

MA 121-003

Wednesday, October 17

BEG:  $\$1,600,000^{00}$

①

ex: F'BALL TICKET REVENUE

let  $x =$  the # of times price is reduced

$$REV = \left( \begin{array}{c} \text{ticket price} \\ 40 - 1 \cdot x \end{array} \right) \cdot \left( \begin{array}{c} \text{number attending} \\ 40,000 + 2,000x \end{array} \right)$$

$$R(x) = (40 - x)(40,000 + 2,000x)$$

$$R(x) = 1,600,000 + 80,000x - 40,000x - 2,000x^2$$

$$R(x) = 1,600,000 + 40,000x - 2,000x^2$$

$$R'(x) = 0 + 40,000 - 4,000x = 0$$

$$40,000 - 4,000x = 0$$

$$\frac{40,000}{4,000} = \frac{4,000x}{4,000}$$

$$10 = x$$

ticket price:  $(40 - x)$   
 $40 - 10 = 30^{00}$

attending game  $(40,000 + 2,000(10))$

$$40,000 + 20,000 = 60,000$$

$$REV: (30^{00})(60,000) = \underline{\underline{\$1,800,000^{00}}}$$

(\$200,000 more in REV)

Since  $r > 0$  we consider only the positive root.

$$r_1 = \sqrt{\frac{A_0}{6\pi}}$$

The second derivative  $V''(r) = -6\pi r$  is negative on the interval  $(0, \infty)$ . Since this interval is our domain of interest, we can apply Theorem 1 to conclude that  $r_1$  is the only critical number of interest and that it is the location of the global maximum. Hence the global maximum volume is

$$V_{max} = V(r_1) = -\pi(r_1)^3 + \frac{1}{2}A_0r_1 = -\pi\left(\sqrt{\frac{A_0}{6\pi}}\right)^3 + \frac{1}{2}A_0\sqrt{\frac{A_0}{6\pi}}$$

Using this value we find the corresponding  $h_1$  to be

$$h_1 = \frac{A_0 - 2\pi r_1^2}{2\pi r_1} = \frac{A_0 - 2\pi\left(\sqrt{\frac{A_0}{6\pi}}\right)^2}{2\pi\sqrt{\frac{A_0}{6\pi}}} = \sqrt{\frac{2A_0}{3\pi}} = \sqrt{\frac{2}{2} \frac{2A_0}{3\pi}} = \sqrt{\frac{4A_0}{6\pi}} = 2\sqrt{\frac{A_0}{6\pi}} = 2r_1$$

Thus the volume of a cylinder is maximized for a fixed surface area when the height is twice the radius.

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**Example 25.** When the ticket price is \$40, the average attendance at the football game is 40,000 people. It has been determined that for every \$1 decrease in the ticket price, an additional 2000 people will purchase tickets and attend the game. Under this arrangement, what price should be charged per ticket to maximize the revenue for the university? How many fans will attend the game at this price? What is the maximum revenue?

(from previous day)

45.) MIN. INVENTORY

COST:

lot size:  $X$



$$\text{COST} = \left( \frac{800}{1} \right) \left( \frac{X}{2} \right) + (5 \cdot X + 10) \left( \frac{360}{X} \right)$$

↑  
ASSUME

# of times to order

$$\frac{N}{1} \cdot \frac{X}{1} = 360$$

$$N = ? = \frac{360}{X}$$

$$N = \frac{360}{30} = 12$$

$3600 X^{-1}$

$$C(X) = 4X + 1800 + \frac{3600}{X}$$

$$C'(X) = 4 + 0 + \frac{-3600}{X^2}$$

$$4 - \frac{3600}{X^2} = 0$$

$-3600 X^{-2}$

$$\frac{4}{1} = \frac{3600}{X^2}$$

$$C''(X) = 0 + \frac{7200}{X^3}$$

$$\frac{3600}{4} = \frac{X^2}{X}$$

$$900 = X^2$$

$$\underline{30 = X}$$

$$C''(X) = \frac{7200}{X^3} = +$$

∴ CONCAVE UP  
∴ MIN

CH 3:

exponential functions:

$y = 2^x$  ;  $y = 5^x$  ;  $y = e^x$  ;  $y = 10^x$

NATURAL EXPONENTIAL FUNCTION:

base "e"

$$e = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k$$

$k = 1,000$

$\left(1 + \frac{1}{1000}\right)^{1000} \approx 2.7169$        $e \approx 2.718$

$k = 100,000$

$\left(1 + \frac{1}{100,000}\right)^{100,000} \approx 2.7182$

$k = 10,000,000$

$\left(1 + \frac{1}{10,000,000}\right)^{10,000,000} \approx \boxed{2.718}$

$y = e^x$

$$y = e^x$$

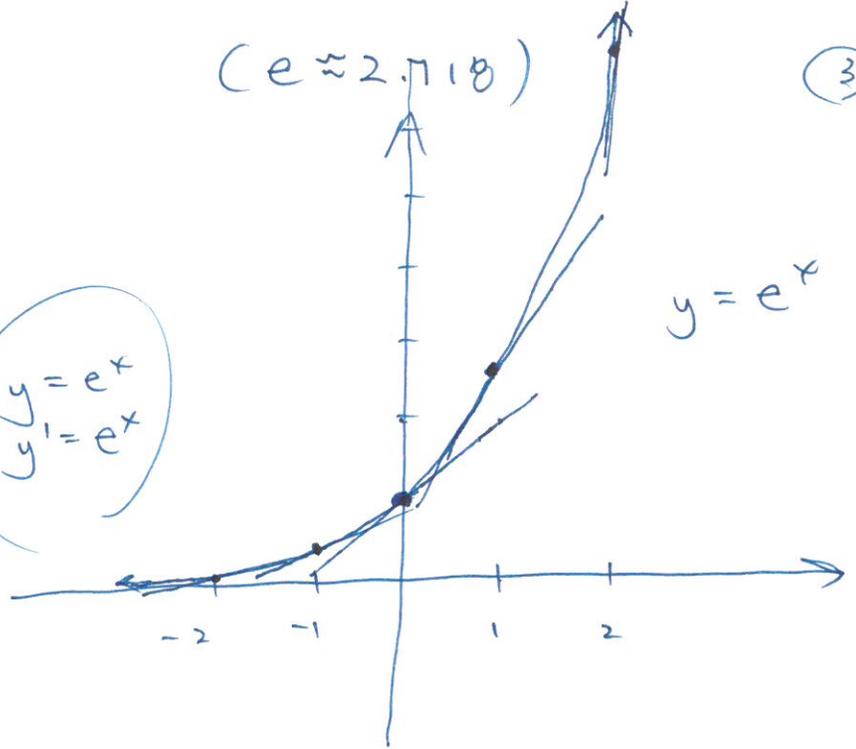
$$(e \approx 2.718)$$

3

x	y
-2	$e^{-2} \approx .135$
-1	$e^{-1} \approx .367$
0	$e^0 = 1$
1	$e^1 \approx 2.718$
2	$e^2 \approx 7.39$

$$y = e^x$$

$$y' = e^x$$



DERIVATIVE OF

$$y = e^x$$

$$y' = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$y' = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

$$y' = \lim_{h \rightarrow 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$y' = e^x \cdot \lim_{h \rightarrow 0} \frac{e^h - 1}{h}$$

$$y' = e^x \cdot \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

$$y' = e^x \cdot 1 = e^x$$

$$\frac{e^{.01} - 1}{.01} \approx 1.005$$

$$\frac{e^{.00001} - 1}{.00001} \approx 1.000005$$

$h \rightarrow 0$

$$\begin{aligned} \frac{dy}{dx} &= e^x \\ \frac{dy}{dx} &= e^x \end{aligned}$$

$$\begin{aligned} y &= e^{f(x)} \\ y' &= e^{f(x)} \cdot f'(x) \end{aligned} \quad \left. \vphantom{\begin{aligned} y &= e^{f(x)} \\ y' &= e^{f(x)} \cdot f'(x) \end{aligned}} \right) \text{chain rule}$$

(4)

$$\begin{aligned} y &= e^{5x} \\ y' &= e^{5x} \cdot \underline{d(5x)} = e^{5x} \cdot 5 = 5 \cdot e^{5x} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= e^{x^2} \\ \frac{dy}{dx} &= e^{x^2} \cdot (2x) = 2x e^{x^2} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= e^{\frac{1}{x}} \\ \frac{dy}{dx} &= e^{\frac{1}{x}} \cdot (-1 \cdot x^{-2}) \\ \frac{dy}{dx} &= \frac{-1}{x^2} \cdot e^{\frac{1}{x}} = \frac{-e^{\frac{1}{x}}}{x^2} \end{aligned} \quad \begin{aligned} \frac{1}{x} &= x^{-1} \\ d(x^{-1}) &= -1 \cdot x^{-2} \\ &= \frac{-1}{x^2} \end{aligned}$$

$$\begin{aligned} y &= \underline{x \cdot e^x} \\ y' &= \underline{x \cdot e^x} + \underline{e^x \cdot (1)} = e^x(x+1) \end{aligned}$$

$$y = e^x$$

INVERSE: (switch  $x \leftrightarrow y$ )

$$\underline{x = e^y}$$

(solve for  $y \dots$ )

$y$  is the power to which "e" is raised to get  $x$

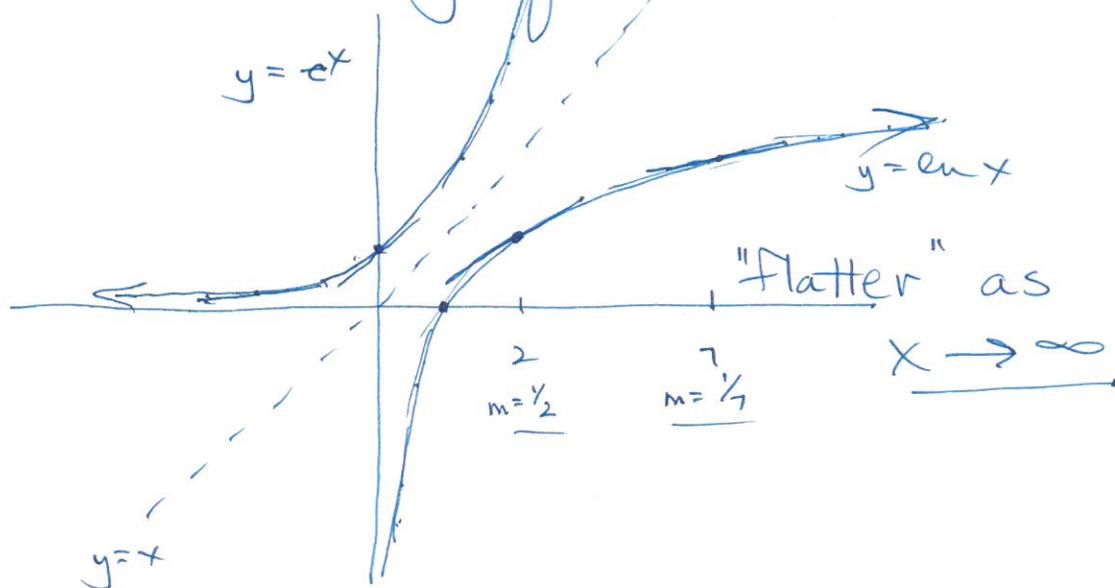
$$y = \log_e x$$

$$\boxed{y = \ln x}$$

← INVERSE OF

$$\boxed{y = e^x}$$

natural log. function



$$y = \ln x \quad y' = ??? = \frac{1}{x}$$

(6)

$$f(x) = \ln(x)$$

rewrite:

$$e^{f(x)} = x$$

find  $f'(x)$

$$\frac{e^{f(x)} \cdot f'(x)}{e^{f(x)}} = \frac{1}{e^{f(x)}}$$

$$f'(x) = \frac{1}{e^{f(x)}} = \frac{1}{x}$$

$$y = \ln(x)$$

$$y' = \frac{1}{x} \checkmark$$

$$\boxed{y = \ln(4x)} \rightarrow y = \boxed{\ln 4} + \ln x$$
$$y' = \frac{1}{4x} \cdot 4 = \frac{1}{x} \quad y' = 0 + \frac{1}{x}$$

$$y = \ln(8x^2 - x + 11)$$

$$y' = \frac{1}{8x^2 - x + 11} \cdot (16x - 1) = \frac{16x - 1}{8x^2 - x + 11}$$

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10/17/18

TEST # 2 RESULTS

A's	<u>82</u>	<u>(37.8%)</u>	}	<u>52.5%</u>
B's	<u>32</u>	<u>(14.7%)</u>		
C's	<u>45</u>	<u>(20.7%)</u>		
D's	<u>21</u>	<u>(9.7%)</u>	}	<u>26.7%</u>
F's	<u>37</u>	<u>(17.1%)</u>		

AVE: 78.438