NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-003 Test #1 Form A Monday, September 24, 2018 Dr. J. Griggs

Put all work and answers in the stamped blue book provided. One problem per page, please. The back of a sheet can serve as a new page. Make sure that your **name, row letter, seat number** and **form of the test** (A or B) are on the outside of your blue book. Calculators may be used; however, not a graphing calculator nor any calculator that does calculus). Eight questions – 12 points each – 4 points for following directions. If you have **not** yet turned in your blue books, please indicate this on the front of your blue book.

1.) Simplify the difference quotient completely:

$$\frac{[4(x+h)^2+5(x+h)-11]-[4x^2+5x-11]}{h}$$

2.) Find $f'(x)$ using the **DEFINITION OF DERIVATIVE**: $f(x)=\dfrac{2}{3x-1}$

(check your answer using the quotient rule)

3.) Find the vertex, all intercepts and graph: $f(x)=x^2-6x+4$

4.) Evaluate the following limits: a.) $\lim\limits_{x\to\infty}\dfrac{2x+5}{3x^2-4x+1}$ b.) $\lim\limits_{x\to 5}\dfrac{2x^2-10x}{x^2-4x-5}$

5.) Graph the following: $f(x)=\begin{cases} 1+x^2 & x<1 \\ 4-x & x\geq 1 \end{cases}$

For this function, find: $\lim\limits_{x\to 1^+} f(x)$ $\lim\limits_{x\to 1^-} f(x)$ $\lim\limits_{x\to 1} f(x)$

Is this function continuous at x=1? (verify; 3 possible steps)

6.) Find the domain of each function (in interval notation):

a.) $f(x)=\sqrt{2x-3}$ b.) $g(x)=\dfrac{2x^3-11}{x^2+5x+6}$

7.) Find the average rate of change of $f(x)$ from x = 1 to x = 4; find the instantaneous rate of change of $f(x)$ at x=3: $f(x)=6x^3-5x^2+2x-4$

8.) Find the equation of the line tangent to $f(x)=3x+\dfrac{8}{x}$ at the point (2,10).

Bonus: (5 points) Write out the words to NCSU's Alma Mater.

MA121 TEST #1 FORM A (12 PTS EACH) J. GRIGGS

1.) $$\frac{\left[4(x+h)^2+5(x+h)-11\right]-\left[4x^2+5x-11\right]}{h}$$

$$= \frac{\cancel{4x^2}+8xh+4h^2+\cancel{5x}+5h-\cancel{11}-\cancel{4x^2}-\cancel{5x}+\cancel{11}}{h}$$

$$= \frac{8xh+4h^2+5h}{h} = \frac{\cancel{h}(8x+4h+5)}{\cancel{h}} = 8x+4h+5 \quad (h\neq 0)$$

2.) $$f'(x) = \lim_{h\to 0}\frac{\left[\frac{2}{3(x+h)-1}\right]-\left[\frac{2}{3x-1}\right]}{h}$$

$$= \lim_{h\to 0}\frac{\left[\frac{2}{3(x+h)-1}\right]\cdot\left(\frac{3x-1}{3x-1}\right)-\left[\frac{2}{3x-1}\right]\cdot\frac{(3(x+h)-1)}{(3(x+h)-1)}}{h}$$

$$= \lim_{h\to 0}\left[\frac{2(3x-1)}{[3(x+h)-1][3x-1]}-\frac{2[3(x+h)-1]}{[3(x+h)-1][3x-1]}\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{2(3x-1)-2[3(x+h)-1]}{[3(x+h)-1][3x-1]}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{6x}-\cancel{2}-\cancel{6x}-6h+\cancel{2}}{[3(x+h)-1][3x-1]\cdot h}$$

$$= \lim_{h\to 0}\frac{-6\cancel{h}}{[3(x+h)-1][3x-1]\cancel{h}} \quad (h\neq 0)$$

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$$= \lim_{h \to 0} \frac{-6}{[3(x+\cancel{h})-1][3x-1]} = \frac{-6}{(3x-1)^2} = f'(x)$$

0

3.) $f(x) = x^2 - 6x + 4$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{6}{2} = 3$

$f(3) = 3^2 - 6(3) + 4$

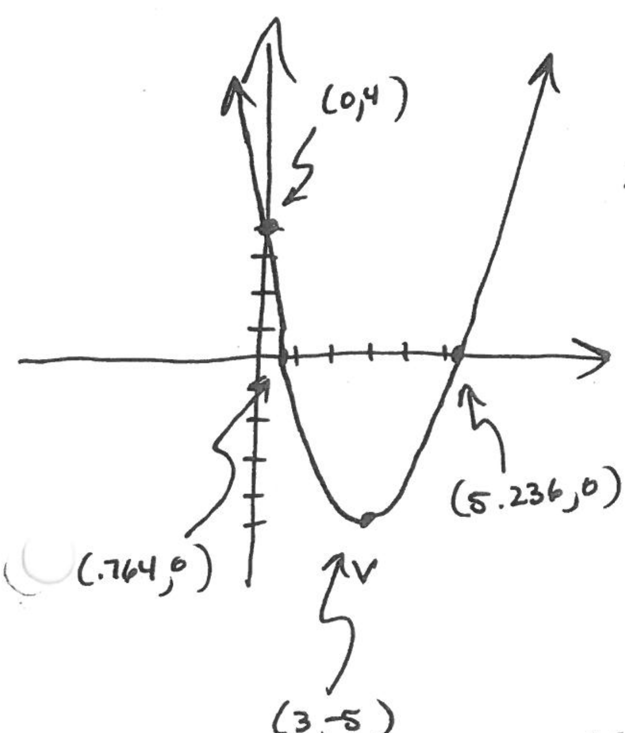
$f(3) = 9 - 18 + 4 = -5$

$V(3, -5)$

intercepts: ① y int: (set $x = 0$)

$f(0) = 0^2 - 6(0) + 4 = 4$

$(0, 4)$



2.) x-int: (set $y = 0$)

$0 = x^2 - 6x + 4$

quad formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(4)}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$x = 3 \pm \sqrt{5} \longrightarrow 5.236$

4.) a.) $\lim\limits_{x \to \infty} \dfrac{2x+5}{3x^2-4x+1} = \lim\limits_{x \to \infty} \dfrac{\frac{2x}{x^2} + \frac{5}{x^2}}{\frac{3x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x}}$

$= \lim\limits_{x \to \infty} \dfrac{\cancelto{0}{\frac{2}{x}} + \cancelto{0}{\frac{5}{x^2}}}{3 - \underbrace{\cancel{\frac{4}{x}}}_{0} + \underbrace{\cancel{\frac{1}{x^2}}}_{0}} = \dfrac{0}{3} = 0$

(OK to use "shortcut")

b.) $\lim\limits_{x \to 5} \dfrac{2x^2-10x}{x^2-4x-5} = \lim\limits_{x \to 5} \dfrac{2x\cancelto{1}{(x-5)}}{\cancelto{1}{(x-5)}(x+1)}$ $(x \neq 5)$

$= \lim\limits_{x \to 5} \dfrac{2x}{x+1} = \dfrac{10}{6} = \dfrac{5}{3}$

5.) $f(x) = \begin{cases} 1+x^2 & x < 1 \\ 4-x & x \geq 1 \end{cases}$

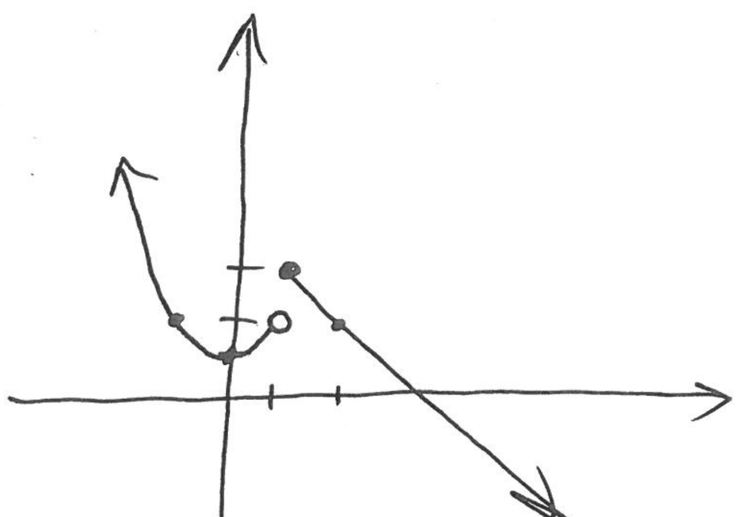
$y = 1+x^2$

x	y
1	2
0	1
−1	2
−2	5

delete

$y = 4-x$

x	y
1	3
2	2
3	1



$\left.\begin{array}{l} \lim\limits_{x \to 1^+} f(x) = 3 \\ \lim\limits_{x \to 1^-} f(x) = 2 \end{array}\right\}$ $\lim\limits_{x \to 1} f(x) =$ DOES NOT EXIST continuous at $x=1$?

1.) $f(1)$ exists? yes, $f(1) = 3$

2.) $\lim_{x \to 1} f(x)$ exists? no, lim D.N.E.

$\therefore f(x)$ is DISCONTIN at $x=1$

6.) $f(x) = \sqrt{2x-3}$

$2x - 3 \geq 0$

$2x \geq 3$

$x \geq 3/2$

$[3/2, +\infty)$

$$g(x) = \frac{2x^3 - 11}{x^2 + 5x + 6} = \frac{2x^3 - 11}{(x+3)(x+2)}$$

$x \in \mathbb{R}, x \neq -3, x \neq -2$

$(-\infty, -3) \cup (-3, -2) \cup (-2, +\infty)$

7.) $f(x) = 6x^3 - 5x^2 + 2x - 4$

average rate of change from $x = 1$ to $x = 4$

$(1, f(1))$ & $(4, f(4))$

$$\frac{f(4) - f(1)}{4 - 1} = \text{ave rate of change}$$

$f(4) = 6(4)^3 - 5(4)^2 + 2(4) - 4$

$f(4) = 384 - 80 + 8 - 4 = 308$

$f(1) = 6(1)^3 - 5(1)^2 + 2(1) - 4$

$f(1) = 6 - 5 + 2 - 4 = -1$

$$\frac{308 - (-1)}{4 - 1} = \frac{309}{3} = 103$$

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instantaneous rate of change at $x = 3$

$f'(x) = 6(3x^2) - 5(2x) + 2(1) - 0$

$f'(x) = 18x^2 - 10x + 2$

$f'(3) = 18(3)^2 - 10(3) + 2$

$f'(3) = 162 - 30 + 2 = 134$

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8) $f(x) = 3x + \frac{8}{x} = 3x + 8x^{-1}$

$f'(x) = m_{TAN} = 3(1) + 8(-1 \cdot x^{-2})$

$f'(x) = m_{TAN} = 3 - \frac{8}{x^2}$ @ $(2, 10)$

$f'(2) = 3 - \frac{8}{(2)^2} = 3 - 2 = 1 = m_{TAN}$

EQ: $y - 10 = 1(x - 2)$

The Alma Mater of NC State

Where the winds of Dixie softly blow o'er the fields of Caroline
There stands ever cherished, N.C. State, as thy honored shrine
So lift your voices! Loudly sing from hill to oceanside!
Our hearts ever hold you, N.C. State in the folds of our love
and pride

Words by Alvin Fountain : Class of '22
Music by Bonnie Norris: Class of '23

Compliments of the **Union Activities Board**

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