

MA 121-003

Monday, November 12

(1)

- interview quiz returned
- Q2 handed out; due WED 11/14 - beg. of class
- finish 4.5: (integration using SUBST)
- begin CH 5: 5.1: consumer surplus; producer surplus
- return TEST #3:

$$\int x \cdot \sqrt[5]{1-x^2} dx$$

$$\left(-\frac{1}{2}\right) \int (1-x^2)^{1/5} \cdot \underline{x} \cdot \underline{dx} \cdot (-2)$$

$$\text{let } u = 1-x^2$$

$$dx \frac{du}{dx} = -2x \cdot dx$$

$$-\frac{1}{2} \int u^{1/5} \cdot du$$

$$du = -2x \cdot dx \quad \checkmark$$

$$-\frac{1}{2} \left[\frac{u^{6/5}}{6/5} \right] + C$$

$$-\frac{1}{2} \cdot \frac{5}{6} \left[1-x^2 \right]^{6/5} + C$$

$$= -\frac{5}{12} (1-x^2)^{6/5} + C$$

4.5: INTEGRATION USING SUBST. (7)

$$\left(\frac{1}{12} \right) \int (3t^4 + 2)^5 \cdot t^3 \cdot \frac{dt}{12} \quad t; dt$$

let $u = 3t^4 + 2$

$$\frac{du}{dt} = 12t^3 \cdot dt$$

$$du = 12t^3 dt$$

$$\frac{1}{12} \int u^5 du \quad u; du$$

$$\frac{1}{12} \frac{u^6}{6} + C$$

$$\frac{1}{72} (3t^4 + 2)^6 + C$$

check: $d \left(\frac{1}{72} (3t^4 + 2)^6 + C \right) \stackrel{??}{=} \dots$

$$(3t^4 + 2)^5 \cdot t^3$$

DEF. INTEGRAL

(2)

$$\int_{x=1}^{x=4} \frac{2x+1}{x^2+x+1} dx$$

$$\begin{array}{l} \underline{x=1} \xrightarrow{u=x^2+x+1} u=3 \\ \underline{x=4} \longrightarrow u=21 \end{array}$$

let $u = x^2 + x + 1$

$du = (2x+1) dx$

~~$\frac{du}{dx} = (2x+1) \cdot dx$~~

$$\frac{du}{u} = \int \frac{1}{u} du$$

$$= \ln|u| \Big|_3^{21}$$

$$= \ln|21| - \ln|3| = \ln \frac{21}{3} = \ln 7$$

OR

$$\int \frac{du}{u} = \int \frac{1}{u} du$$

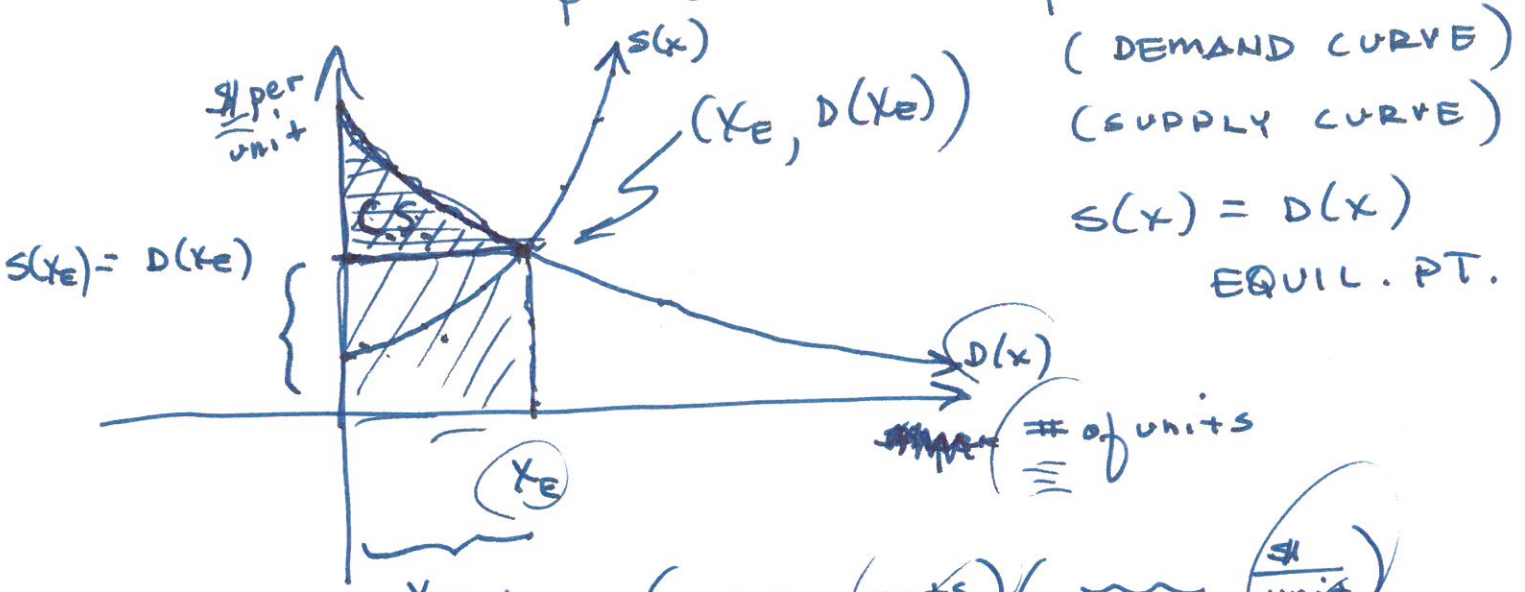
$$\ln|u| \Big|_1^4 = \ln|x^2+x+1| \Big|_1^4$$

$$= \ln|4^2+4+1| - \ln|1^2+1+1|$$

$$= \ln|21| - \ln(3) = \ln \frac{21}{3}$$

$$= \ln 7$$

S.1: consumer surplus & producer surplus



(DEMAND CURVE)
 (SUPPLY CURVE)
 $S(x) = D(x)$
 EQUIL. PT.

$x_e A = (\text{units}) (\frac{\$}{\text{unit}})$

$A = \$ (\text{units})$

ex: $(14 \text{ items}) (3^{\text{op}} / \text{item})$

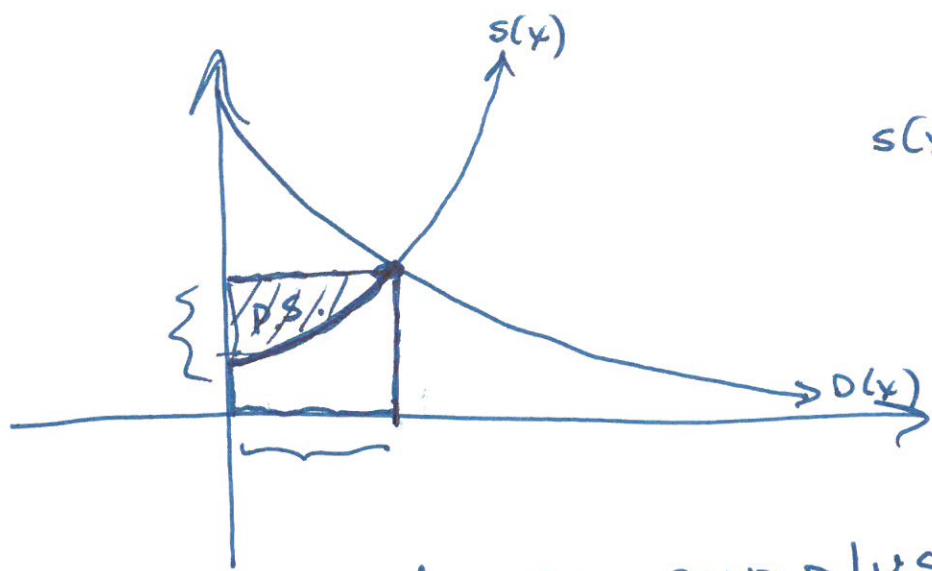
money box = $= \$42^{00}$

C.S. \rightarrow not real money.

C.S. = $\int_0^{x_e} D(x) dx$ — money box

what is it? C.S. = 2.61

good feeling or sense that you "got ~~the~~ more than you paid for" (not \$)



$$S(x) = D(x)$$

p.s = real \$

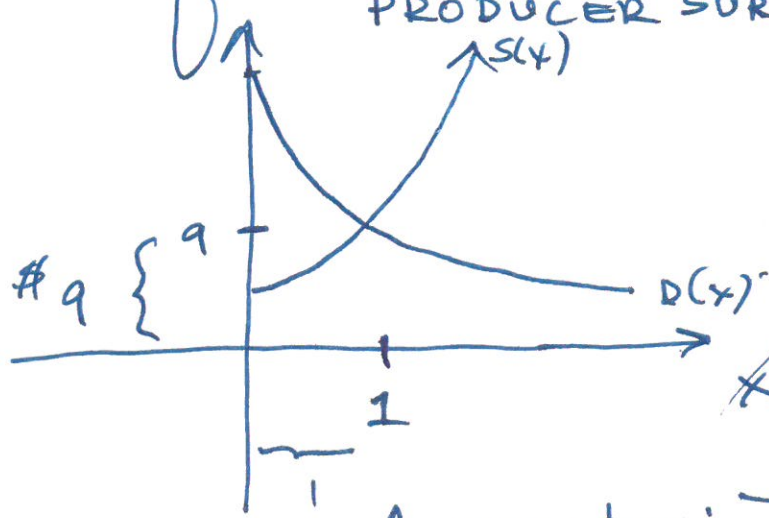
producer surplus:

$$P.S. = (\text{money box}) - \int_0^{x_E} S(x) dx$$

producer surplus - real \$ - part of the revenue (maybe the profit) for the seller.

ex: $D(x) = (x-4)^2$ $S(x) = x^2 + 2x + 6$

find EQ. PT; CONSUMER SURPLUS; PRODUCER SURPLUS



EQ. PT:

$$D(x) = S(x)$$

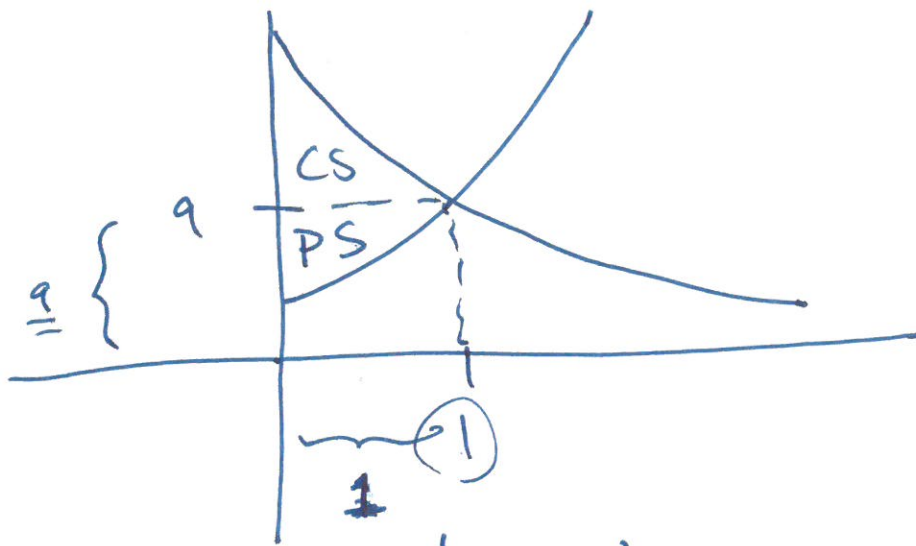
$$(x-4)^2 = x^2 + 2x + 6$$

$$\begin{array}{r} x^2 - 8x + 16 = x^2 + 2x + 6 \\ \hline -8x + 16 = 2x + 6 \\ \hline -8x - 2x = 6 - 16 \\ -10x = -10 \\ \hline 10 = 10x \\ 1 = x \end{array}$$

A money box:
 $(1)(9) = 9$

$$\begin{array}{r} 10 = 10x \\ 1 = x \end{array}$$

(5)



$$C.S. = \int_0^1 (x-4)^2 dx - (1)(a)$$

$$= \int_0^1 (x^2 - 8x + 16) dx - 9$$

$$= \left[\frac{x^3}{3} - \frac{8}{2}x^2 + 16x \right]_0^1 - 9$$

$$= \left(\frac{1}{3} - 4 + 16 \right) - 9 = 3\frac{1}{3} \approx 3.33$$

$$P.S. = (1)(a) - \int_0^1 (x^2 + 2x + 6) dx$$

$$= 9 - \left[\frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 6x \right]_0^1$$

$$= 9 - \left[\left(\frac{1}{3} + 1 + 6 \right) - (0) \right]$$

$$= 9 - 7\frac{1}{3} = 1\frac{2}{3} \approx 1.67$$

Name _____ Row ___ Seat ___

NORTH CAROLINA STATE UNIVERSITY

Department of Mathematics

MA121-003 Quiz #2 Due Wednesday, November 14 (at the beginning of class) J. Griggs

Three points per question (1 point for following directions); you are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet. (use the back, if necessary; no additional pages, please)

1.) Find the area bounded by the two curves: $f(x) = x^2 - x - 5$ and $g(x) = x + 10$.

2.) Find the average value of the function $f(x) = x^2 - x + 1$ on $[0, 2]$.

3.) Integrate using substitution: $\int (2t^5 - 3)^2 t^4 dt$

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TEST #3 RESULTS:

A's :	<u>82</u>	(40.4%)	}	<u>60.6%</u>
B's :	<u>41</u>	(20.2%)		
C's :	<u>51</u>	(25.1%)		
D's :	<u>16</u>	(7.9%)	}	<u>14.3%</u>
F's :	<u>13</u>	(6.4%)		

AVE: 82.6