MA 121-003

Monday, October 29

4.1: ANTIDERIVATIVE (INTEGRAL)

1. $$\int a x^n \, dx = \frac{a \cdot x^{n+1}}{n+1} + C$$

"family of curves"

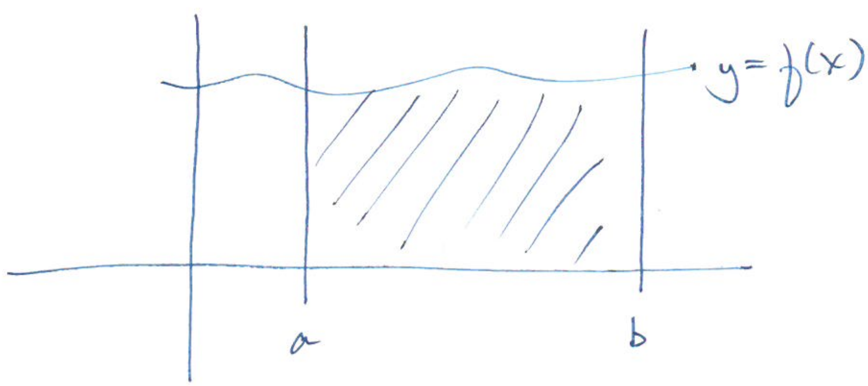
"indefinite integral"

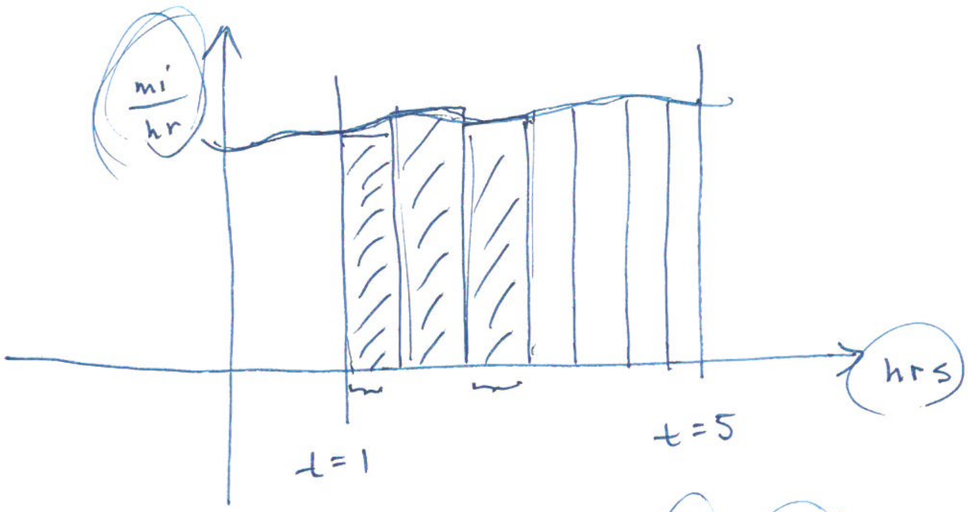
ex: $$\int 2x \, dx = x^2 + C$$

2. $$\int a \cdot x^{-1} \, dx = \int a \cdot \left(\frac{1}{x}\right) dx = a \cdot \ln|x| + C$$

3. $$\int a \cdot e^{bx} \cdot dx = a \cdot \frac{1}{b} e^{bx} + C$$

4.2: AREAS (UNDER CURVES)





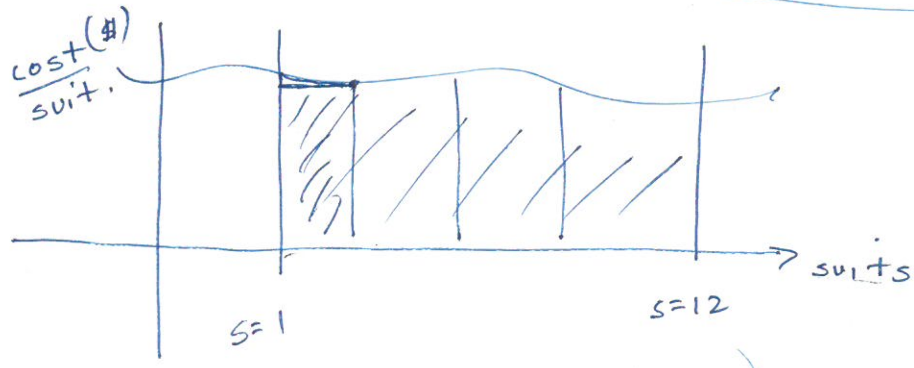
AREAS: $(\sim \cancel{\text{hrs}})(\sim \frac{\text{mi}}{\cancel{\text{hr}}})$

WIDE

TALL

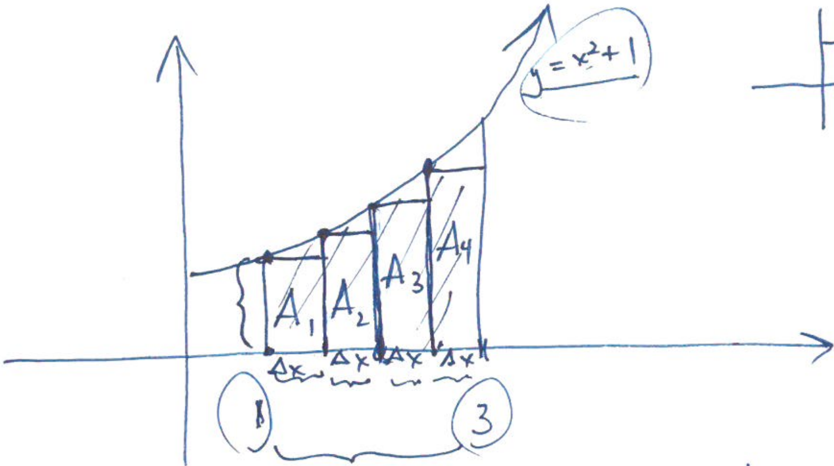
$A = (\boxed{\sim} \text{ mi})$

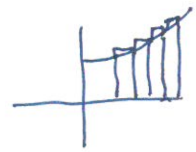
area a VELOCITY curve $\left(\frac{\text{mi}}{\text{hr}}\right)$ is DISTANCE



$A = \left(\sim \frac{\text{cost}(\$)}{\text{suit}} \right) \left(\sim \text{suits} \right)$

$= \left(\sim \text{cost}(\$) \right)$





find the area (APPROX the area) under the curve $y = x^2 + 1$ from $x = 1$ to $x = 3$

$$\text{AREA} \approx A_1 + A_2 + A_3 + A_4$$

$$A_1 = \left(\tfrac{1}{2}\right)(1^2+1) = \left(\tfrac{1}{2}\right)(2) = 1$$

$$A_2 = \left(\tfrac{1}{2}\right)\left(\left(\tfrac{3}{2}\right)^2+1\right) = \left(\tfrac{1}{2}\right)\left(\tfrac{13}{4}\right) = \tfrac{13}{8}$$

$$A_3 = \left(\tfrac{1}{2}\right)(2^2+1) = \left(\tfrac{1}{2}\right)(5) = \tfrac{5}{2}$$

$$A_4 = \left(\tfrac{1}{2}\right)\left(\left(\tfrac{5}{2}\right)^2+1\right) = \left(\tfrac{1}{2}\right)\left(\tfrac{29}{4}\right) = \tfrac{29}{8}$$

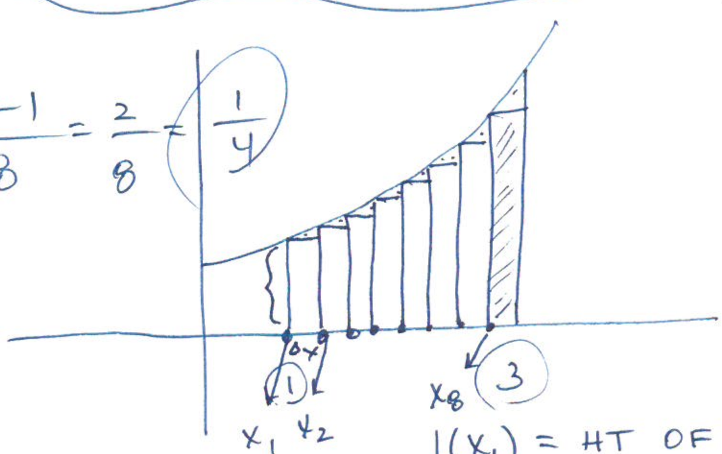
Δx = WIDTH

$$\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$A = 1 + \frac{13}{8} + \frac{5}{2} + \frac{29}{8} = \frac{8}{8} + \frac{13}{8} + \frac{20}{8} + \frac{29}{8}$$

$$A = \frac{70}{8} = \frac{35}{4}$$

$$\Delta x = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$$



4 rect → 8 rect.

more rectangles → higher accuracy.

$f(x_1)$ = HT OF FIRST RECTANGLE

$f(x_2)$ = HT OF SECOND RECTANGLE

⋮

$f(x_8)$ = HT OF EIGHTH RECTANGLE (LAST)

$$\text{AREA} = \underbrace{f(x_1)\cdot\Delta x}_{A_1} + \underbrace{f(x_2)\cdot\Delta x}_{A_2} + \underbrace{f(x_3)\cdot\Delta x}_{A_3} + \underbrace{f(x_4)\cdot\Delta x}_{A_4}$$

$$+ \underbrace{f(x_5)\cdot\Delta x}_{A_5} + \underbrace{f(x_6)\cdot\Delta x}_{A_6} + \underbrace{f(x_7)\cdot\Delta x}_{A_7} + \underbrace{f(x_8)\cdot\Delta x}_{A_8}$$

(APPROX)

$$\text{AREA} \approx \sum_{i=1}^{8} \left[f(x_i)\cdot\Delta x \right]$$

better . . . . . .

$$A \approx \sum_{i=1}^{7188} \left[f(x_i)\cdot\Delta x \right]$$

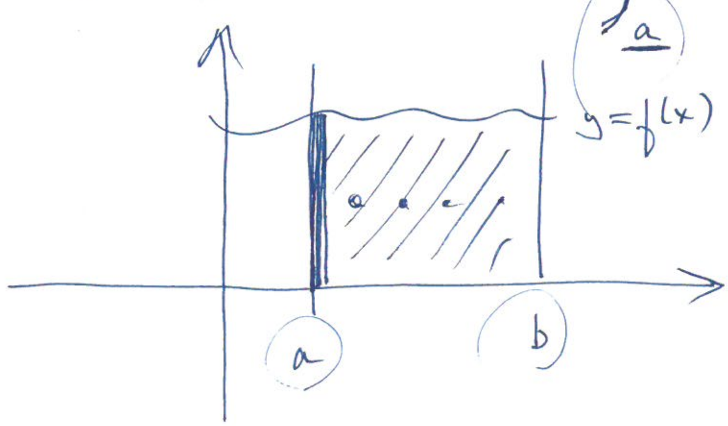
$$A = \sum_{i=1}^{\infty} \left[\underbrace{f(x_i)}_{\substack{\uparrow \\ \text{HT}}} \cdot \underbrace{\Delta x}_{\substack{\uparrow \\ \text{WDTH}}} \right]$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[f(x_i)\cdot\Delta x \right]$$

n rectangles
take limit
as $n \to \infty$

DEF:

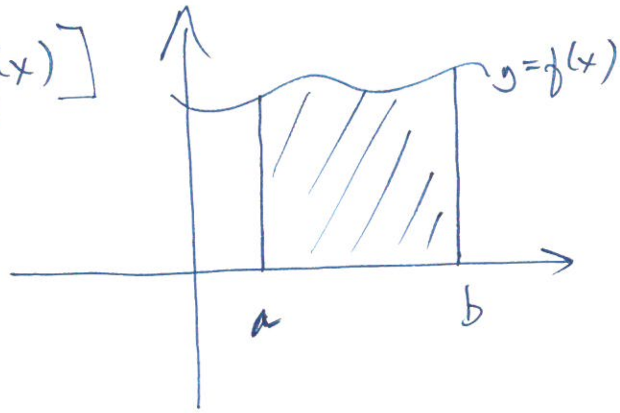
$$A = \lim_{\substack{n\to\infty \\ \boxed{\Delta x \to 0}}} \sum_{i=1}^{n} \left[f(x_i)\cdot\Delta x \right] = \int_a^b f(x)\cdot dx$$



$$A = \int_a^b f(x)\,dx = F(x)\Big]_a^b = \underbrace{F(b) - F(a)}$$

a number (which is the area under the curve $y = f(x)$ from $x = a$ to $x = b$)

$[F'(x) = f(x)]$



ex: find the area under the curve $y = x^2 + 1$ from $x = 1$ to $x = 3$. (APPROX: $\frac{35}{4}$)

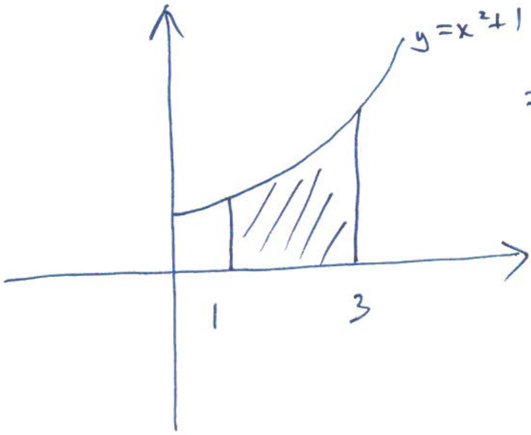
$$A = \int_1^3 \underset{\text{HT}}{\underline{(x^2+1)}} \cdot \underset{\text{WDTH}}{\underline{dx}} = \frac{x^3}{3} + x \Big]_1^3$$

$$= \left[\frac{(3)^3}{3} + (3)\right] - \left[\frac{(1)^3}{3} + (1)\right]$$

$$= 9 + 3 - \frac{1}{3} - 1 = 11 - \frac{1}{3} = 10\tfrac{2}{3}$$

$$= \frac{32}{3} = \text{AREA}$$

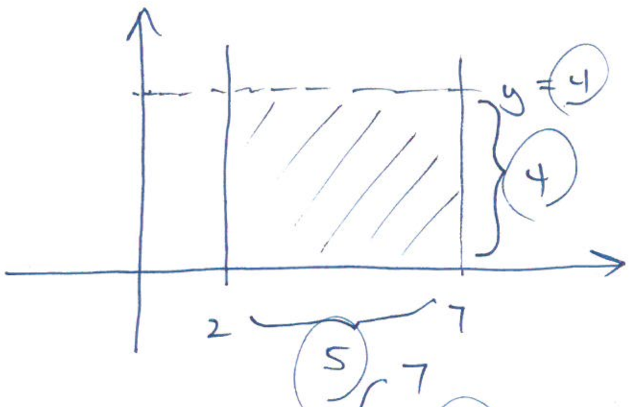
$\frac{35}{4}$



$10\frac{2}{3}$ (sq. units)

$8\frac{3}{4}$ (only 4 rectangles)

$$A = \int_a^b f(x) \cdot dx$$

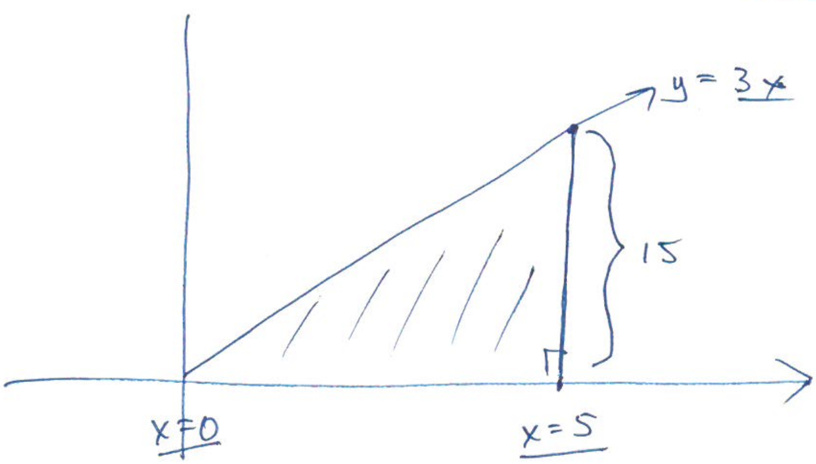


area under the "curve" $y=4$ from $x=2$ to $x=7$.

$$A = \int_2^7 4 \cdot dx = 4x\Big]_2^7 = 4(7) - 4(2)$$

$$= 28 - 8 = 20$$

check: $A = (5)(4) = 20$



$$A = \int_0^5 (3x) \cdot dx$$

$$A = 3 \cdot \frac{x^2}{2}\Big]_0^5$$

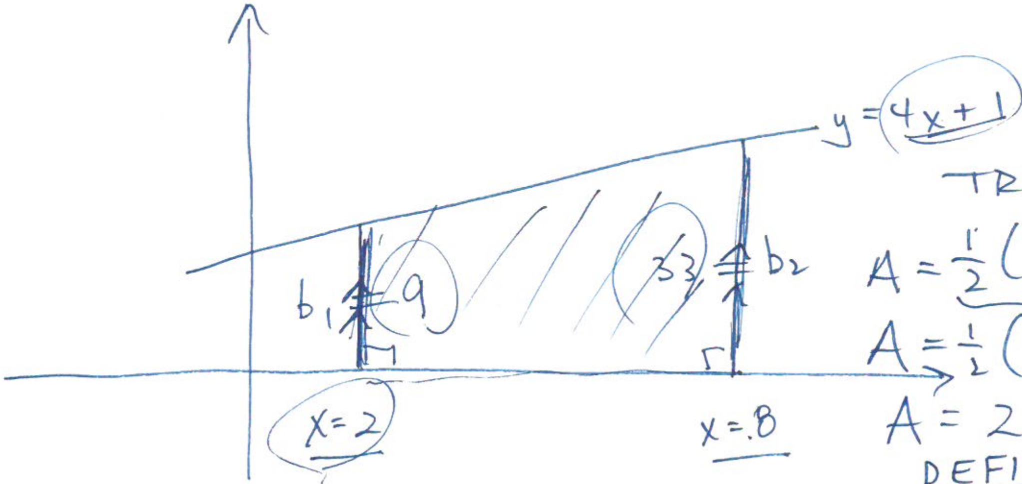
$$A = \frac{3}{2}(5)^2 - \cancel{\frac{3}{2}(0)^2}$$

$$A = \frac{75}{2}$$

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2}(5)(15)$$

$$A = \frac{75}{2}$$



TRAPEZOID:

$$A = \underbrace{\frac{1}{2}(b_1+b_2)} \cdot \underline{h}$$

$$A = \frac{1}{2}(9+33) \cdot 6$$

$$A = 21 \cdot 6 = 126$$

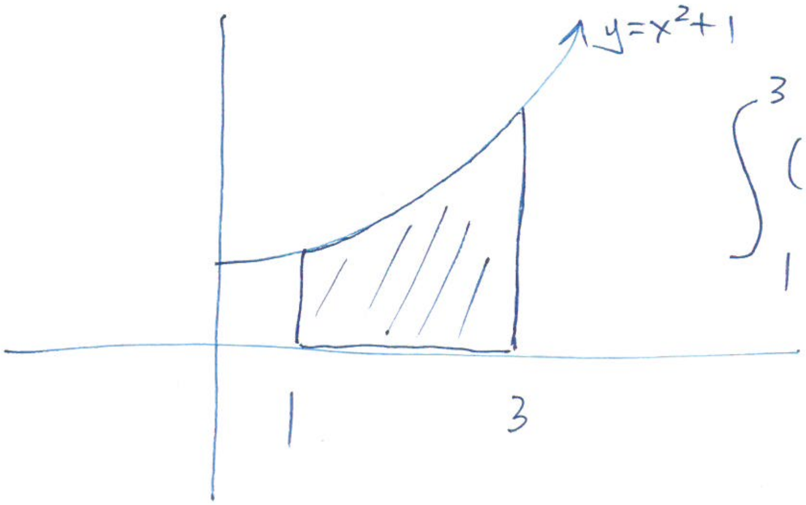
DEFINITE INTEGRAL

$$A = \int_2^8 (4x+1) \cdot dx$$

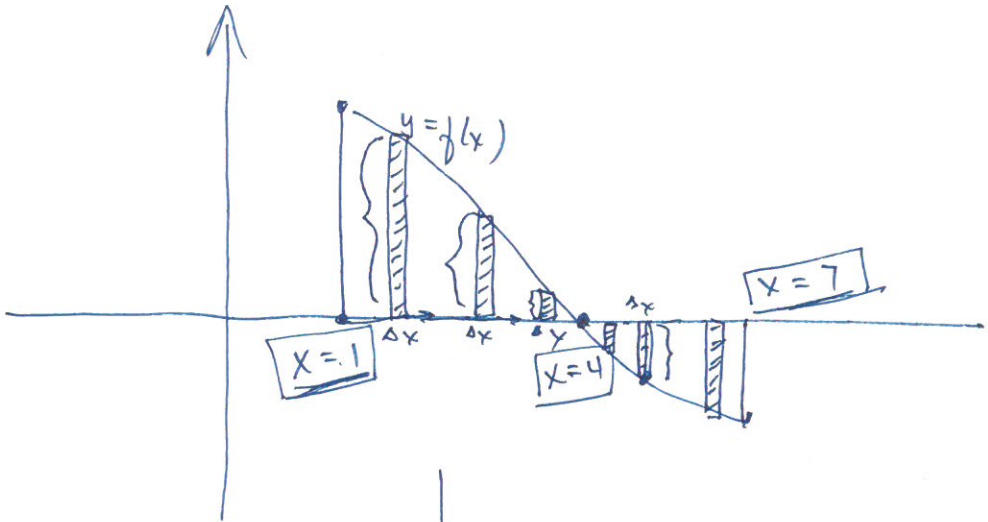
$$A = 4 \cdot \frac{x^2}{2} + x \Big]_2^8 = 2x^2 + x \Big]_2^8$$

$$A = \left[2(8)^2 + 8\right] - \left[2(2)^2 + 2\right]$$

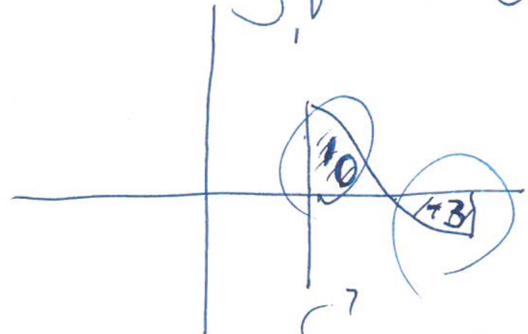
$$A = 128 + 8 - 8 - 2 = 126$$

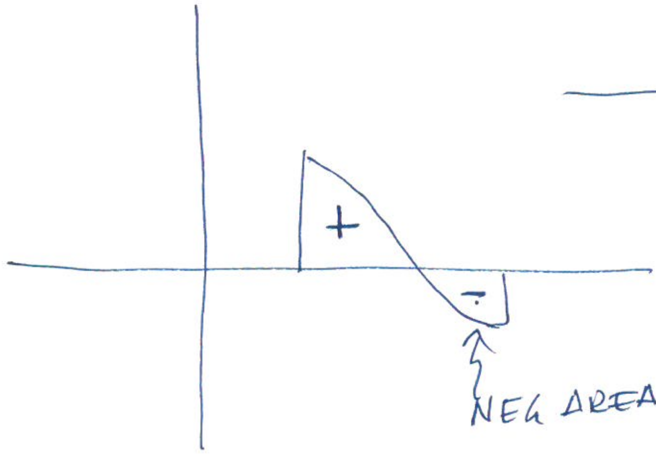


$$\int_1^3 (x^2+1) \cdot dx = A$$



$$\int_1^4 f(x)\,dx + \int_4^7 f(x)\,dx$$





$$A = \int_1^7 (f(x))\,dx$$

$$= 7$$