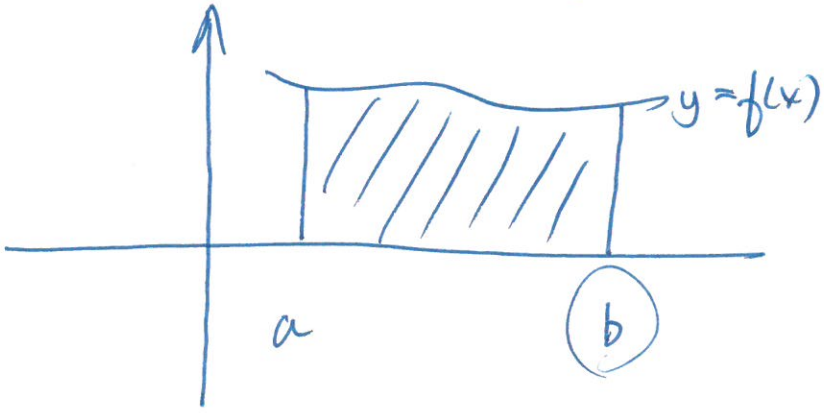
MA 121 - 003
Monday, November 19

5.3: improper integrals
5.6: volumes of solids of revolution

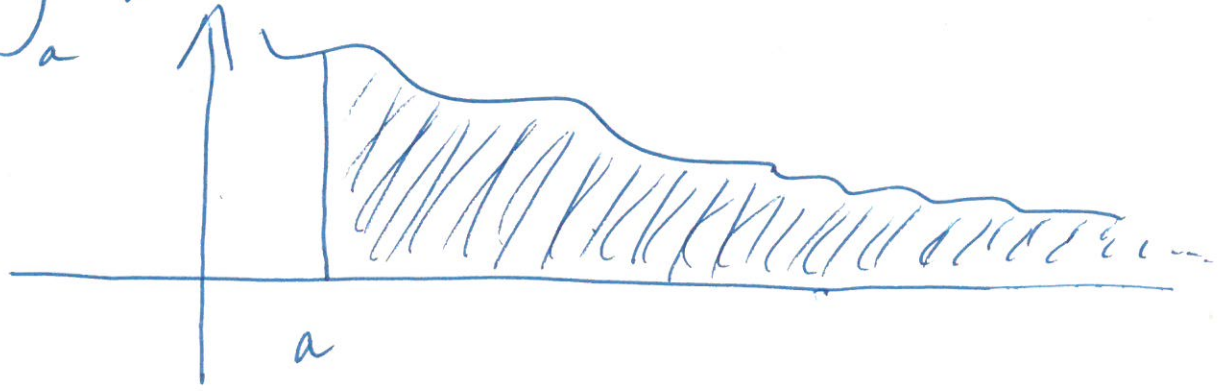
(handouts Q3 – due mon, NOV 26th)
(Q2 returned; solutions posted)

5.3: IMPROPER INTEGRALS

Proper: $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

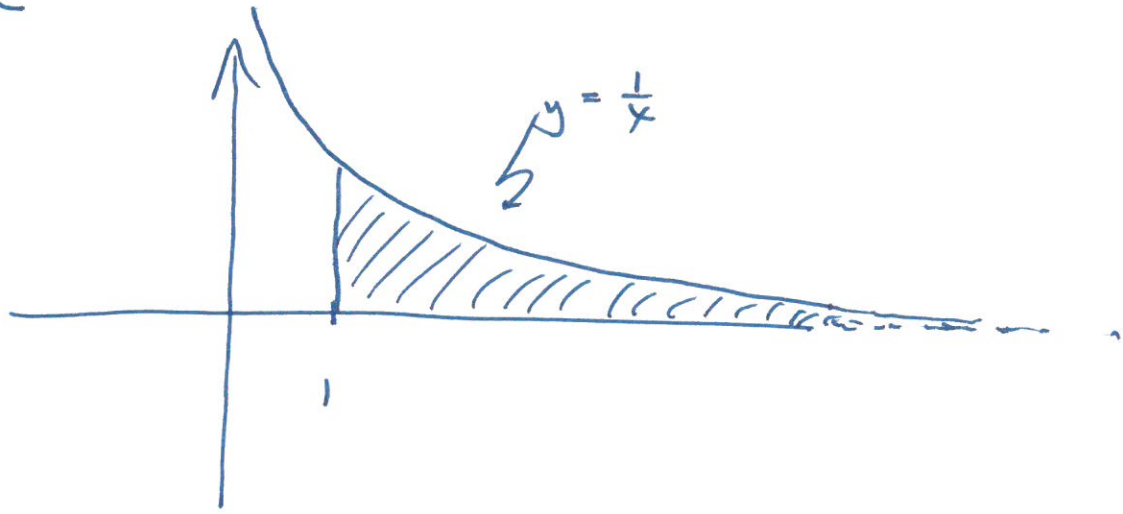


$\int_a^{\infty} f(x)\,dx = F(x)\Big]_a^{\infty} =$ _____



ex: $y = \frac{1}{x}$ from $x = 1$ to $x \to \infty$

$$\int_1^{\infty} \frac{1}{x}\,dx =$$



$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{A\to\infty} \int_1^{A} \frac{1}{x}\,dx$$

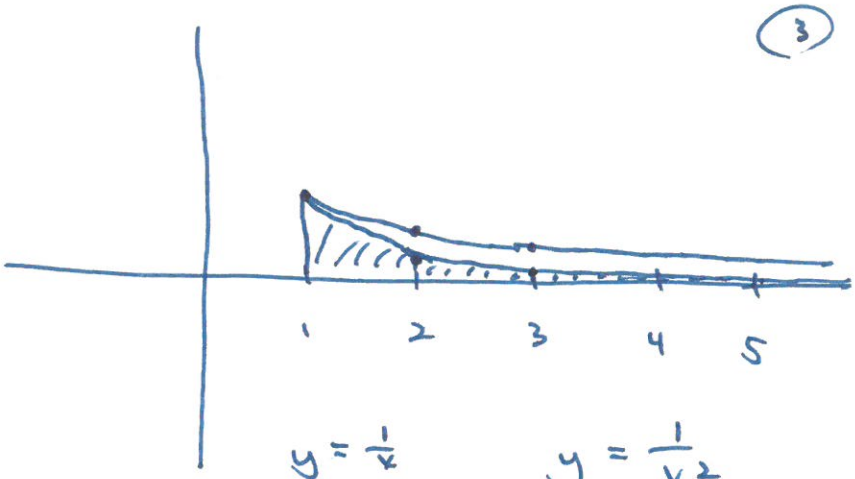
$$= \lim_{A\to\infty} \Big[\ln|x| \Big]_1^{A} = \lim_{A\to\infty} \left(\ln|A| - \underbrace{\ln|1|}_{\to 0} \right)$$

$$= \lim_{A\to\infty} (\ln A) = \text{NO LIMIT}$$

$A = 100$, $\ln 100$; $A = 10{,}000$, $\ln 10{,}000$; $A = 100{,}000{,}000$, $\ln 100{,}000{,}000$

Integral DIVERGES

$$\int_1^\infty \frac{1}{x^2}\,dx$$



$$\lim_{B\to\infty} \int_1^B \frac{1}{x^2}\,dx$$

$y = \frac{1}{x}$

x	y
1	1
2	1/2
3	1/3
	1/4

$y = \frac{1}{x^2}$

x	y
1	1
2	1/4
3	1/9
	1/16

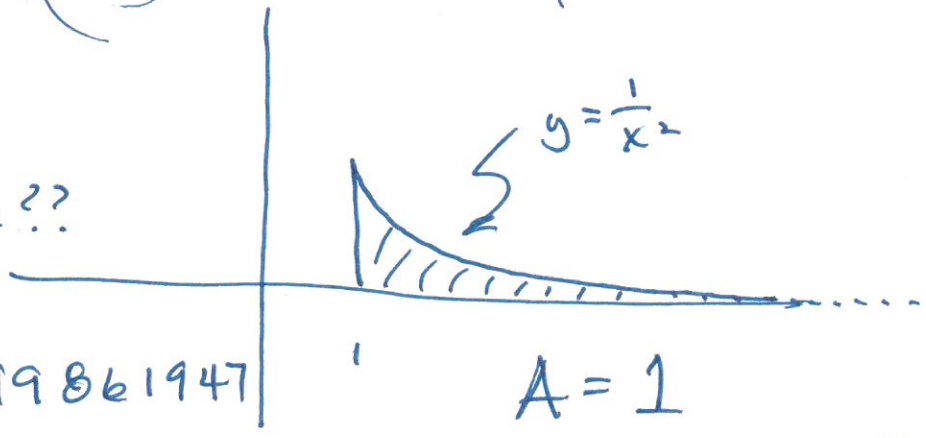
$$= \lim_{B\to\infty} \int_1^B x^{-2}\,dx$$

$$= \lim_{B\to\infty} \left[\frac{x^{-1}}{-1}\right]_1^B$$

$$= \lim_{B\to\infty} \left[\frac{-1}{x}\right]_1^B = \lim_{B\to\infty} \left[\left(\frac{-1}{B}\right) - \left(\frac{-1}{1}\right)\right]$$

$$= \left\{\lim_{B\to\infty} \left(1 - \frac{1}{B}\right) \to 0 = 1\right\}$$

$$\int_1^{628{,}795} \frac{1}{x^2}\,dx = ??$$



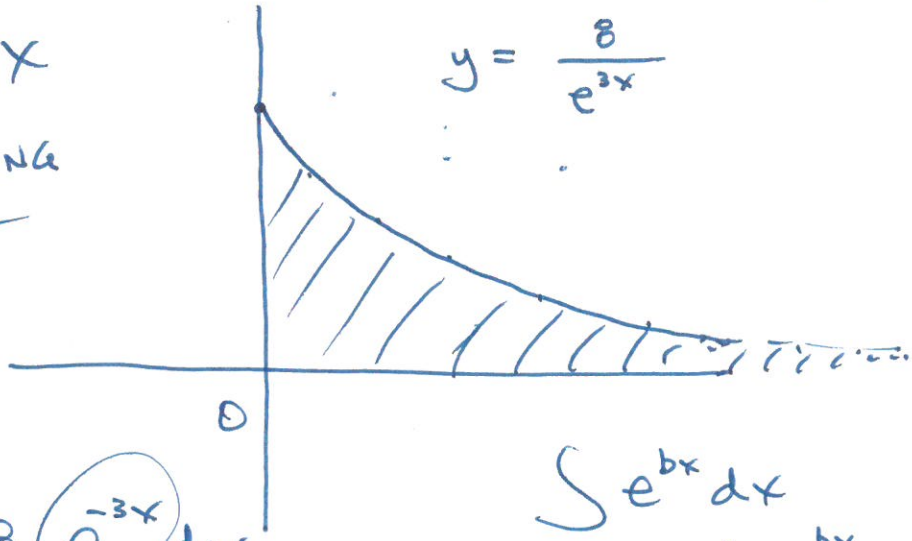
$= .999998619\overline{47}$

$A = 1$

$\therefore$ this integral CONVERGES

$$\int_0^\infty 8 \cdot e^{-3x}\,dx$$

↑ DECREASING



$$\int e^{bx}\,dx = \frac{1}{b}e^{bx}$$

$$\lim_{A\to\infty} \int_0^A 8e^{-3x}\,dx$$

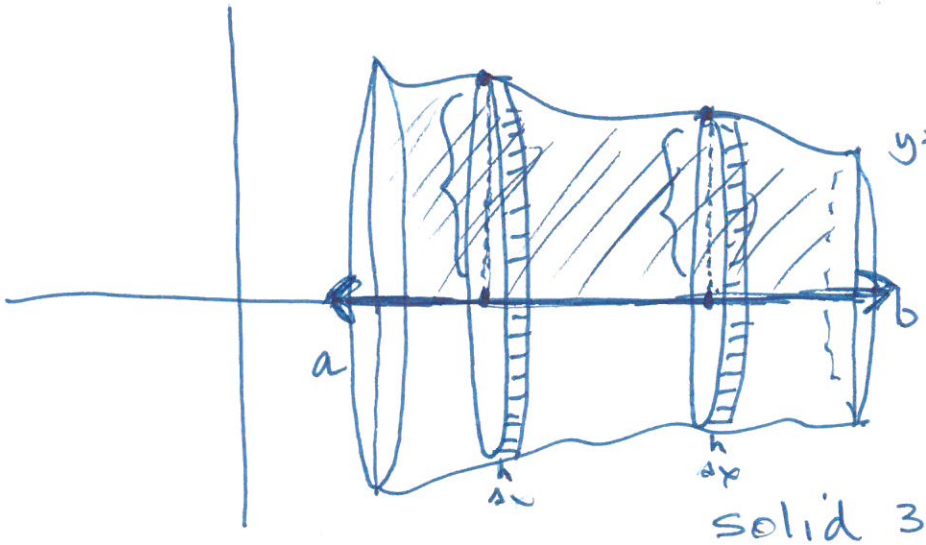
$$= \lim_{A\to\infty} \left[8 \cdot \frac{1}{-3}e^{-3x}\right]_0^A$$

$$= \frac{8}{-3} \lim_{A\to\infty} \left[\frac{1}{e^{3x}}\right]_0^A = -\frac{8}{3} \lim_{A\to\infty} \left[\frac{1}{e^{3A}} - \frac{1}{e^{3\cdot 0}}\right]$$

$$= -\frac{8}{3} \lim_{A\to\infty} \left[\frac{1}{e^{3A}} - 1\right] = \frac{-8}{3}(-1) = \frac{8}{3}$$

this integral converges

5.6: VOLUMES OF SOLIDS OF REVOLUTION



solid 3D figure

revolve about (around) the x-axis

↑ AXIS OF REVOLUTION

CYLINDER

ACCUMULATION MODEL:

$\int f(x)\,dx$

VOL OF DISK:

$$V = \underbrace{\pi r^2}_{\text{area of base}} \cdot h$$

$r = ? = y = f(x)$

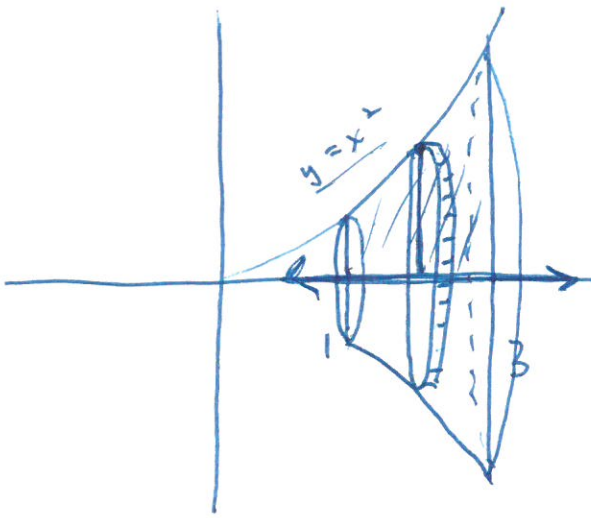
$h = ? = \Delta x \to dx$

$$VOL = \int_a^b \pi r^2 \cdot h$$

$$V = \int_a^b \pi (f(x))^2 \cdot dx$$

$y = x^2$ $\quad x = 1$ to $x = 3$

revolve this bounded region about the x-axis



$$V = \int_1^3 \pi \cdot r^2 \cdot h$$

$r = y = x^2$

$h = \Delta x \to dx$

$$V = \int_1^3 \pi (x^2)^2 \cdot dx$$

$$V = \pi \int_1^3 x^4 \, dx$$

$$V = \pi \left[\frac{x^5}{5} \right]_1^3$$

$$V = \frac{\pi}{5} \left[3^5 - 1^5 \right]$$

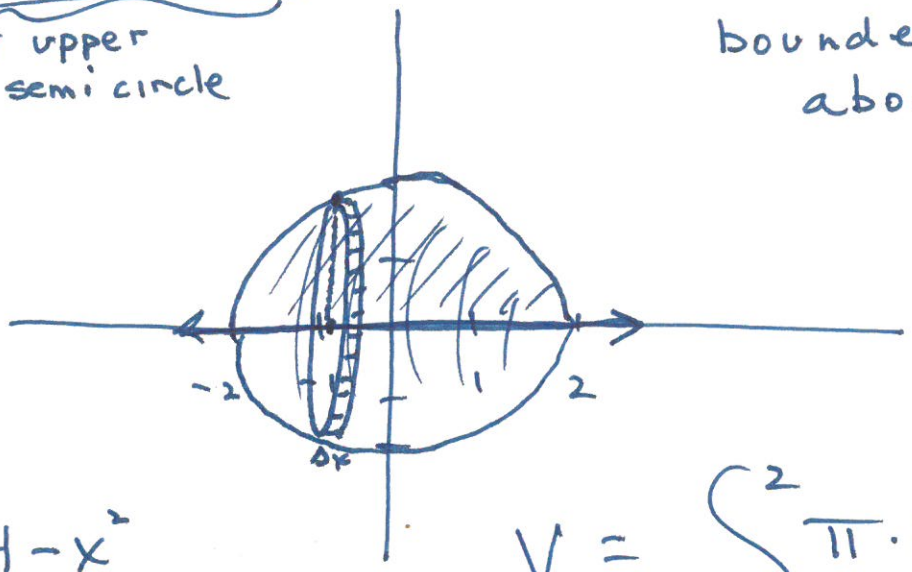
$$V = \frac{\pi}{5} [243 - 1] = \frac{242\pi}{5}$$

$y = \sqrt{4 - x^2}$

upper semi circle

$x = -2$ to $x = 2$

bounded region about the x-axis



$r = y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

circle; $C(0,0)$; $r = 2$

$$V = \int_{-2}^{2} \pi \cdot r^2 \cdot h \qquad h = \Delta x \Rightarrow dx$$

$$V = \pi \int_{-2}^{2} \left(\sqrt{4-x^2}\right)^2 \cdot h$$

$$V = \pi \int_{-2}^{2} (4 - x^2) \cdot dx$$

$$V = \pi \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$V = \pi \left[\left(4 \cdot 2 - \frac{8}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$V = \pi \left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right]$$

$$V = \pi \left[16 - \frac{16}{3}\right] = \pi \left(\frac{48}{3} - \frac{16}{3}\right)$$

$$V = \pi \cdot \frac{32}{3} = \frac{32\pi}{3}$$

sphere

Sphere:

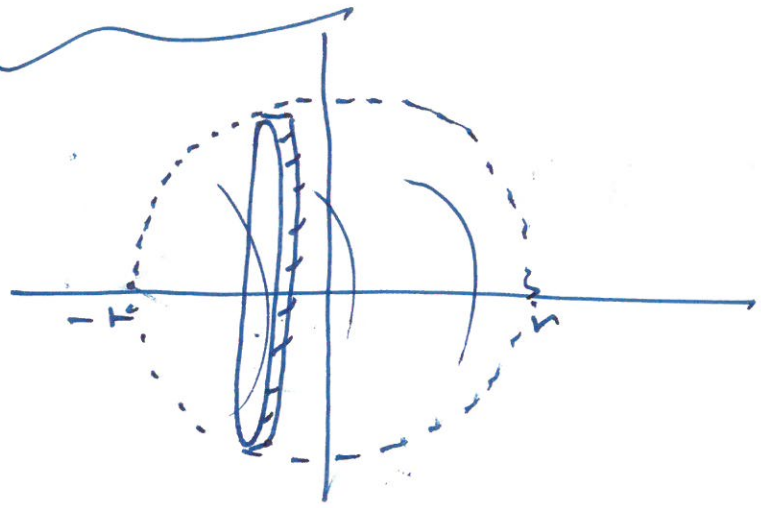
$$VOL = \frac{4}{3}\pi \cdot r^3$$

$$V = \frac{4}{3}\pi (2)^3$$

$$V = \frac{32\pi}{3}$$

sphere

$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \frac{4}{3}\pi r^3$$



NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA121 Quiz #2 — J. Griggs

Three points per question; you are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet.

1.) Find the area bounded by the two curves: $f(x)=x^2-x-5$ *and* $g(x)=x+10$.

3 PTS

pts of int: $x^2-x-5 = x+10$

$x^2-x-5-x-10=0$

$x^2-2x-15=0$

$(x+3)(x-5)=0$

$x=-3$ and $x=5$

$$A=\int_{-3}^{5}\left[(x+10)-(x^2-x-5)\right]dx$$

$$=\int_{-3}^{5}(-x^2+2x+15)\,dx$$

$$=\left[\frac{-x^3}{3}+x^2+15x\right]_{-3}^{5}$$

$$=\left[\frac{-5^3}{3}+5^2+15(5)\right]-\left[\frac{-(-3)^3}{3}+(-3)^2+15(-3)\right]$$

$$=\left(\frac{-125}{3}+25+75\right)-\left(\frac{27}{3}+9-45\right)$$

$$=145-18-\frac{125}{3}$$

$$=\frac{256}{3}=A=85\tfrac{1}{3}$$

3 PTS

2.) Find the average value of the function $f(x)=x^2-x+1$ *on* $[0,2]$.

$$\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{2-0}\int_0^2(x^2-x+1)\,dx=\frac{1}{2}\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^2$$

$$=\frac{1}{2}\left[\left(\frac{2^3}{3}-\frac{2^2}{2}+2\right)-(0)\right]=\frac{1}{2}\left[\frac{8}{3}-2+2\right]=\frac{4}{3}=f_{AVE}$$

3 PTS

3.) Integrate using substitution: $\int(2t^5-3)^2\,t^4\,dt$

let $u=2t^5-3$

$du=\left[2(5t^4)-0\right]dt$

$du=10t^4\,dt$

$$\frac{1}{10}\int\underbrace{(2t^5-3)^2}_{u^2}\cdot\underbrace{t^4\cdot dt\cdot 10}_{du}$$

$$\frac{1}{10}\int u^2\cdot du=\frac{1}{10}\cdot\frac{u^3}{3}+C$$

$$=\frac{1}{30}(2t^5-3)^3+C$$

NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-003 Quiz #3 Due Monday, November 26, 2018 (at the beginning of class) J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

1.) Evaluate the improper integral $\int_2^\infty 7x^{-2}\,dx$. Doe this integral converge or diverge?

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5 \;\; for -5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of $200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?