MA 121 - 003

Monday, November 26

5.7: SEPARABLE DIFFERENTIAL EQUATIONS

(D.E.'s) has a DERIV in it.

ex: $y' = 11x$ find y:

$$\int 11x\,dx = 11\frac{x^2}{2} + C$$

$$y = \frac{11}{2}x^2 + C \qquad y(2) = 7$$

$x = 2$
$y = 7$

$$7 = \frac{11}{2}(2)^2 + C$$

$$7 = \frac{11}{2}(4) + C$$

$$7 = 22 + C$$

$$-22 \qquad -22$$

$$-15 = C$$

$$y = \frac{11}{2}x^2 - 15$$

ex: $y'' - 2y' + y = 0$ (2nd order D.E.)

solution: $y = \boxed{-2e^x + x \cdot e^x}$

verify the solution:

$$y' = \boxed{-2e^x + x\cdot(e^x) + (e^x)(1)}$$

$$y'' = \boxed{-2e^x + x(e^x) + (e^x)(1) + e^x}$$

$$(-2e^x + xe^x + 2e^x) - 2(-2e^x + xe^x + e^x) + (-2e^x + xe^x) \stackrel{?}{=} 0$$

$$-2e^x + xe^x + 2e^x + 4e^x - 2xe^x - 2e^x + -2e^x + xe^x \stackrel{?}{=} 0$$

$0 = 0$

$\xleftarrow{y's}$ $\xrightarrow{x's}$

$$\cancel{dx}\,\frac{dy}{\cancel{dx}} = \frac{2x}{y}\cdot d\cancel{x} \quad (\text{SEP. D.E.})$$

solve for y . . . .

sep. x's & dx's from the y's & dy's

① sep x's & dx's from y's & dy's

② integrate both sides

$$y\,dy = \frac{2x}{\cancel{y}}\cdot dx\cdot\cancel{y}$$

$$y\,dy = 2x\,dx$$

$$\int y\,dy = \int 2x\,dx$$

$$\cancel{2}\,\frac{x^2}{\cancel{2}}$$

$$\frac{y^2}{2} + \underbrace{C_1}_{-C_1} = x^2 + \underbrace{C_2}_{-C_1}$$

$$\frac{y^2}{2} = x^2 + C$$

$$y^2 = 2x^2 + K$$

$$y = \pm\sqrt{2x^2 + K}$$

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = 5x^4 \cdot y \cdot dx$$

$$\frac{1}{y} \cdot dy = 5x^4 \cdot \cancel{y} \cdot dx \cdot \frac{1}{\cancel{y}}$$

$$\frac{1}{y} \cdot dy = 5x^4 \, dx$$

$$\int \frac{1}{y} \, dy = \int 5x^4 \cdot dx$$

$$\ln y = \cancel{5} \cdot \frac{x^5}{\cancel{5}} + C$$

$$\ln y = x^5 + C$$

solve for y . . . . .

$$e^{\ln y} = e^{x^5 + C}$$

$$y = e^{x^5 + C}$$

$$y = e^{x^5} \cdot e^{C}$$

$$(e^C = K)$$

$$\boxed{y = K \cdot e^{x^5}}$$

ex:

$$\frac{dP}{dt} = K \cdot P$$

(exp. GROWTH)

$$\cancel{dt} \, \frac{dP}{\cancel{dt}} = K \cdot P \cdot dt$$

$$\frac{1}{P} \, dP = K \cdot \cancel{P} \cdot dt \cdot \frac{1}{\cancel{P}}$$

$$\frac{1}{P}\,dP = K \cdot dt$$

$$\int \frac{1}{P}\,dP = \int \underline{K} \cdot dt$$

$$\ln P = \underline{k \cdot t} + C$$

solve for P:

$$e^{\ln P} = e^{\overline{kt+c}}$$

$e^c = A$

$$P = e^{kt} \cdot e^c$$

$$\boxed{P = A \cdot e^{kt}}$$

$t = 0$
$P = P_0$

$$P_0 = A \cdot e^{k(0)}$$

$$P_0 = A$$

$$\underline{P = P_0 \cdot e^{kt}}$$ exp. growth

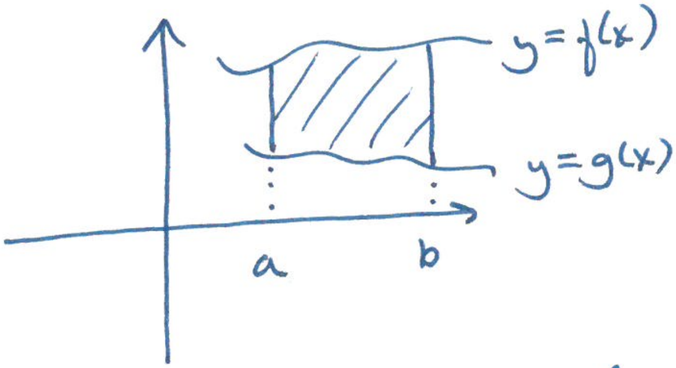
$$y' = \underline{2x - xy}$$

$$\frac{1}{2-y}\,\cancel{dx}\,\frac{dy}{\cancel{dx}} = x(2-y)\cdot dx\;\frac{1}{2-y}$$

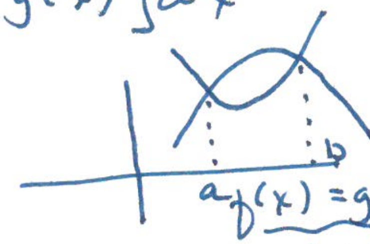
$$\int \frac{1}{2-y}\,dy = \int x\,dx$$

MA121 — TEST #4:

4.4: • area between 2 curves:



$$\int_a^b \left[f(x)-g(x)\right]dx$$



$f(x)=g(x)$
solve for x
a.....b

• average value of a function

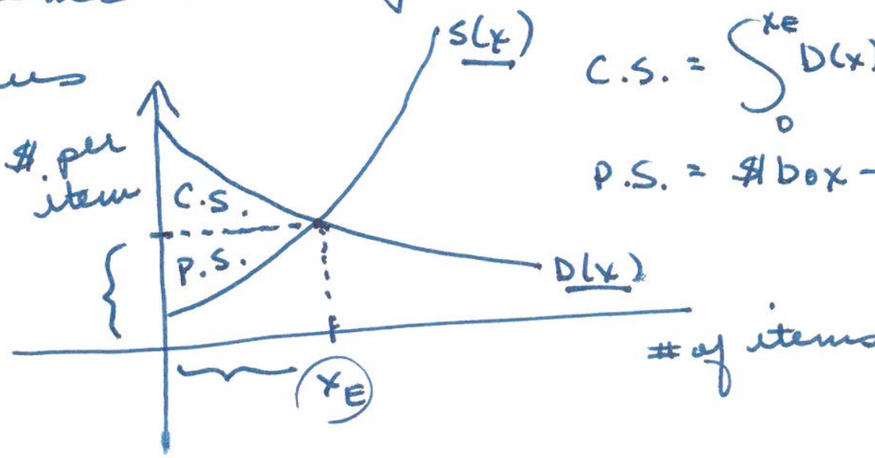
AVE HT. $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

4.5: • integration using SUBSTITUTION

let $u =$ ____

$du = 8x^4 \cdot dx$

5.1: • consumer surplus and producer surplus



$$C.S. = \int_0^{x_E} D(x)\,dx - \$\text{ box}$$

$$P.S. = \$\text{ box} - \int_0^{x_E} S(x)\,dx$$

EQ. PT: $D(x) = S(x)$

$x_E =$ ____

5.2: • accumulation models

① one-time contribution:

$$y = y_0 \cdot e^{kt}$$

② multiple (yearly) contributions:

$$\int_{T_0}^{T_1} y_0 e^{kt}\, dt \qquad \underset{\text{F.V.}}{\text{ACC}} = \int_0^{22} (10{,}000)\, e^{.015t}\, dt$$

(accumulated future value of a continuous income stream)

5.3: • improper integrals

$$\int_3^{\infty} f(x)\, dx \rightarrow \lim_{A \to \infty} \int_3^{A} f(x)\, dx$$

① delay ∞ 'til end of prob. (lim ...)

② integrate

③ evaluate

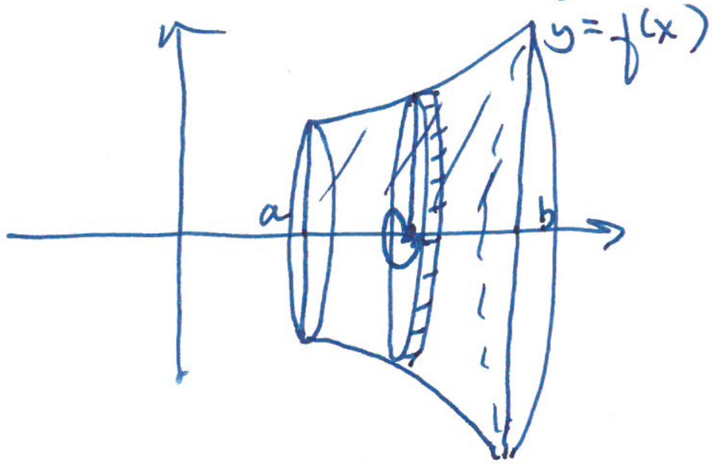
④ simplify

⑤ take limit. ($A \to \infty$)

⑥ converge or diverge

(page 3)

<u>5.6:</u> • volumes of solids of revolution



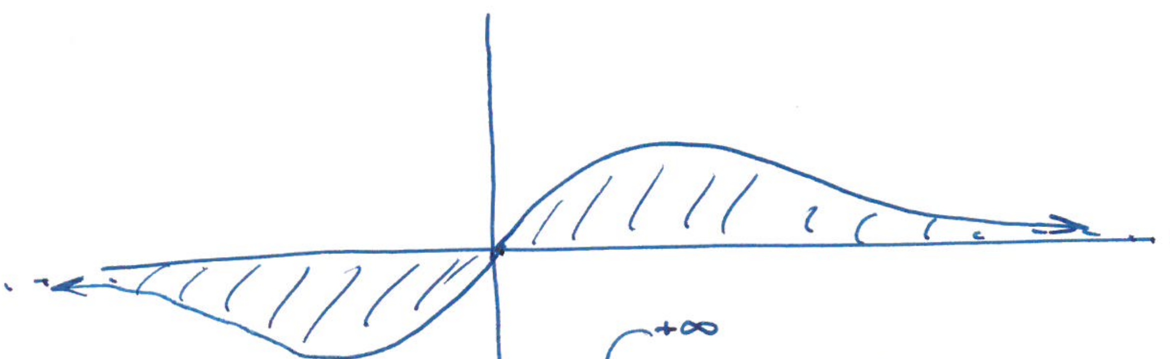
$$V = \int_a^b \pi \left(f(x)\right)^2 \cdot dx$$

<u>5.7:</u> • separable differential equations

✓ ① separate x's & dx's from y's & dy's.

② integrate both sides (+C)

③ if possible, solve for y



$$\int_{-\infty}^{+\infty} f(x)\,dx$$

$$= \boxed{\int_{-\infty}^{0} f(x)\,dx} + \boxed{\int_{0}^{\infty} f(x)\,dx}$$

NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-003 Quiz #3 Due Monday, November 26, 2018 (at the beginning of class) J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on **this** sheet – use the back of this sheet, if necessary.

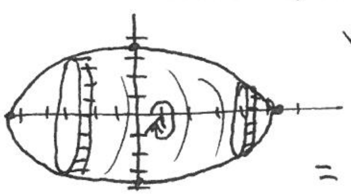
1.) Evaluate the improper integral $\int_2^\infty 7x^{-2}\,dx$. Does this integral converge or diverge?

$$\lim_{A\to\infty} 7\int_2^A x^{-2}\,dx = 7\cdot\lim_{A\to\infty}\left[\frac{x^{-1}}{-1}\right]_2^A = 7\cdot\lim_{A\to\infty}\left[\frac{-1}{x}\right]_2^A$$

$$= 7\cdot\lim_{A\to\infty}\left[\left(\frac{-1}{A}\right)-\left(\frac{-1}{2}\right)\right] = 7\cdot\lim_{A\to\infty}\left[\frac{1}{2}-\frac{1}{A}\xrightarrow{}0\right] = 7\cdot\frac{1}{2} = \frac{7}{2}$$

∴ integral converges

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions – i.e. the New England Patriots). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5$ $for -5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.



$$\text{VOL} = \int_{-5.5}^{5.5} \pi\,(f(x))^2\cdot dx = \pi\int_{-5.5}^{5.5}(-.116x^2+3.5)^2\,dx$$

$$= \pi\int_{-5.5}^{5.5}(.013456x^4 - .812x^2 + 12.25)\,dx$$

$$= \pi\left[.013456\frac{x^5}{5} - .812\frac{x^3}{3} + 12.25x\right]_{-5.5}^{5.5} = \pi\left[\left(\frac{.013456}{5}(5.5)^5\right.\right.$$

$$\left.- \frac{.812}{3}(5.5)^3 + (12.25)(5.5)\right) - \left(\frac{.013456}{5}(-5.5)^5 - \frac{.812}{3}(-5.5)^3 + (12.25)(-5.5$$

$$\approx 71.774\pi \approx 225.48 \text{ in}^3$$

3.) At age 31, Kelli earns her MBA and accepts a position as the creative team leader at Netflix. Assume that she will retire at the age of 65, having received an annual salary of \$200,000 per year, and that the interest rate is 2.9%, compounded continuously. What is the accumulated future value of her earnings at her new job?

$$\int_0^{34} 200{,}000\cdot e^{.029t}\,dt = 200{,}000\left[\frac{1}{.029}e^{.029t}\right]_0^{34}$$

$$= \frac{200{,}000}{.029}\left[e^{.029(34)} - e^{.029(0)}\right] = \frac{200{,}000}{.029}\left[2.68049 - 1\right]$$

$$\approx \$11{,}589{,}593.^{35}$$