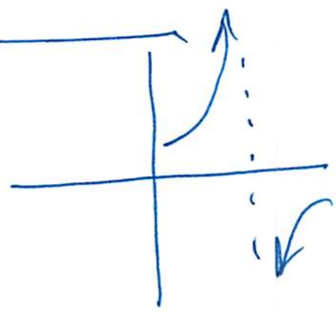
$(-1, 0)$ is <u>not</u> a point of infl.

2.3: COMPREHENSIVE GRAPHING

(1) VERTICAL ASYMP:

$$y = \frac{f(x)}{g(x)}$$ no LIKE factors

$g(x) = 0 \qquad x = K$



$$y = \frac{5}{x-4} \qquad \text{V.A.: } x = 4$$

(2) "HOLE" IN THE GRAPH:

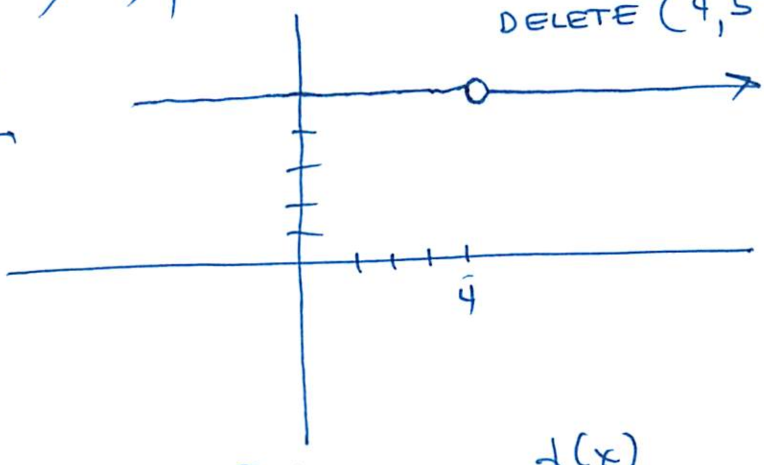
$x = 4.001$

$$\boxed{y =} \frac{5\cancel{(x-4)}}{\cancel{(x-4)}} \qquad x \neq 4$$

(no vertical asymp.)

DELETE (4,5)

$y = 5$



(3) HORIZONTAL ASYMP: $y = \frac{f(x)}{g(x)}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = R \qquad y = R \text{ is H.A.}$$

$$y = \frac{2x-1}{5x+2} \qquad \text{V.A.: } x = -2/5 \qquad \text{H.A.: } y = 2/5$$

$$\lim_{x\to\infty} \frac{2x-1}{5x+2} = 2/3$$

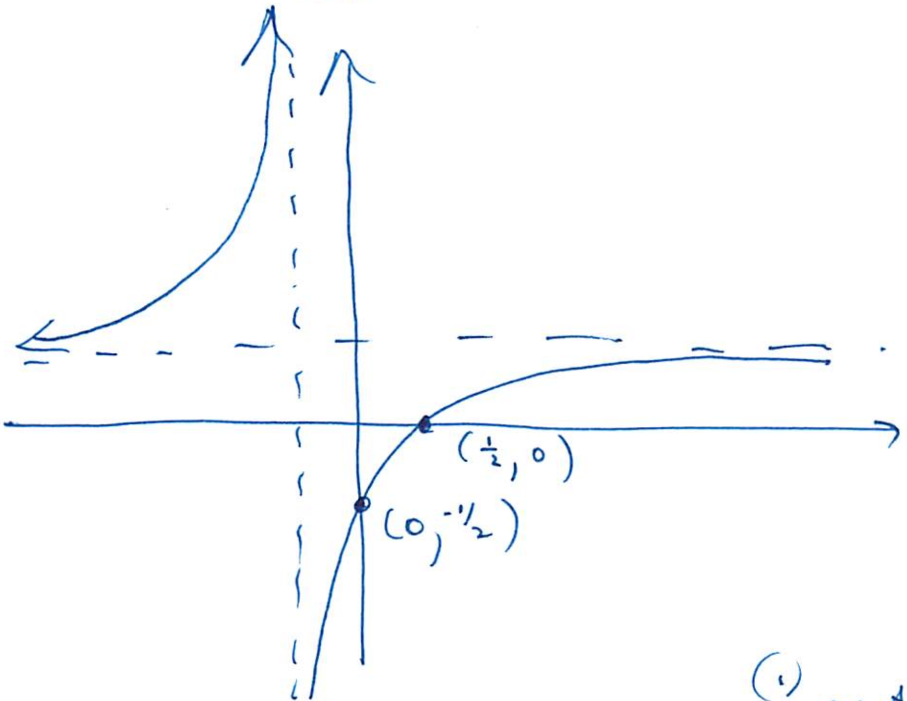
$$y = \frac{2x-1}{5x+2}$$

$$y' = \frac{(5x+2)\cdot 2 - (2x-1)\cdot 5}{(5x+2)^2}$$

$$y' = \frac{\cancel{10x}+4-\cancel{10x}+5}{(5x+2)^2}$$

$$y' = \frac{9}{(5x+2)^2} = +$$

$f(x)$ INCR.



(4) INTERCEPTS: (1) set $x=0$ $(0, ?)$
(2) set $y=0$ $(?, 0)$

$$y = \frac{2x-1}{5x+2} \quad (0, \frac{-1}{2}) \quad (\frac{1}{2}, 0)$$

$$0 = \frac{2x-1}{5x+2} \text{ when } x = \frac{1}{2}$$

(5) SLANT (OBLIQUE) ASYMPTOTES

$$y = \frac{x^2+2x-3}{x+5}$$

H.A.:

$$\lim_{x\to\infty} \frac{x^2+2x-3}{x+5} = \text{DNE}$$

when the degree of the numerator is ONE larger than the degree of the DENOM.

$$x - 3 + \frac{12}{x+5}$$

$$x+5 \overline{\smash{\big)}\, x^2 + 2x - 3}$$

$$-(x^2 + 5x)$$

$$-3x - 3$$

$$-(-3x - 15)$$

$$12$$

$$\frac{x^2+2x-3}{x+5} = \underline{x-3} + \cancel{\frac{12}{x+5}} \to 0$$

$x \to \infty$

$$\boxed{y = x - 3}$$

slant asymptote

$$f(x) = \frac{x^2+2x-3}{x+5}$$