MA 121 - 003

Wednesday, October 24

NEWTON'S LAW OF COOLING:

- the rate of change of temp (is) directly prop. to the diff. in temp. between the object and the surrounding medium.

T = Temp. (of object) m = temp. of surr. med.

t = time

$$\frac{dT}{dt} = k \cdot (T - m) \qquad \frac{dy}{dx}$$

rate of change of temp

$$T = a \cdot e^{kt} + m$$

① $t = 0$ $T = 400°$

$m = 70°$

$t = 10$ min. $T = 325°$

$t = ?$ $T = 150°$

$$T = a \cdot e^{kt} + m$$

$$T = a \cdot e^{kt} + 70$$

$$400 = a \cdot \boxed{e^{k \cdot 0}} + 70$$

$$400 = a + 70$$

$$400 - 70 = a = 330$$

$$T = 330 \cdot e^{kt} + 70 \qquad \begin{array}{l} t=10 \\ T=325 \end{array}$$

$$325 = 330 \cdot e^{k(10)} + \cancel{70}$$

$$-70 \qquad\qquad -\cancel{70}$$

$$\frac{255}{330} = \frac{\cancel{330}\, e^{10k}}{\cancel{330}}$$

$$\frac{255}{330} = e^{10k}$$

$$\frac{\ln\left(\frac{255}{330}\right)}{10} = \frac{\cancel{10}k}{\cancel{10}} \approx -.0258$$

$$T = 330 \cdot e^{-.0258t} + 70 \qquad \begin{array}{l} t = ?? \\ T = 150 \end{array}$$

$$150 = 330\, e^{-.0258t} + 70$$

⋮

solve for t

⋮

$t \approx$ ______

radiocarbon dating:

$k = -.000001205$

$$y = \boxed{y_0} \cdot e^{kt}$$

half-life of C_{14} : 5230 yrs.

$$60\% = 100\% \cdot e^{k(t)}$$

lost 40% of its C_{14}

$$.6 = 1 \cdot e^{-.000001205t}$$

3.5:

$y = e^x \quad y' = e^x$

$y = e^{f(x)} \quad y' = e^{f(x)} \cdot f'(x)$

$y = 2^x \quad y = 10^x \quad y' = ??$

$y = \ln x \quad y' = \frac{1}{x}$

$y = \ln(f(x)) \quad y' = \frac{1}{f(x)} \cdot f'(x)$

$y = \log_2 x \quad y = \log_{10}(x^2+x-1)$

$y' = ??$

$y = 2^x \quad y = 8^x \quad y = 10^x$

$e^{\ln u} = u$

general: $\boxed{y = a^x}$ $\quad y' = ??$

rewrite:

$y = e^{\ln a^x}$

$y = e^{x \ln a}$

$y = e^{x \cdot \ln a}$

$y' = \boxed{e^{x \cdot \ln a}} \cdot (\ln a)$

$\boxed{y' = a^x \cdot (\ln a)}$

$y = 5^x \qquad y' = 5^x \cdot \ln 5$

$y = 8^{2x+1} \qquad y' = 8^{2x+1} \cdot \ln 8 \cdot (2)$

$y = e^x$

$y' = e^x \cdot \ln e$

$y' = e^x \, (1)$

$\boxed{\begin{array}{l} y = a^x \\ y' = a^x \cdot \ln a \end{array}}$ *

$y = \log_a x$

$y = \log_2 x$

$y = \log_{10} x$ *

$\boxed{f(x) = \log_a x}$

rewrite in exponential form

$a^{f(x)} = x$

DERIV:

$$\frac{\cancel{a^{f(x)} \cdot \ln a} \cdot f'(x)}{\cancel{a^{f(x)} \cdot \ln a}} = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{x \cdot \ln a}$$ ✓

$y = \log_3 x$

$y' = \dfrac{1}{x \cdot \ln 3}$

$y = \log_{10}(x^2+x+8)$

$y' = \dfrac{1}{(x^2+x+8)\cdot \ln 10} \quad (2x+1)$ chain rule

$y' = \dfrac{2x+1}{(x^2+x+8)\cdot \ln 10}$

4.1: ANTIDERIVATIVES (integrals)

DERIV:

$y = x^2 - 8x + 11$
$y' = 2x - 8$ ✓

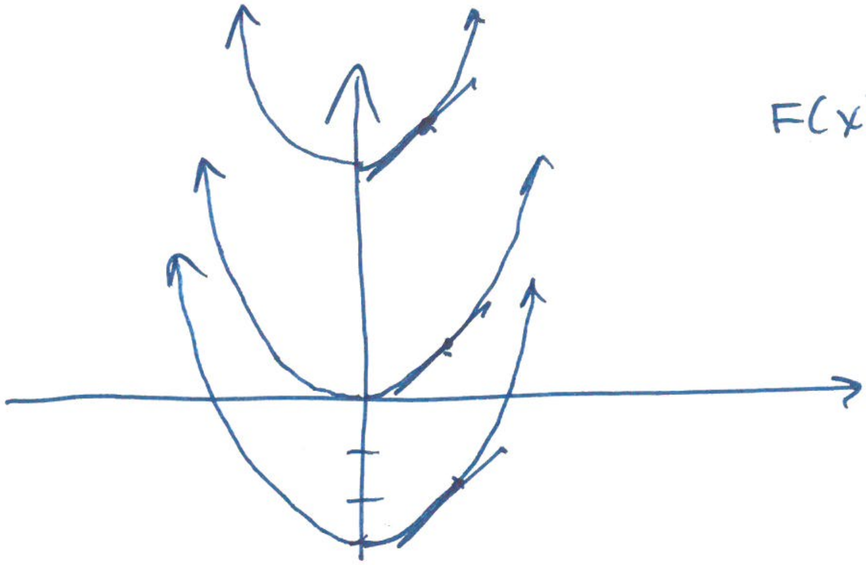
ANTIDERIV:

$f(x) = 2x - 8$

$[F'(x) = f(x)] \quad F(x) = x^2 - 8x + C$ indefinite integral

"family of curves"

$f(x) = x^2 - 8x + 11$
$f(x) = x^2 - 8x - 5$
$f(x) = x^2 - 8x + 3$



$F(x) = x^2 + C$

$f(x) = x^2$

$f(x) = x^2 - 3$

$f(x) = x^2 + 7$

$\vdots$

3 ANTIDERIVATIVE "RULES":

$\int 4\, dx = 4x + C$

$\int 4\, dr = 4r + C$

(1) $\boxed{\int} \left(2x - 8 \right) dx$

anti diff. with respect to x

$= x^2 - 8x + C$

$$\int a\, x^n \cdot dx = a \cdot \frac{x^{n+1}}{n+1} + C$$

ex: $\int \boxed{2x}\, dx = \frac{\cancel{2} \cdot x^2}{\cancel{2}} + C$

$$\int 11 \cdot x^5 \, dx$$

$$= \frac{11 \cdot x^6}{6} + C$$

check:

$$d\left[\frac{11}{6} x^6 + C\right] \overset{??}{=} 11x^5$$

$$= \frac{11}{\cancel{6}} \cdot \cancel{6} x^5$$

(1) $$\int a \cdot x^n \, dx = \frac{a \cdot x^{n+1}}{n+1} + C$$

(for all n, $n \neq -1$)

$\boxed{n=-1}$

(2) $$\int a \, x^{-1} \, dx = \int a \frac{1}{x} dx = a \cdot \ln|x| + C$$

(3) $$\int a \cdot e^x \cdot dx = a \cdot e^x + C$$

$$\int a \cdot e^{k \cdot x} dx = \frac{a}{k} \cdot e^{k \cdot x} + C$$

$$\int e^{5x}\,dx = \underline{e^{5x} + C}$$

check: $d\left(e^{5x} + c\right) \overset{?}{=} e^{5x} \cdot 5$??

$$\int 7e^{5x}\,dx = 7 \cdot \frac{e^{5x}}{5} + C$$

check $d\left(\frac{7}{5} e^{5x} + c\right) = \frac{7}{\cancel{5}} \cdot e^{5x} \cdot \cancel{5}$

3.4: exponential decay

$$\boxed{y = y_0 \cdot e^{kt}} \quad (\text{K is NEG.})$$

$\boxed{100\text{ lbs}}$. radioactive

(half-life: 1 year) $\rightarrow$ K = ____

how long?, sample decays to 5 lb.

$$y = 100 \cdot e^{kt}$$

$$\frac{50}{100} = \frac{\cancel{100} \cdot e^{k(1)}}{\cancel{100}} \qquad \frac{1}{2} = e^{k}$$

$$\ln(1/2) = K$$

$$K \approx -.693$$

$$\boxed{\frac{5}{100} = \frac{\cancel{100} \cdot e^{-.693(t)}}{\cancel{100}}}$$

$$\ln e^{u} = u$$

$$\left\{ \frac{5}{100} = e^{-.693t} \right.$$

(take LN of both sides)

$$\frac{\ln\left(\frac{5}{100}\right)}{-.693} = \frac{\cancel{-.693}\, t}{\cancel{-.693}} \approx \underline{4.323 \text{ yr.}}$$

100 (1) 50 (1) 25 (1) 12.5 (1) 6.25