MA121-003

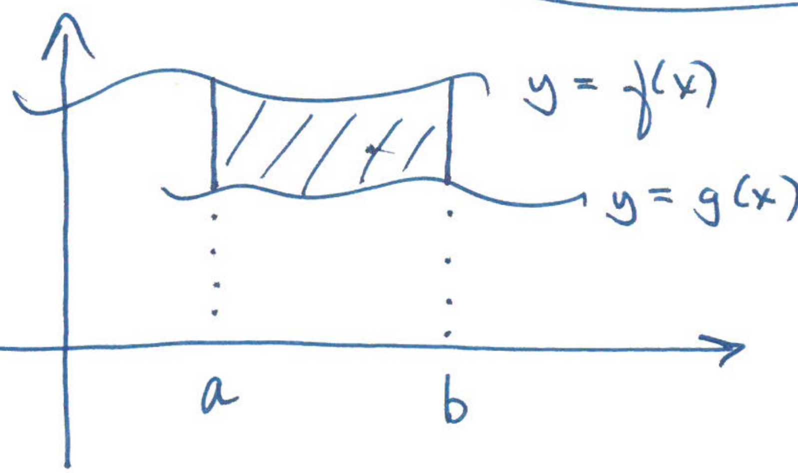
Wednesday, November 7

4.4: area between 2 curves

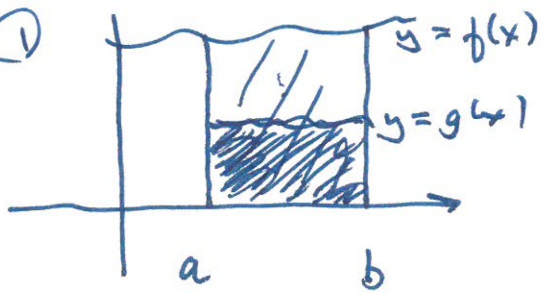
average value of a function

4.5: integration using SUBSTITUTION

4.4:



(1)

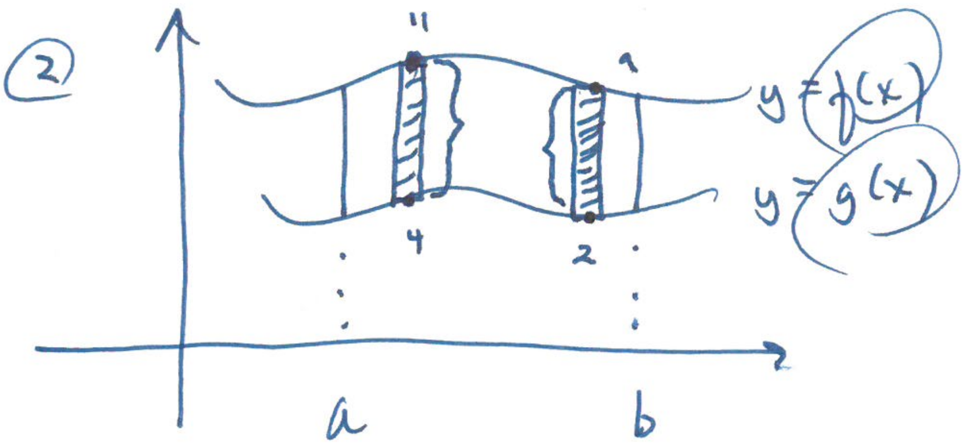


$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

$$\int_a^b g(x)\,dx = G(x)\Big]_a^b = G(b) - G(a)$$

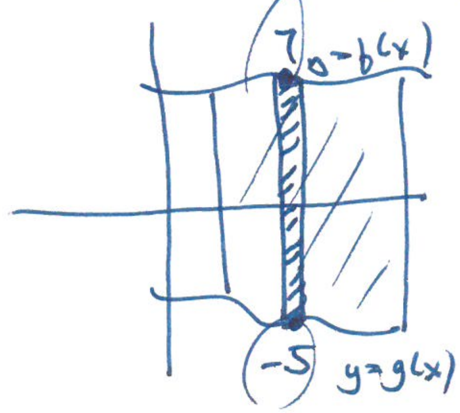
$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \left[f(x) - g(x)\right]dx$$

2

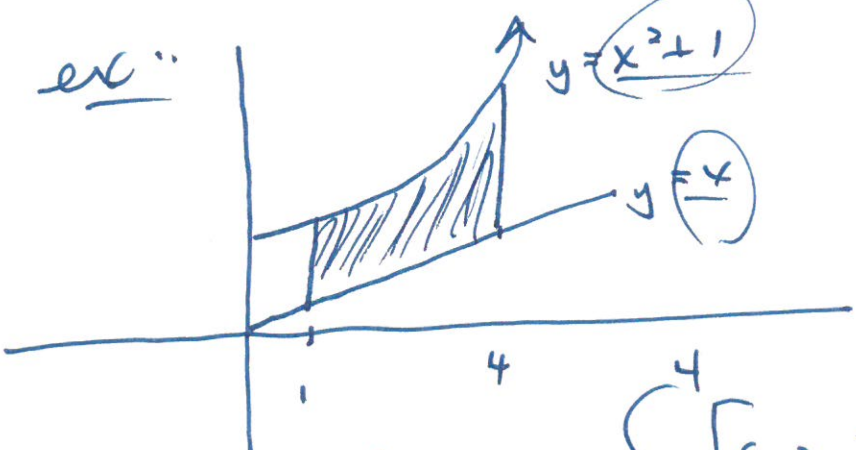


$$\Sigma \qquad \underbrace{\int_a^b \underbrace{\left(f(x) - g(x)\right)}_{\text{HEIGHT}} \; \underbrace{dx}_{\text{WIDTH}}}_{\text{AREA OF RECTANGLE}}$$

HT: 7 − (−5)



ex:



$$\text{AREA} = \int_1^4 \left[(x^2+1) - (x)\right] dx$$

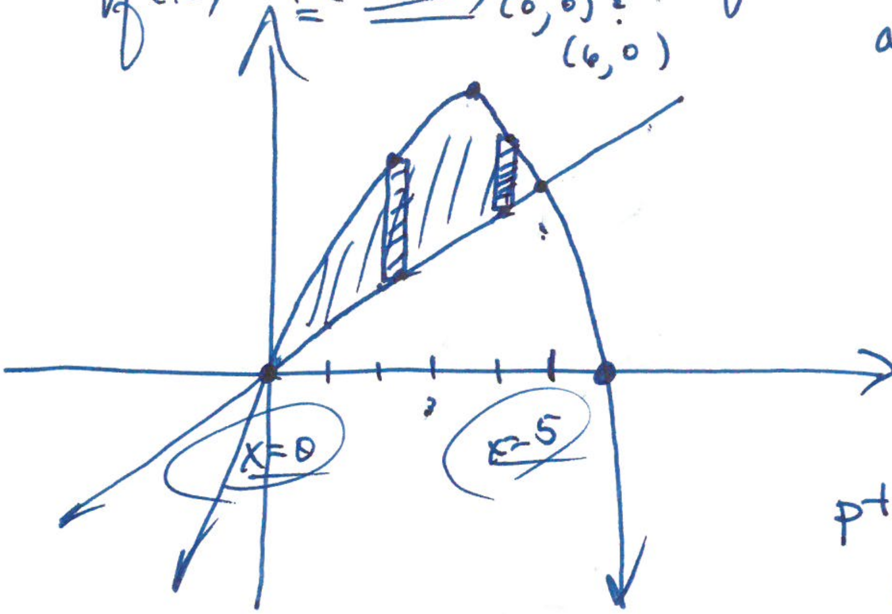
$$= \int_1^4 (x^2 + 1 - x)\, dx$$

$$= \frac{x^3}{3} + x - \frac{1x^2}{2}\Bigg]_1^4 = \left(\frac{4^3}{3} + 4 - \frac{(4)^2}{2}\right) - \left(\frac{1^3}{3} + 1 - \frac{(1)^2}{2}\right)$$

$f(x) = 6x - x^2$ $\qquad f(x) = x$

$f(x) = x(6 - x)$ $\quad$ (0,0) & (6,0)

a = ? $\quad$ b = ?



$v(3, ?) = (3, 9)$

$f(3) = 6(3) - 3^2$

$= 18 - 9 = 9$

pts of int:

$6x - x^2 = x$

$0 = x^2 + x - 6x$

$0 = x^2 - 5x$

$0 = x(x - 5)$

$x = 0$ $\qquad x - 5 = 0$

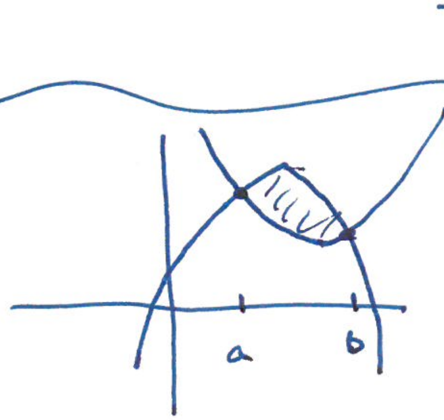
$\qquad\qquad x = 5$

$$A = \int_0^5 \left(\left[6x - x^2 \right] - \left[x \right] \right) dx$$

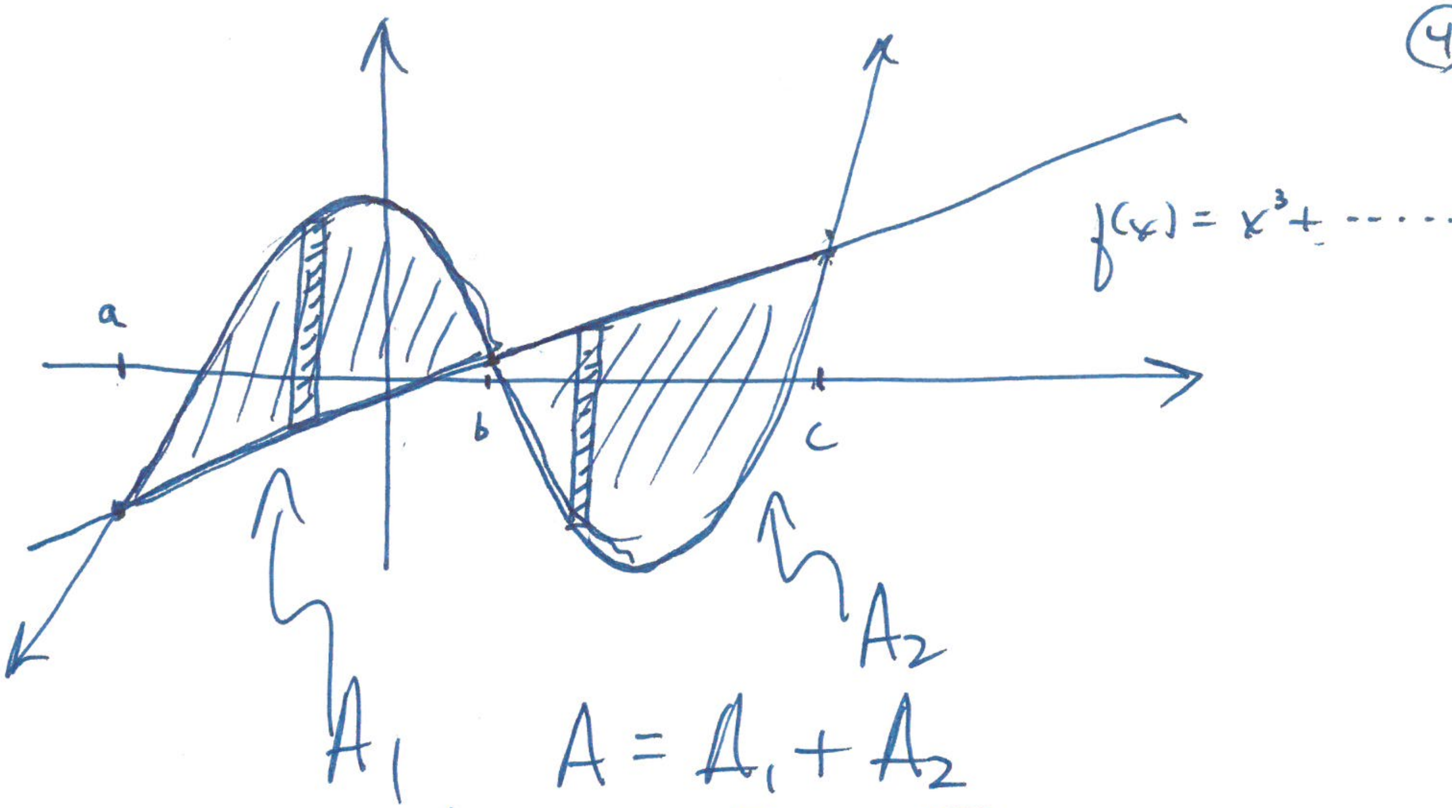
$$\int_0^5 (5x - x^2)\, dx = \frac{5}{2}x^2 - \frac{x^3}{3} \Big]_0^5$$

$$= \left(\frac{5}{2}(5)^2 - \frac{(5)^3}{3} \right) - (0)$$

$$= \underline{\qquad\qquad}$$



set $f(x) = g(x)$



$$\underbrace{A = A_1 + A_2}$$

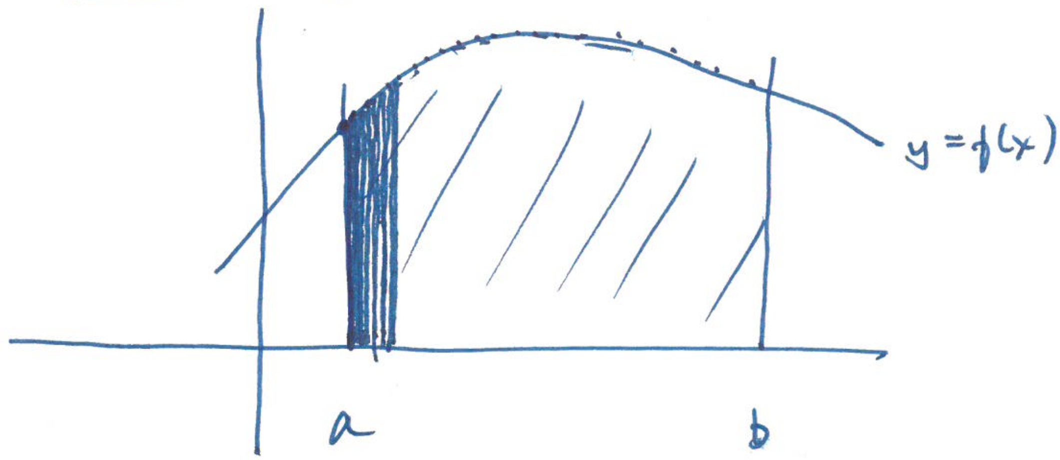
$$\underline{A_1} = \int_a^b \left[(\text{cubic polynom}) - (\text{line})\right] dx$$

$$\underline{A_2} = \int_b^c \left[(\text{line}) - (\text{cubic polynom})\right] dx$$

AVERAGE VALUE OF A FUNCTION:

(AVE. Y-VALUE)

(AVE. HEIGHT)



$$\frac{\text{AREA UNDER THE CURVE}}{\text{WIDTH}}$$

$$\frac{\int_a^b f(x)\,dx}{b-a} = \text{AVE. HT}.$$

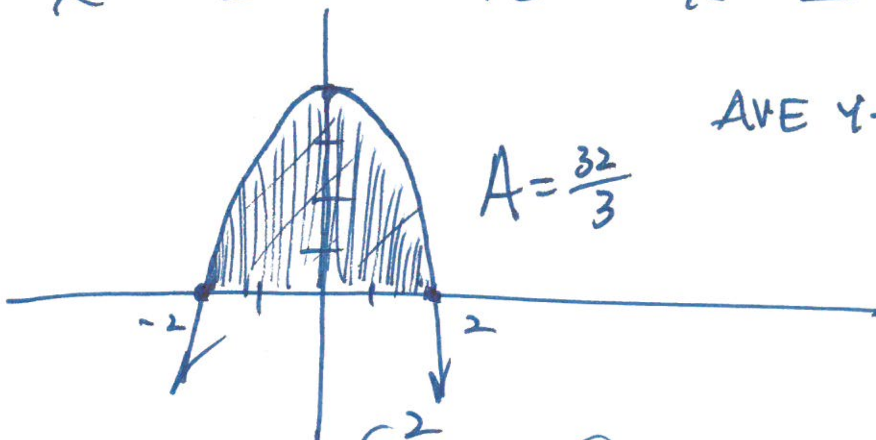
$$\frac{1}{b-a}\left[\int_a^b f(x)\,dx\right] = \text{AVE HT}$$

(AVE Y-VAL.)

$f(x) = 4 - x^2$

find the average value of the function from

$x = -2$ to $x = 2$



$A = \frac{32}{3}$

AVE Y-VALUE:

$\cancel{2}$ (GUESS)

$8/3$

$$\text{AVE Y-VAL} = \frac{1}{4}\int_{-2}^{2} (4 - x^2)\,dx$$

$$= \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{1}{4}\left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$= \frac{1}{4}\left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right] = \frac{1}{4}\left[16 - \frac{16}{3}\right]$$

$$= \frac{1}{4}\left[\frac{48}{3} - \frac{16}{3}\right]$$

$$= \frac{1}{4}\left[\frac{32}{3}\right] = \frac{8}{3}$$

4.5: INTEGRATION USING SUBST.

$$\frac{1}{12}\int (3t^4+2)^5 \cdot t^3 \cdot dt \cdot 12 \qquad t;\, dt$$

let $u = 3t^4+2$

$$dt \frac{du}{dt} = 12t^3 \cdot dt$$

$$du = 12t^3 \cdot dt$$

$$\frac{1}{12}\int u^5 \cdot du \qquad u;\, du$$