MA121-003

Monday, October 22

10/22 : 3.3 ; 3.4
10/24 : 3.5 ; 4.1

10/29 : 4.2 ; 4.3
10/31 : 4.3 ; review

11/5 : TEST #3

LOGS & EXPONENTIALS :

$$\underline{e}^{.08t} = 143$$

(solve for t)

nat. log of both sides

$$\underline{\ln e^{.08t}} = \ln 143$$

$$\boxed{.08t = \ln 143}$$

$$\frac{\cancel{.08}t}{\cancel{.08}} = \frac{\ln 143}{.08} \approx \_\_\_\_\_\_$$

$y = \frac{\boxed{\ln x}}{x^2}$ DERIV find y':

$$y' = \frac{(x^2)\cdot\frac{1}{x} - (\ln x)(2x)}{[x^2]^2}$$

$$y' = \frac{x - 2x\cdot\ln x}{x^4} = \frac{\cancel{x}(1 - 2\ln x)}{\cancel{x}\cdot x^3}$$

$$\boxed{y' = \frac{1 - ②\ln x}{x^3}}\ \square$$

$-\ln x^{②}$ $\square$

$-2\ln x$

$y = \ln\left(\frac{x^2+x}{x+3}\right)$ $y' = ??$

rewrite . . .

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$y = \ln(x^2+x) - \ln(x+3)$

$y' = \frac{1}{x^2+x}\cdot(2x+1) - \frac{1}{x+3}(1)$

$y' = \frac{2x+1}{x^2+x} - \frac{1}{x+3}$

(4)

$y = \frac{x^2}{\ln x}$ [illegible] [illegible] y :

$$y' = \frac{(x^2)\cdot\frac{1}{x} - (\ln x)(2x)}{[x^2]^2}$$

$$y' = \frac{x - 2x\cdot\ln x}{x^4} = \frac{\cancel{x}(1 - 2\ln x)}{\cancel{x}\cdot x^3}$$

$$y' = \boxed{\frac{1 - \textcircled{2}\ln x}{x^3}}$$

$-\ln x^2$

$-2\ln x$

$y = \ln\left(\frac{x^2+x}{x+3}\right)$ $y' = ??$

$= \ln\left(\frac{a}{b}\right)$ [illegible] . . .

$y = \ln(x^2+x) - \ln(x+3)$ $\ln a - \ln b$

$$y' = \frac{1}{(x^2+x)}\cdot(2x+1) - \frac{1}{x+3}(1)$$

$$y' = \left(\frac{2x+1}{x^2+x}\right) - \left[\frac{1}{x+3}\right]$$

$$y = \ln x \qquad y' = \frac{1}{x}\cdot 1$$

$$y = \ln(x^2+x) \qquad y' = \frac{1}{x^2+x}(2x+1)$$

$\underline{3.3}$: exponential growth (BASE "e")

growing exp: its rate of growth is directly prop. to the amount present at any time

$$\boxed{\frac{dy}{dt} = k\cdot y}$$

(solve for y)

$$\boxed{y = \underline{C}\cdot \underline{e^{kt}}}$$

check:

$$y' = \frac{dy}{dt} = C\cdot\left[e^{kt}\cdot k\right] = k\cdot(C\cdot e^{kt})$$

$$\frac{dy}{dt} = k\cdot y$$

exp GROWTH: $y = C\cdot e^{kt}$

at time zero $(t=0)$ amount of y y_0

$$y_0 = C\cdot e^{k(0)}$$

$$y_0 = C\cdot 1 \qquad y_0 = C$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$** \ \boxed{y = \underset{\uparrow}{y_0}\, e^{kt}} \ **$$

Initial amount of y

(y-value when $t=0$)

"standard" exponential growth/decay

$$P = \underline{P_0} \cdot e^{kt}$$

$$y = y_0 \cdot e^{kt}$$

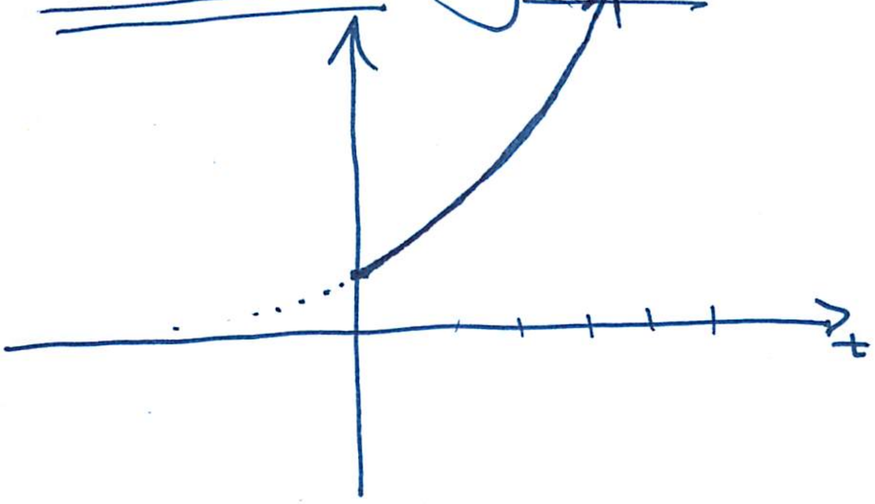
$$* \ \boxed{\underline{A} = \underline{P} \cdot e^{rt}} \Leftarrow$$ contin comp. int.

$P = \$10{,}000^{00}$ $\quad r = .03$

$t = 10$ yr

$$A = \underbrace{(10{,}000) \cdot e^{(.03)(10)}}$$

uninhibited growth



$\underline{\underline{y}} = \boxed{\underline{y_0}} \cdot e^{kt}$ $\left(\frac{dy}{dt} = \boxed{k} \cdot \underline{y} \right)$

INDIANAPOLIS, IN:

$\left\{ \begin{array}{l} \underline{2012}: \underline{844{,}000} \\ \underline{2018}: 865{,}000 \end{array} \right.$

predict pop in $\underline{\underline{2025}}$ $(t = 13)$

① $t = 0$ (2012)

$y_0 = 844{,}000$

$y = (844{,}000) \cdot e^{kt}$

② $t = 6$ (2018)

pop: 865,000

$$\frac{865{,}000}{844{,}000} = \frac{\cancel{844{,}000} \cdot \boxed{e^{\underset{\sim}{k(6)}}}}{\cancel{844{,}000}}$$

$$\frac{865{,}000}{844{,}000} = e^{6k}$$

$$\frac{\ln\left(\frac{865{,}000}{844{,}000}\right)}{6} = \frac{\cancel{6}\boxed{k}}{\cancel{6}} \approx \underline{.0041}$$

③ $y = \underline{844{,}000} \left(e^{\underline{.0041t}}\right)$

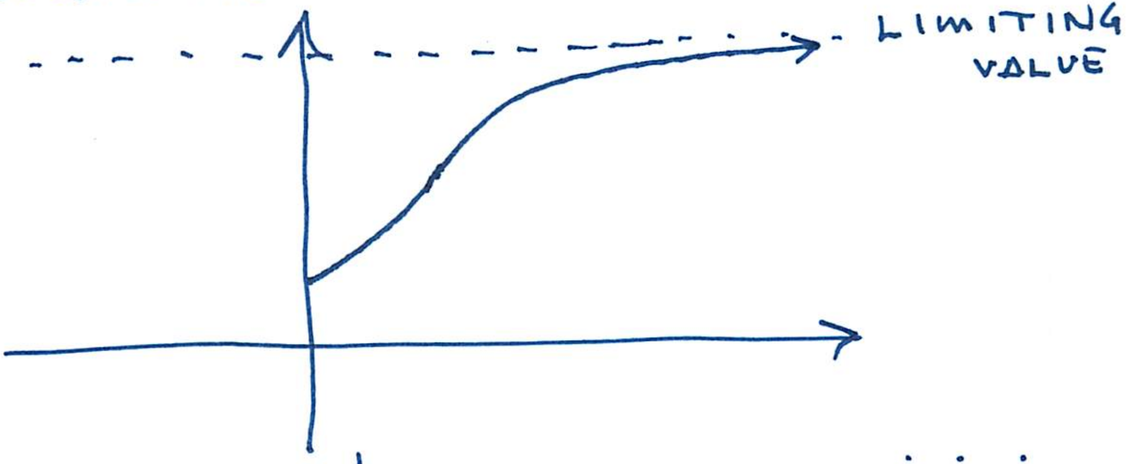
$y = 844{,}000 \left(e^{\underline{.0041(13)}}\right)$

$y \approx \underline{890{,}205}$

844,000
⋮
⑥
⋮
865,000
⋮
⑦
⋮
890,205

long-term model:

LOGISTIC GROWTH



$$* \quad y = \frac{L}{1+\underline{\underline{b}}\cdot e^{k(t)}}$$

L = limiting value

TARHEEL, NC:

2010: 5,218 (t=0)

2015: 6,100

max. supp. pop: 10,000

find pop (predict) in 2020

① $y = \dfrac{10{,}000}{1+b\cdot e^{kt}}$

$\underline{t=0}$; pop = $\underline{5218}$

$$5218 = \frac{10{,}000}{1+b\cdot e^{k(0)}}$$

$$\frac{5218}{1} = \frac{10{,}000}{1+b}$$

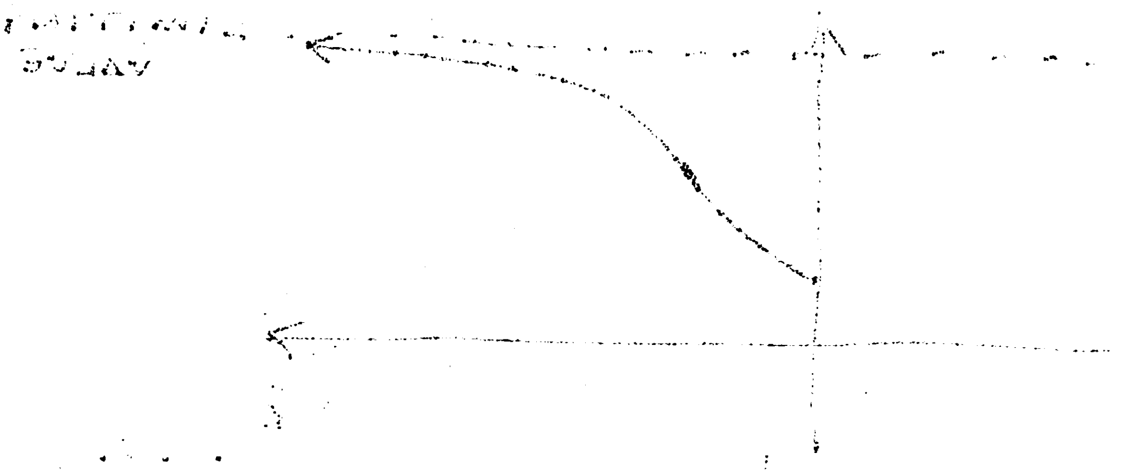
$$\frac{10{,}000}{5{,}218} - 1 = \frac{\cancel{5218}(\cancel{1}+b)}{\cancel{5218}} \cancel{-1}$$

$$\frac{10{,}000}{5{,}218} - 1 = b$$

$b \approx \underline{.916}$

[illegible] growth model:

LOGISTIC GROWTH



[illegible] $= \frac{[illegible]}{1 + P \cdot e^{[illegible]}}$

ASSUMPTIONS:

[illegible]

$\frac{10\,000}{1 + P \cdot e^{[illegible]}}$

[illegible] $- 1 = P$

$\frac{10\,000}{1 + P} = [illegible]$

[illegible]

⑦

$$y = \frac{10,000}{1 + (.916)\cdot e^{kt}}$$

2010: 5218

2015: 6100

$\underline{t=5}$; $y = 6100$

2020: ???

$t=10$

$$\frac{6100}{1} = \frac{10,000}{1 + (.916)e^{k(5)}}$$

$$\frac{61\cancel{0}\cancel{0}\left(1 + .916e^{5k}\right)}{61\cancel{0}\cancel{0}} = \frac{10,000}{6,100}$$

$$\begin{array}{l}\cancel{1} + \dfrac{\boxed{\cancel{.916}\, e^{5k}}}{\cancel{.916}} \\ -\cancel{1}\end{array} = \frac{\dfrac{10,000}{6,100} - 1}{.916}$$

$$e^{5k} = \frac{\dfrac{10,000}{6,100} - 1}{.916}$$

$$\frac{\cancel{5}k}{\cancel{5}} = \frac{\ln\left[\dfrac{\dfrac{10,000}{6,100} - 1}{.916}\right]}{5}$$

$$k \approx \underline{-.072}$$

$$y = \frac{10,000}{1 + (.916)e^{-.072t}}$$

($y = ??$ when $t=10$)

$$y = \frac{10,000}{1 + (.916)e^{-.072(10)}} \approx \underline{6,916}$$

3.4: exponential decay

$$y = y_0 \cdot e^{\boxed{K}t} \quad (\text{K is NEG.})$$

$\boxed{100 \text{ lbs}}$. radioactive

(half-life: 1 year) → K = ____

how long? sample decay to 5 lb.

$$y = 100 \cdot e^{Kt}$$

$$\frac{50\cancel{y}}{100} = \frac{\cancel{100} \cdot e^{K(1)}}{\cancel{100}} \qquad \frac{1}{2} = e^{K}$$

$$\underline{\ln(1/2)} = K$$

$$K \approx -.693$$

$$\frac{5\cancel{y}}{100} = \frac{\cancel{100} \cdot e^{-.693(t)}}{\cancel{100}}$$

$$\frac{5}{100} = e^{-.693t}$$

(take LN of both sides)