

MA121-003

(1)

Monday, October 22

{ 10/22 : 3.3 ; 3.4

{ 10/24 : 3.5 ; 4.1

{ 10/29 : 4.2 ; 4.3

{ 10/31 : 4.3 ; review

11/5 : TEST #3

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LOGS & EXPONENTIALS :

$$\underline{e}^{.08t} = 143$$

(solve for t)

nat. log of both sides

$$\underline{\ln e^{.08t}} = \ln 143$$

$$\underline{.08t} = \ln 143$$

$$\frac{.08t}{.08} = \frac{\ln 143}{.08} \approx \underline{\quad}$$

1.

800 - 1000

... ..

<u>P.S</u>	<u>S.S</u>	: 200
1.4	3.8	: 100

<u>E.P</u>	<u>A.P</u>	: 100
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...	<u>S.P</u>	: 100
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...	<u>T.P</u>	: 100
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... ..

$$E.P = 100$$

(... ..)

... ..

$$E.P = 100$$

$$E.P = 100$$

$$E.P = 100$$

$y = \frac{\ln x^2}{x^2}$  DERIV find  $y'$ :

$$y' = \frac{(x^2) \cdot \left(\frac{1}{x}\right) - (\ln x)(2x)}{[x^2]^2}$$

$$y' = \frac{x - 2x \cdot \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x \cdot x^3}$$

$$y' = \frac{1 - 2 \ln x}{x^3}$$

$- \ln x$   
 $- 2 \ln x$

$y = \ln \left( \frac{x^2 + x}{x + 3} \right)$

$y' = ??$

rewrite ...

$\ln \left( \frac{a}{b} \right) = \ln a - \ln b$

$y = \ln(x^2 + x) - \ln(x + 3)$

$$y' = \frac{1}{x^2 + x} \cdot (2x + 1) - \frac{1}{x + 3} (1)$$

$$y' = \frac{2x + 1}{x^2 + x} - \frac{1}{x + 3}$$

3

$$\lim_{x \rightarrow 0} \frac{(x^2)(x^2) - \frac{1}{x} \cdot (x^2)}{[x^2] \cdot x} = \frac{0}{0}$$

$$\frac{(x^2 - 1) \cdot x}{x \cdot x} = \frac{x^2 - 1}{x} = \frac{(x-1)(x+1)}{x}$$

lim x → 0  
 lim x → 0

$$\lim_{x \rightarrow 0} \left( \frac{x^2 + x}{x + x} \right) = \frac{0}{0}$$

$$= \left( \frac{2x}{2} \right) = x$$

$$\lim_{x \rightarrow 0} \left( \frac{x^2 + x}{x + x} \right) - \lim_{x \rightarrow 0} \left( \frac{x^2 + x}{x + x} \right) = 0$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} - \lim_{x \rightarrow 0} \frac{1}{x^2} = \frac{1}{0} - \frac{1}{0}$$

$$\left( \frac{1}{x^2} \right) - \left( \frac{1+x^2}{x^2} \right) = \frac{1}{x^2} - \frac{1}{x^2} - \frac{x^2}{x^2} = -1$$

$$y = \ln(x)$$

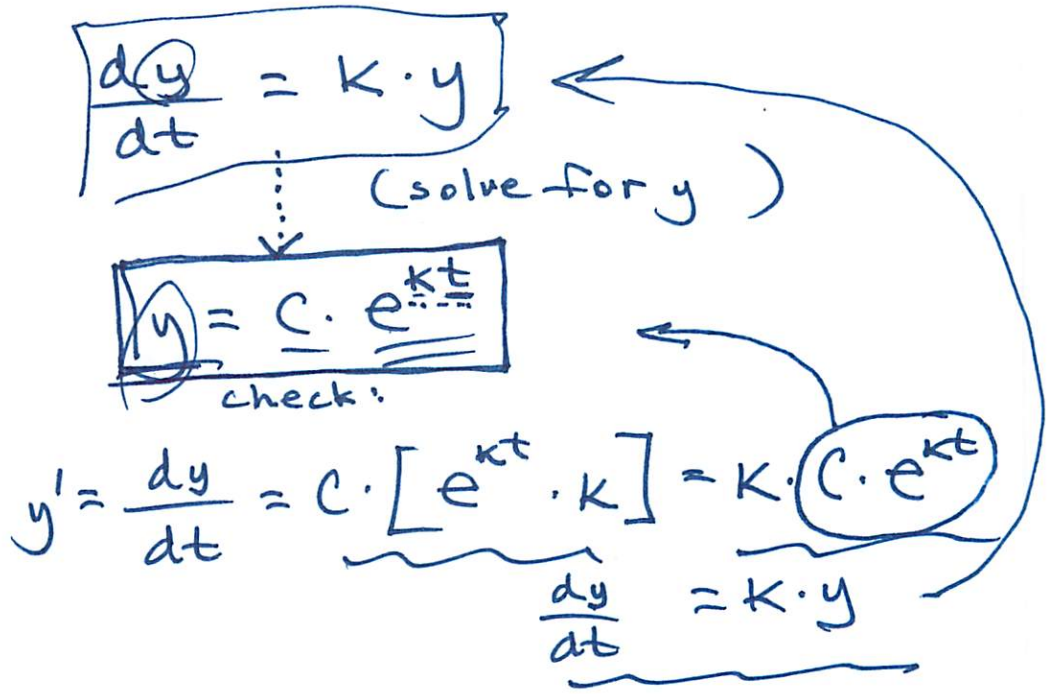
$$y' = \frac{1}{x} \cdot 1$$

$$y = \ln(x^2 + x)$$

$$y' = \frac{1}{x^2 + x} (2x + 1)$$

### 3.3: exponential growth (BASE "e")

growing exp: its rate of growth is directly prop. to the amount present at any time



exp GROWTH:  $y = C \cdot e^{kt}$

at time zero ( $t=0$ ) amount of y

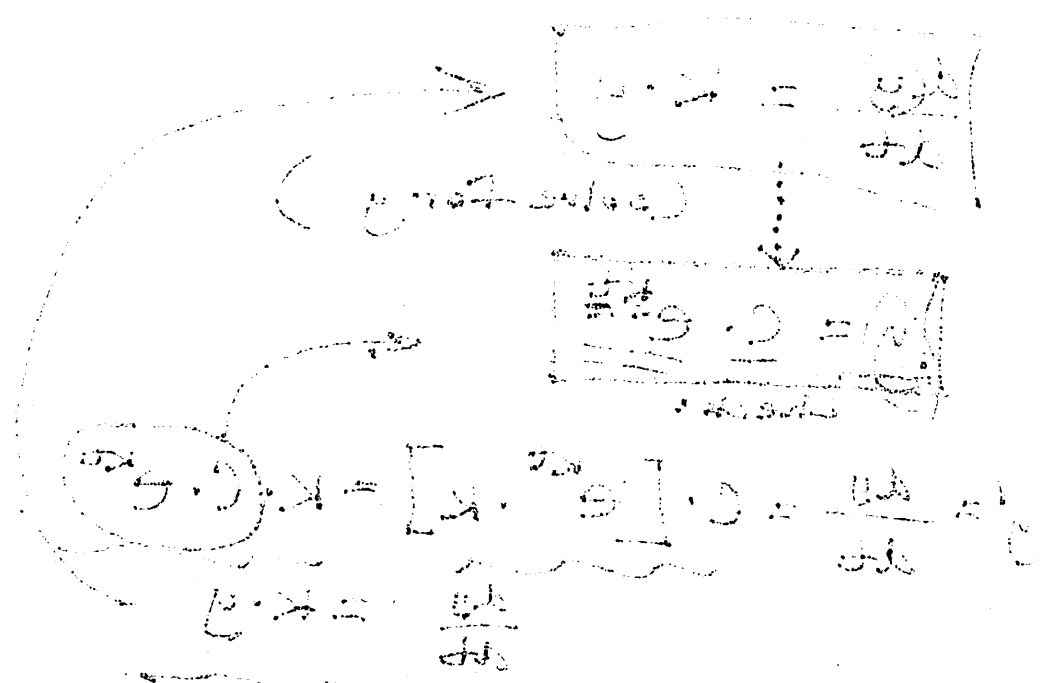
$y_0 = C \cdot e^{k(0)}$

$y_0 = C \cdot 1 \quad y_0 = C$

$$\left. \begin{aligned} (y + x) \cdot x &= p \\ (1+x) \cdot \frac{1}{y+x} &= 1 \end{aligned} \right\} \text{And } \left. \begin{aligned} (y + x) \cdot x &= p \\ (1+x) \cdot \frac{1}{y+x} &= 1 \end{aligned} \right\}$$

stasiun kultural : E.C  
 ("S" SEAS)

faktor de : apa yang  
 + ...  
 pada ke ...



$p \cdot x = \frac{p}{1+x}$  : #TRUCED qye

of ... (S = #) ...

$$p \cdot x = \frac{p}{1+x} \Rightarrow x = \frac{p}{1+x}$$



$$y = y_0 \cdot e^{kt}$$

initial amount  
of y

(y-value when  $t=0$ )

"standard" exponential  
growth/decay

$$P = P_0 \cdot e^{kt}$$

$$y = y_0 \cdot e^{kt}$$

$$A = P \cdot e^{rt}$$

continuous comp. int.

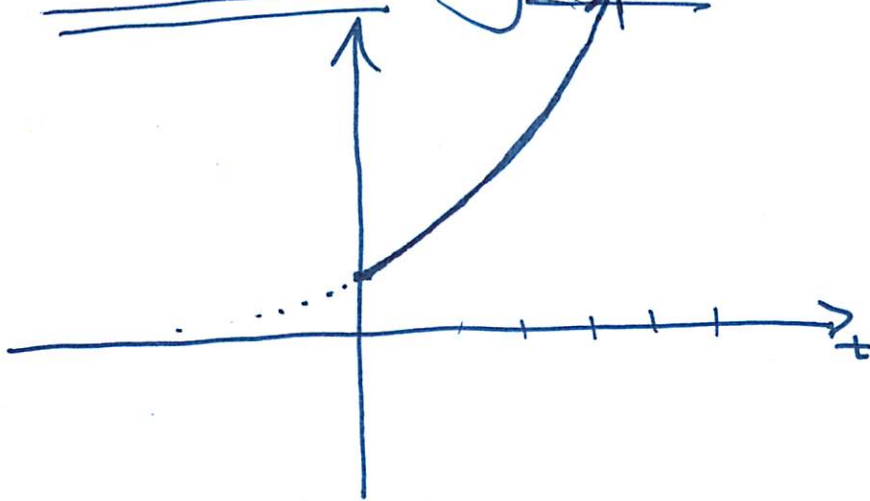
$$P = 10,000$$

$$r = .03$$

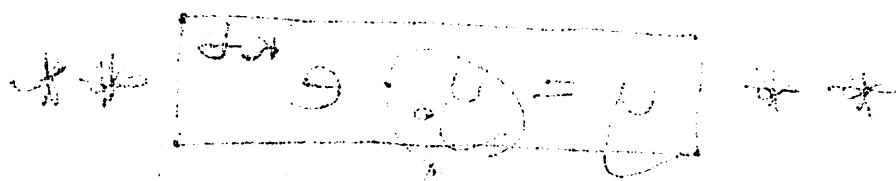
$$A = (10,000) \cdot e^{(.03)(10)}$$

$t = 10 \text{ yr}$

uninhibited growth



(1)



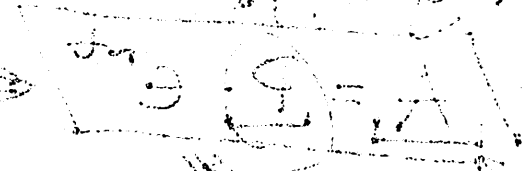
Initial amount  
 $P = P_0$

( $0 = \text{Final amount} - P_0$ )

"Standard" exponential  
 growth/decay

$$KT \cdot G = P = P_0$$

$$KT \cdot G = U = U_0$$



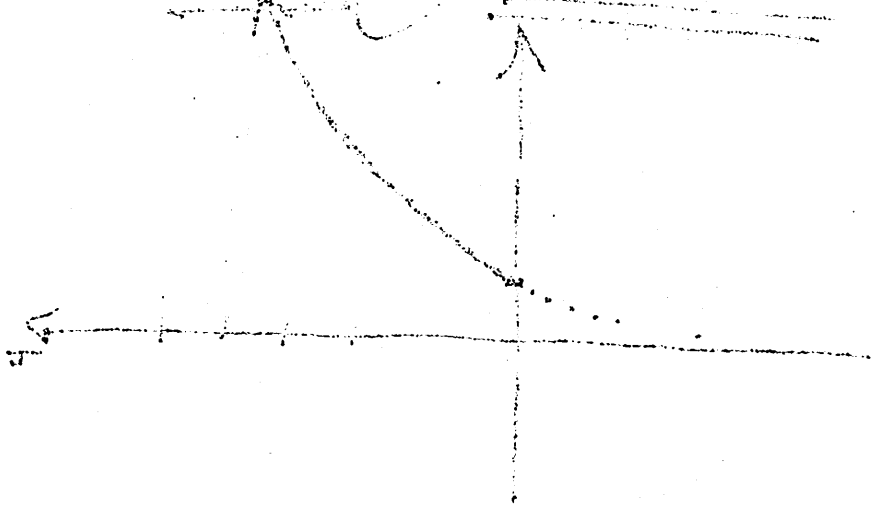
Final amount

$$P_0 = 1$$

$$U_0 = 1000$$

$$A = (1000, 0) \cdot (0.0001, 0.0001) = A$$

Standard exponential





$$y = y_0 \cdot e^{kt} \quad \left( \frac{dy}{dt} = k \cdot y \right) \quad (5)$$

INDIANAPOLIS, IN:

$$\begin{cases} 2012: 844,000 \\ 2018: 865,000 \end{cases}$$

predict pop in 2025 ( $t=13$ )

①  $t=0$  (2012)

$$y_0 = 844,000$$

$$y = (844,000) \cdot e^{kt}$$

②  $t=6$  (2018)

pop: 865,000

$$\frac{865,000}{844,000} = \frac{844,000 \cdot e^{k(6)}}{844,000}$$

$$\frac{865,000}{844,000} = e^{6k}$$

$$\frac{\ln\left(\frac{865,000}{844,000}\right)}{6} = k \approx \underline{\underline{.0041}}$$

③  $y = 844,000 \left( e^{.0041t} \right)$

$$y = 844,000 \left( e^{.0041(13)} \right)$$

$$y \approx \underline{\underline{890,205}}$$



3

$$\left( \frac{1}{2} \right) = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

INDIVIDUALS:

$$\left. \begin{aligned} 000 \text{ PPP} &= 1100 \\ 000 \text{ PPP} &= 0100 \end{aligned} \right\}$$

(SI = 0) 2000 ni qay tashkent

(1100) 0000

$$000 \text{ PPP} = 0$$

$$\frac{1}{2} \cdot (000 \text{ PPP}) = 0$$

(0100) 0000

$$000 \text{ PPP} = 000$$

$$\left( \frac{1}{2} \right) \cdot 000 \text{ PPP} = \frac{000 \text{ PPP}}{000 \text{ PPP}}$$

$$= \frac{000 \text{ PPP}}{000 \text{ PPP}}$$

$$\frac{1000}{1000} = \frac{1000}{1000}$$

$$\left( \frac{1}{2} \right) \cdot 000 \text{ PPP} = 0$$

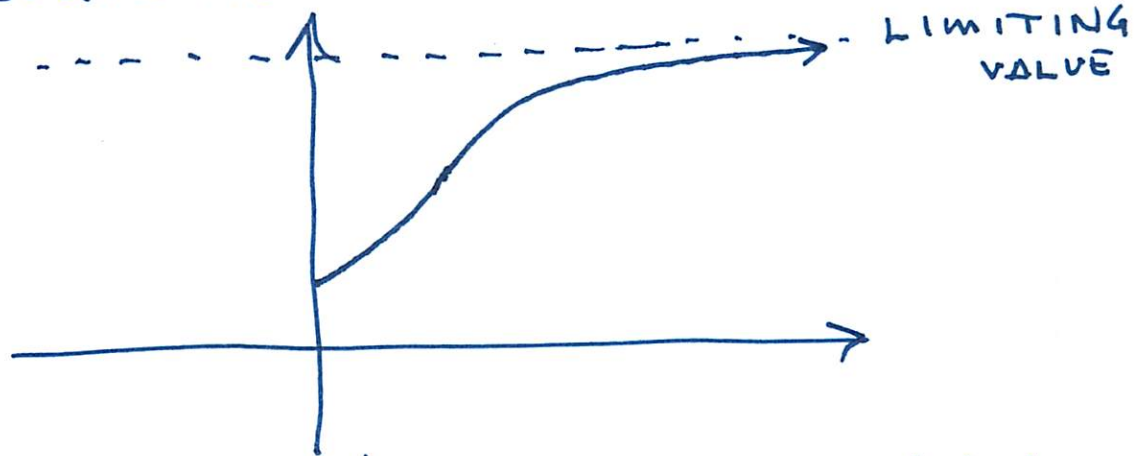
$$\left( \frac{1}{2} \right) \cdot 000 \text{ PPP} = 0$$

$$\frac{000 \text{ PPP}}{000 \text{ PPP}} = 1$$

000 PPP  
000 PPP  
000 PPP

long-term model:

LOGISTIC GROWTH



$$y = \frac{L}{1 + \underline{b} \cdot e^{kt}}$$

$L = \text{limiting value}$

TARHEEL, NC:

2010: 5,218 ( $t=0$ )

2015: 6,100

max. supp. pop: 10,000

find pop (predict)  
in 2020

$$\textcircled{1} \quad y = \frac{10,000}{1 + b \cdot e^{kt}}$$

$$\underline{t=0} ; \text{ pop} = \underline{5218}$$

$$\frac{10,000}{5,218} - 1 = b$$

$$b \approx \underline{.916}$$

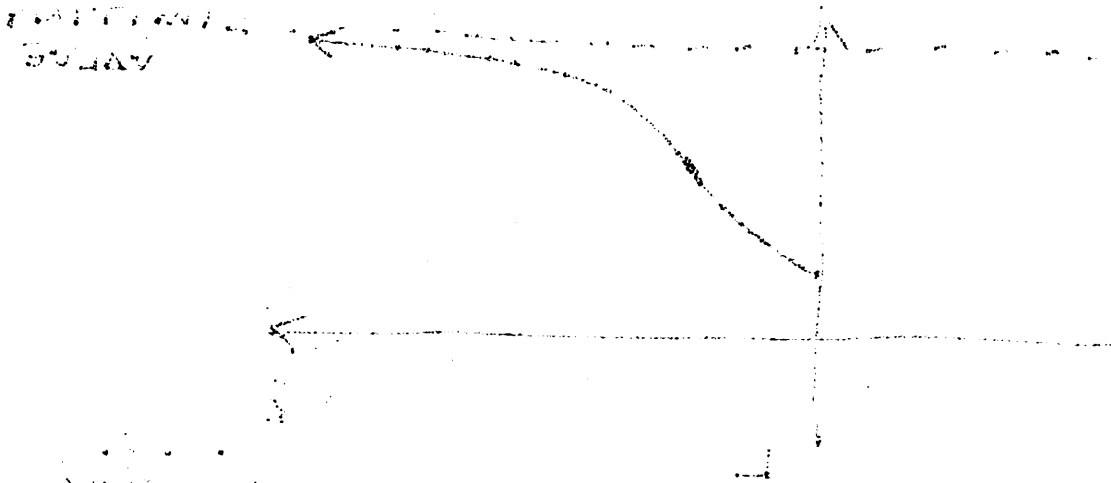
$$5218 = \frac{10,000}{1 + b \cdot e^{k(0)}}$$

$$\frac{5218}{1} = \frac{10,000}{1 + b}$$

$$\frac{10,000}{5,218} - 1 = \frac{5218(1+b)}{5218}$$

... ..

HTWGRD ... ..



... ..

$$1 + p \cdot \frac{1}{2} = \dots$$

... ..

... ..

... ..

... ..

$$\frac{10000}{1 + p \cdot \frac{1}{2}} = \dots$$

$$10000 = \dots$$

$$2518 = \frac{10000}{1 + p \cdot \frac{1}{2}}$$

$$2518 = \frac{10000}{1 + p}$$

$$(1 + p) \cdot 2518 = 10000$$

$$p = \frac{10000}{2518} - 1 = \dots$$

$$p = \dots$$

$$y = \frac{10,000}{1 + (.916) \cdot e^{kt}}$$

2010: 5218  
2015: 6100

t=5; y=6100  
2020: ???  
t=10

$$\frac{6100}{1} = \frac{10,000}{1 + (.916) e^{k(5)}}$$

$$\frac{6100 (1 + .916 e^{5k})}{6100} = \frac{10,000}{6,100}$$

$$\cancel{1} + \frac{.916 e^{5k}}{\cancel{.916}} = \frac{\frac{10,000}{6,100} - 1}{.916}$$

$$e^{5k} = \frac{\frac{10,000}{6,100} - 1}{.916}$$

$$\cancel{5k} = \ln \left[ \frac{\frac{10,000}{6,100} - 1}{.916} \right]$$

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$$k \approx \underline{\underline{-0.072}}$$

$$y = \frac{10,000}{1 + (.916) e^{-0.072t}}$$

(y = ??  
when  
t=10)

$$y = \frac{10,000}{(1 + (.916) e^{-0.072(10)})} \approx \underline{\underline{6,916}}$$



3.4: exponential decay

$$y = y_0 \cdot e^{kt} \quad (k \text{ is NEG.})$$

100 lbs. radioactive

(half-life: 1 year)  $\rightarrow k = \underline{\hspace{2cm}}$

how long? sample decays to 5 lbs.

$$y = 100 \cdot e^{kt}$$

$$\frac{50}{100} = \frac{100 \cdot e^{k(1)}}{100}$$

$$\frac{1}{2} = e^k$$

$$\frac{5}{100} = \frac{100 \cdot e^{-.693(t)}}{100} \quad \frac{\ln(1/2)}{k} = k$$

$$k \approx -.693$$

$$\frac{5}{100} = e^{-.693t}$$

(take LN of both sides)



Expt. 1: exponential decay  
 (k is in  $\text{hr}^{-1}$ )

initial concentration

Half-life:  $t_{1/2} \rightarrow k = \dots$

Expt. 2: sample decay to  $\frac{1}{10}$

$$\frac{200}{100} = \frac{100 \cdot e^{-k \cdot 100}}{100 \cdot e^{-k \cdot 0}}$$

$$2 = e^{-k \cdot 100}$$

$$\ln(2) = -k \cdot 100$$

$$k = -\frac{\ln(2)}{100}$$

$$\frac{2}{100} = \frac{100 \cdot e^{-k \cdot 100}}{100 \cdot e^{-k \cdot 0}}$$

$$2 = e^{-k \cdot 100}$$

(value of k is same)