NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA121-003 Test #4 Form B Wednesday, November 28, 2018 Dr. John Griggs

Please put all work and solutions in the stamped blue book provided. Begin each new problem on a new page (the back of the sheet can be a new page). Put your **name, form of test** (A or B), **row letter,** and **seat number** on the outside of the blue book. Be sure to include proper units on the answers when appropriate. Fold the test copy and turn it in with your blue book.
(14 points per question – 2 points for following directions – show all work)

1.) Solve for y: $\frac{dy}{dx} = 6xy$

2.) Evaluate the improper integral; is the integral convergent or divergent? $\int_0^{\infty} 4e^{-3x}\,dx$

3.) Find the volume generated when the region bounded by $y = 3\sqrt{x}$, the x-axis, x=2 and x=4 is revolved about the x-axis.

4.) Find the equilibrium point, the consumer surplus (labeled CS) and producer surplus (labeled PS) for the given supply and demand functions: $D(x) = -.05x + 50 \quad S(x) = .1x + 20$

5.) Find the accumulated future value of a continuous income stream when \$4000 is invested each year for 17 years, and the interest rate is 2.6% compounded continuously.

6.) Find the area of the region bounded by the two curves: $y = 4x - x^2 \quad and \quad y = x^2 - 6x + 8$

7.) Integrate (using substitution) and evaluate: $\int_0^2 3x^2 e^{x^3}\,dx$

8.) A typist's speed over a 5-minute interval is given by $W(t) = -6t^2 + 12t + 90$, t in [0, 5], where W(t) is the speed in words per minute at time t. Find the average speed of the typist (average value of the function) over the 5-minute interval.

Bonus: (5 points) In 2013, annual world demand for crude oil was approximately 33.3 billion barrels, and it was projected to increase by 1.5% per year. World reserves of crude oil in 2013 were approximately 1635 billion barrels. Assuming that no new oil reserves are found, when will the reserves be depleted?

MA 121 TEST #4 FORM B J. GRIGGS

(12 points each)

1.) $\frac{dy}{dx} = 6xy$ solve for y ...

12 PTS

$$\frac{1}{y} \cdot \cancel{dx} \cdot \frac{dy}{\cancel{dx}} = 6x\cancel{y} \cdot dx \cdot \frac{1}{\cancel{y}}$$

$$\frac{1}{y} dy = 6x\,dx$$

$$\int \frac{1}{y} dy = \int 6x\,dx$$

$$\ln y = 6\left(\frac{x^2}{2}\right) + C$$

$$\ln y = 3x^2 + C$$

$$e^{\ln y} = e^{3x^2+C}$$

$$y = e^{3x^2} \cdot e^{C} \quad (e^{C} = A)$$

$$y = A \cdot e^{3x^2}$$

2.) $\int_0^{\infty} 4 \cdot e^{-3x} dx = \lim_{A \to \infty} \int_0^A 4 \cdot e^{-3x} dx$

12 PTS

$$= \lim_{A \to \infty} 4 \cdot \int_0^A e^{-3x} dx = 4 \cdot \lim_{A \to \infty} \left[\frac{1}{-3} e^{-3x}\right]_0^A$$

$$= \frac{4}{-3} \cdot \lim_{A \to \infty} \left[\frac{1}{e^{3x}}\right]_0^A = -\frac{4}{3} \lim_{A \to \infty} \left[\frac{1}{e^{3A}} - \frac{1}{e^{3(0)}}\right]$$

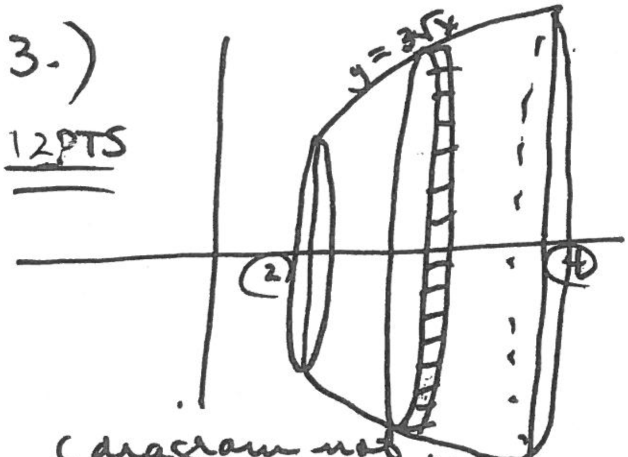
(page 2)

$$= \frac{-4}{3} \lim_{A\to\infty} \left[\cancel{\frac{1}{e^{3A}}}^{\to 0} - 1 \right] = \frac{-4}{3} \cdot (0-1) = \frac{4}{3}$$

$\therefore$ integral converges

3.)

12 PTS



(diagram not necessary)

$$\int \pi r^2 h \to \pi \int (f(x))^2 \cdot dx$$

$r = y = f(x)$

$h = \Delta x \to dx$

$$\pi \int_2^4 (3\sqrt{x})^2 dx = \pi \int_2^4 9x\,dx = 9\pi \left(\frac{x^2}{2}\right)\Big]_2^4$$

$$= \frac{9\pi}{2}\left[4^2 - 2^2\right] = \frac{9\pi}{2}\left[16-4\right] = \frac{9(\cancel{12}^{6})\pi}{\cancel{2}} = 54\pi$$

≈ 169.65

4.) $D(x) = -.05x + 50$

12 PTS $S(x) = .1x + 20$

EQ. PT.: $-.05x + 50 = .1x + 20$

$+.05x - 20 \quad +.05x - 20$

$30 = .15x \qquad x = \frac{30}{.15} = 200$

$D(200) = -.05(200) + 50 = 40$

$S(200) = .1(20) + 20 = 40$

$(200, 40^{00})$

C.S.:

$$\int_0^{200}(-.05x+50)\,dx - \left[(200)(40)\right]$$

$$C.S. = \left[-.05\frac{x^2}{2}+50x\right]_0^{200} - 8000$$

$$C.S. = \left[\frac{-.05}{2}(40,000)+50(200)\right] - 8000$$

$$C.S. = \left[-1,000 + 10,000\right] - 8000 = 10,000 - 9,000$$

$$C.S. = 1000^{\underline{00}}$$

P.S.:

$$\left[(200)(40)\right] - \int_0^{200}(.1x+20)\,dx$$

$$P.S. = 8,000 - \left[\frac{.1x^2}{2}+20x\right]_0^{200}$$

$$P.S. = 8,000 - \left[\frac{.1}{2}(200)^2+20(200)\right]$$

$$P.S. = 8,000 - \left[2,000 + 4,000\right] = 8,000 - 6,000$$

$$P.S. = 2000^{\underline{00}}$$

) 12 PTS

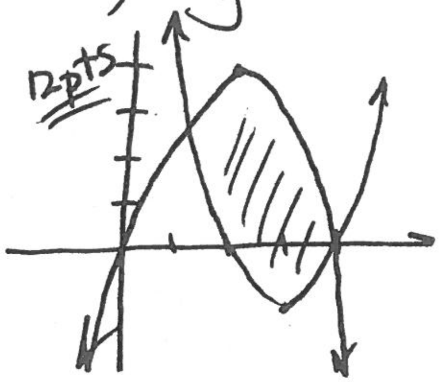
$$\int_0^{17}(4000)e^{.026t}\,dt = 4000\int_0^{17}e^{.026t}\,dt$$

$$= 4000\left[\frac{1}{.026}e^{.026t}\right]_0^{17} = \frac{4000}{.026}\left[e^{.026(17)} - e^{.026(0)}\right]$$

$$\approx \frac{5000}{.028}\left[1.6553-1\right] = \frac{5000\,[.6553]}{.028} \approx \$117{,}023^{\underline{10}}$$

6.) $y = 4x - x^2$ and $y = x^2 - 6x + 8$

12pts



graph not required

pts of int: $4x - x^2 = x^2 - 6x + 8$

$$\begin{array}{r} -4x + x^2 \quad +x^2 - 4x \\ \hline 0 = 2x^2 - 10x + 8 \\ 0 = 2(x^2 - 5x + 4) \\ 0 = 2(x-4)(x-1) \end{array}$$

$x = 1 \quad x = 4$

$$\text{AREA} = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8)\,dx = -2\frac{x^3}{3} + 10\cdot\frac{x^2}{2} - 8x\Big]_1^4$$

$$= \frac{-2}{3}x^3 + 5x^2 - 8x\Big]_1^4 = \left[\frac{-2}{3}(4)^3 + 5(4)^2 - 8(4)\right] - \left[\frac{-2}{3}(1)^3 + 5(1)^2 - 8(1)\right]$$

$$= \left(\frac{-2}{3}(64) + 5(16) - 32\right) - \left(\frac{-2}{3} + 5 - 8\right)$$

$$= \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8 = 9$$

7.) $\int_0^2 3x^2 \cdot e^{x^3}\,dx$

12pts

$u = x^3$

$du = 3x^2\,dx$

$x = 0 \xrightarrow{u = x^3} u = 0$

$x = 2 \longrightarrow u = 8$

$$\int_0^8 e^u\,du = e^u\Big]_0^8 = e^8 - e^0 = e^8 - 1$$

≈ 2979.96

(page 5)

3.) AVE Y-VALUE $= \frac{1}{5-0}\int_0^5 (-6t^2+12t+90)\,dt$

12pts

$= \frac{1}{5}\left[-6\left(\frac{t^3}{3}\right)+12\left(\frac{t^2}{2}\right)+90t\right]_0^5$

$= \frac{1}{5}\left[-2t^3+6t^2+90t\right]_0^5 = \frac{1}{5}\left[\left(-2(5)^3+6(5)^2+90(5)\right)-0\right]$

$= \frac{1}{5}\left[-250+150+450\right] = \frac{1}{5}\left[350\right] = 70$ words per minute

BONUS: (5 PTS)

$$1635 = \int_0^T 33.3\, e^{.015t}\, dt$$

(find T)
(amounts in BILLIONS)

$$1635 = \frac{33.3}{.015}\left[e^{.015T} - e^{.015(0)}\right]$$

$$1635 = \frac{33.3}{.015}\left[e^{.015T} - 1\right]$$

$$\frac{(.015)1635}{33.3} = e^{.015T} - 1$$

$$\frac{(.015)(1635)}{33.3} + 1 = e^{.015T}$$

$$\ln\left(\frac{.015(1635)}{33.3} + 1\right) = .015T$$

$$T = \frac{\ln\left(\frac{.015(1635)}{33.3} + 1\right)}{.015} \approx 36.8 \text{ years (after 2013)}$$