MA 141-012

Wednesday, September 19

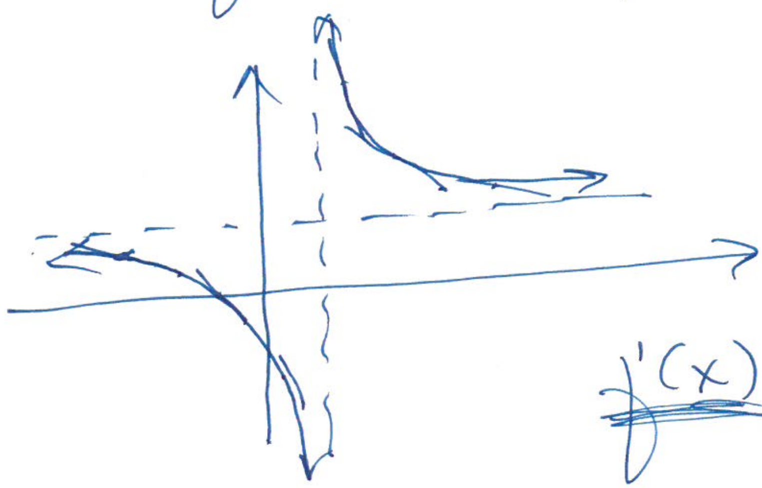
$$\lim_{x \to \infty} \frac{2x+5}{3x-1} = \frac{2}{3}$$

$$\lim_{x \to \infty} \frac{\frac{2x}{x} + \frac{5}{x}}{\frac{3x}{x} - \frac{1}{x}} = \lim_{x \to \infty} \frac{2 + \cancel{\frac{5}{x}}^{\,0}}{3 - \cancel{\frac{1}{x}}_{\,0}} = \frac{2}{3}$$

$$f(x) = \frac{2x+5}{3x-1}$$

H.A.: $y = 2/3$

V.A.: $x = 1/3$



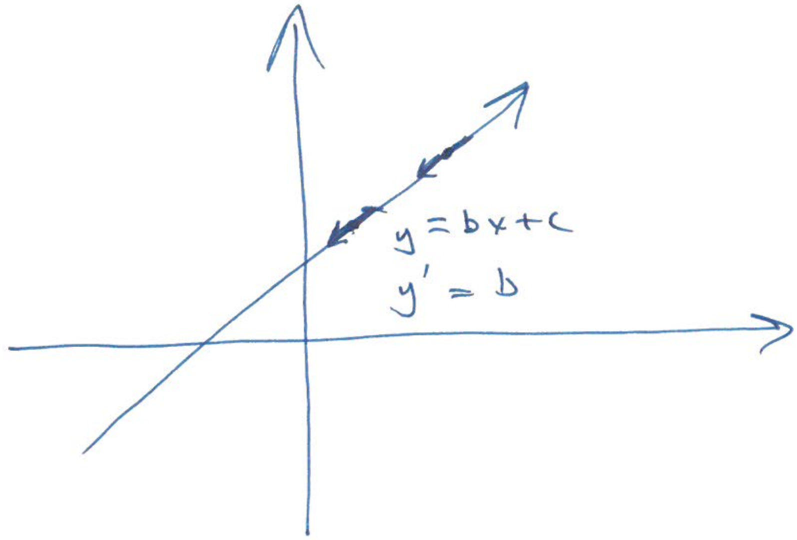
$$f'(x) = \frac{-10}{(3x-1)^2} = \text{NEG}$$

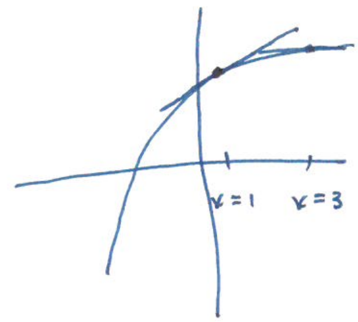
m_{TAN}

DEF. OF DERIV

$$f'(x) = m_{TAN} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\textcircled{2}$





$y = \sqrt{x}$?

$y = 3x^2 - 5x + 8$ ✓

$y = \dfrac{2x}{3x+4}$?

$$\frac{3}{15} - \frac{1}{15} = \frac{3-1}{15}$$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{2(x+h)}{3(x+h)+4} - \dfrac{2x}{3x+4}}{h}$$

$$= \lim_{h \to 0} \left[\frac{2(x+h)\,(3x+4)}{\underbrace{[3(x+h)+4](3x+4)}} - \frac{2x\,(3(x+h)+4)}{\underbrace{(3x+4)(3(x+h)+4)}} \right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{(2x+2h)(3x+4) - (2x)\overset{(3x+3h+4)}{(3(x+h)+4)}}{[3(x+h)+4] \cdot [3x+4] \cdot h}$$

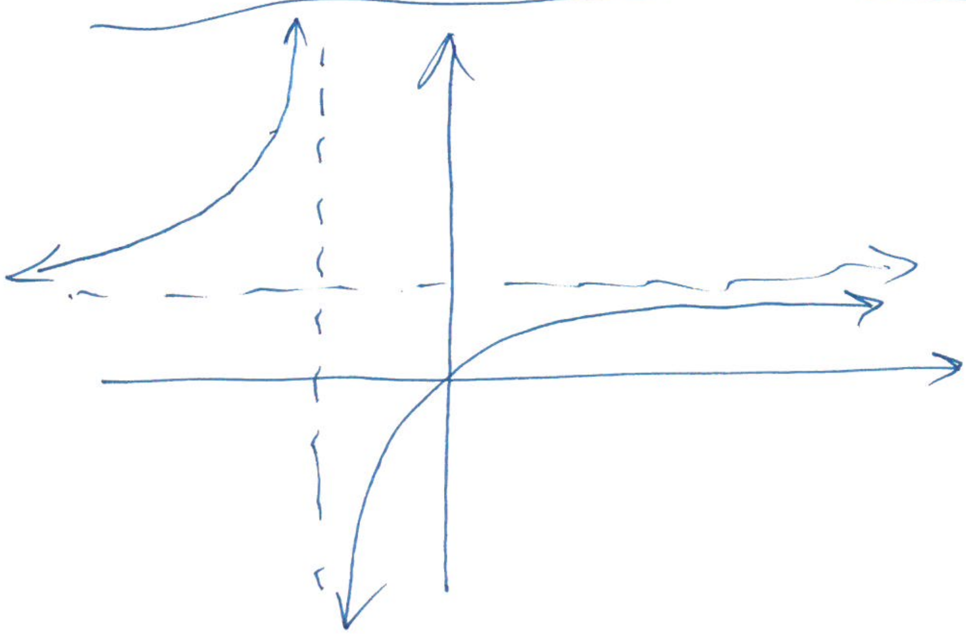
$$= \lim_{h \to 0} \frac{\cancel{6x^2} + \cancel{8x} + \cancel{6xh} + 8h - \cancel{6x^2} - \cancel{6xh} - \cancel{8x}}{[3(x+h)+4] \cdot [3x+4] \cdot h}$$

$$= \lim_{h \to 0} \frac{8 \cdot \cancel{h}^{1}}{[3(x+h)+4][3x+4] \cdot \cancel{h}_{1}} \qquad (h \neq 0)$$

$$= \lim_{h \to 0} \frac{8}{[3(x+h)+4]\cdot[3x+4]} =$$

$$\frac{8}{[3x+4]\cdot[3x+4]} = \frac{8}{(3x+4)^2}$$

$$f(x) = \frac{2x}{3x+4} \qquad f'(x) = \frac{8}{(3x+4)^2} = +$$



$f(x) = \sqrt{x}$

$(a-b)(a+b) = a^2 - b^2$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \boxed{\sqrt{x}})(\sqrt{x+h} + \boxed{\sqrt{x}})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{(\cancel{x}+h-\cancel{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}^{\,1}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} \qquad (h \neq 0)$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+\underset{0}{h}} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f(x) = \frac{2x}{3x+4}$$ ✓

$$f'(x) = \frac{8}{(3x+4)^2}$$

find the equation of the line tangent to this $f(x)$ at $(2, 2/5)$

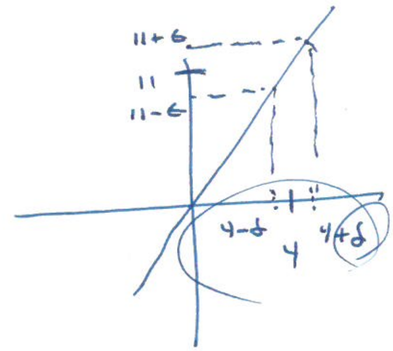
$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 2/5 = 2/25\,(x-2)}$$ *

$$f'(2) = \frac{8}{(3(2)+4)^2} = \frac{8}{100} = \frac{2}{25}$$

$f(x) = 3x - 1$

$x \to 4$



$\lim_{x \to 4} (3x - 1) = 11$

choose δ (in terms of ϵ)

we want: $|(3x-1) - 11| < \epsilon$

$|3x - 12| < \epsilon$

$3|x-4| < \epsilon$

$|x-4| < \epsilon/3$

choose $\delta = \epsilon/3$

proof:

if $|x-4| < \delta$ choose $\delta = \epsilon/3$

$|x-4| < \epsilon/3$

$3|x-4| < \epsilon$

$|3x-12| < \epsilon$

then $|(3x-1) - 11| < \epsilon$ $\quad |f(x) - L| < \epsilon$

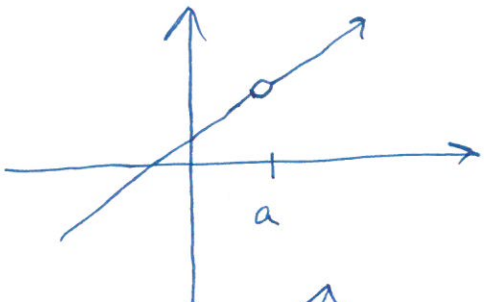
$|f(x) - L| < \epsilon$

1.3:

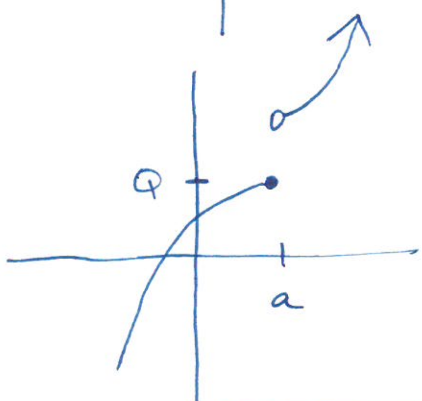
CONTINUITY: (at $x=a$)

1.) ¿ $\boxed{f(a)}$ EXISTS?

¿ is there a point plotted there?



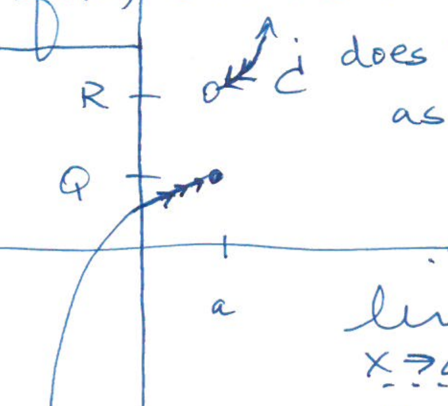
$(a, ?)$ ∴ DISCON.



(a, Q) passes test #1

2.) ¿ $\boxed{\lim_{x \to a} f(x)}$ EXISTS?

¿ does f(x) have a limit as x appr "a"?

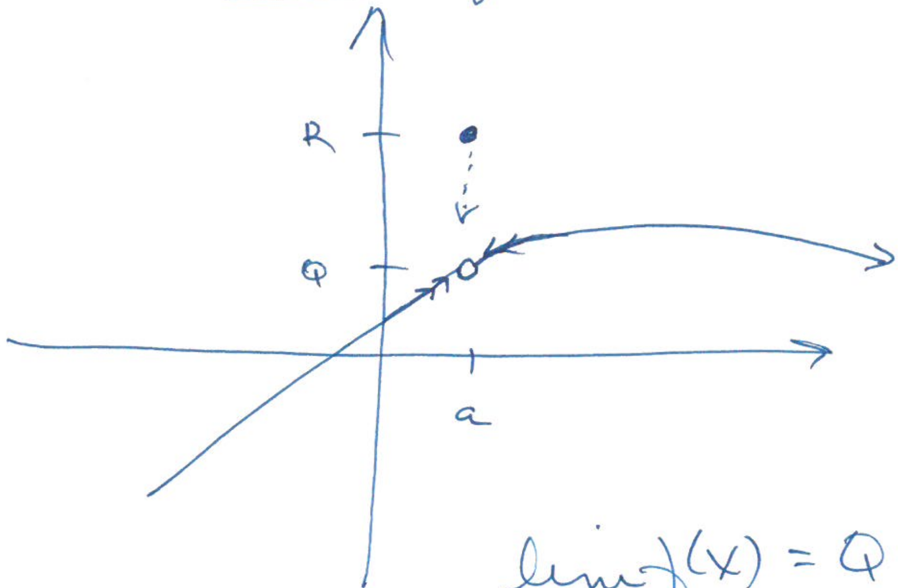


$\lim_{x \to a} f(x) =$ D.N.E.

∴ DISCON.

$$\left.\begin{array}{l} \lim_{x \to a^+} f(x) = R \\ \lim_{x \to a^-} f(x) = Q \end{array}\right\}$$

3.) $\boxed{\lim_{x \to a} f(x)} \overset{??}{=} f(a)$



$\lim_{x \to a} f(x) = Q$

$f(a) = R$

$\lim_{x \to a} f(x) \overset{?}{=} f(a)$

$Q \overset{??}{=} R$

$\therefore$ DISCON.

$$f(x) = \begin{cases} x^2+2, & x \le 1 \\ 3x+5, & x > 1 \end{cases}$$

¿ contin at $x = 1$?

1.) $f(1) = 3$ ✓

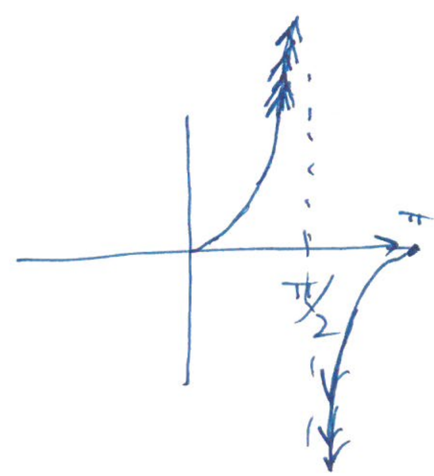
2.) $\lim_{x \to 1} f(x)$ exists

$$\lim_{x \to 1} f(x) = \text{D.N.E.} \left\{ \begin{array}{l} \lim_{x \to 1^+} f(x) = \underline{8} \\ \lim_{x \to 1^-} f(x) = \underline{3} \end{array} \right\}$$

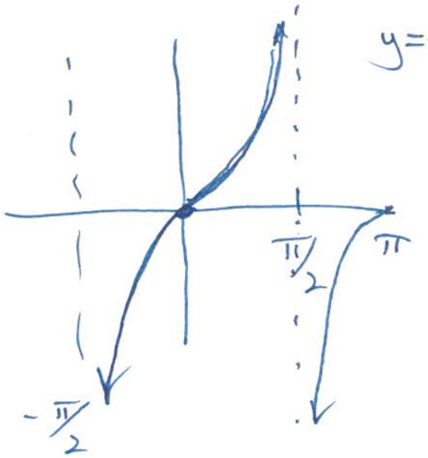
∴ DISCON.

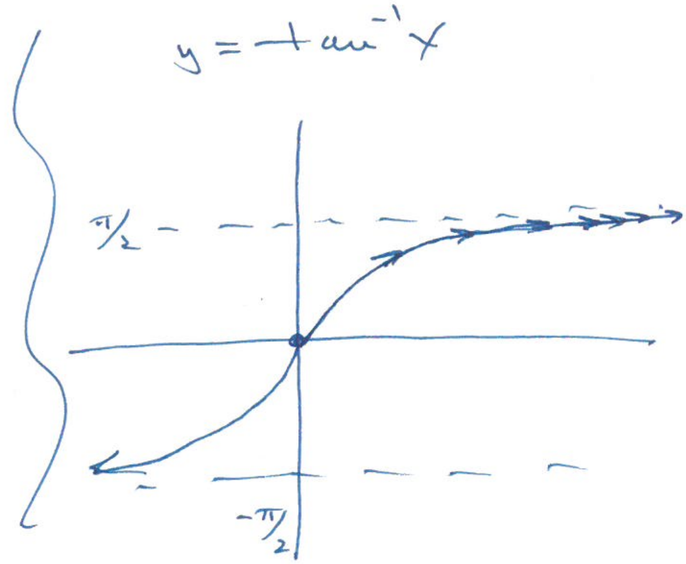
$$\lim_{\theta \to \pi/2} \sin\theta = 1$$

$$\lim_{\theta \to \pi/2} \tan\theta = \text{D.N.E}$$

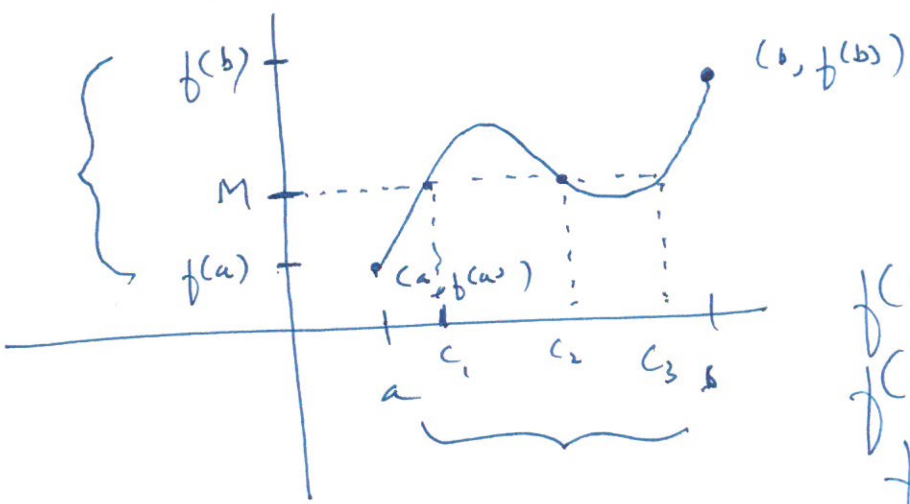


$$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$





INTERMEDIATE VALUE THEOREM:

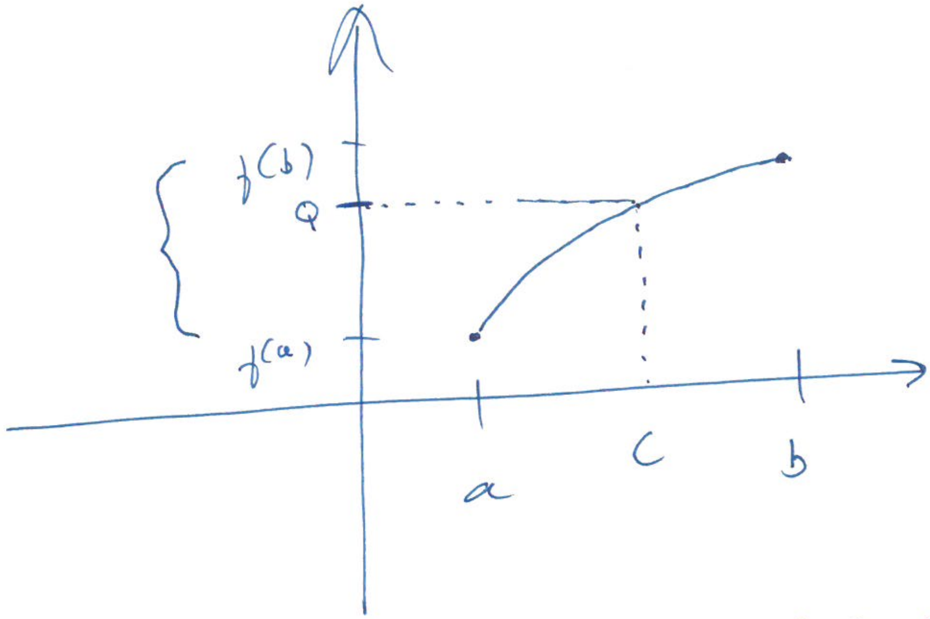


$[a, b]$

$f(c_1) = M$

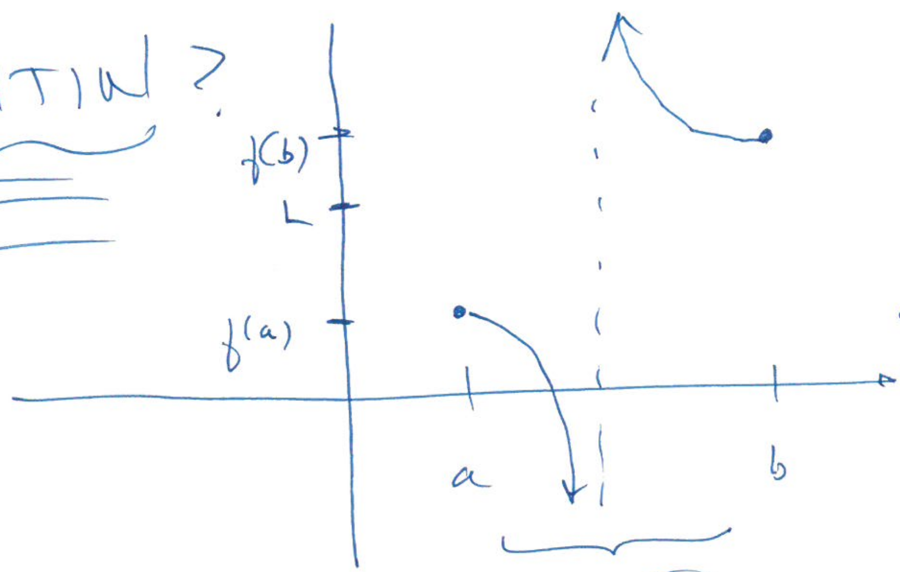
$f(c_2) = M$

$f(c_3) = M$



$a < c < b$
such that
$f(c) = Q$

CONTIN ?



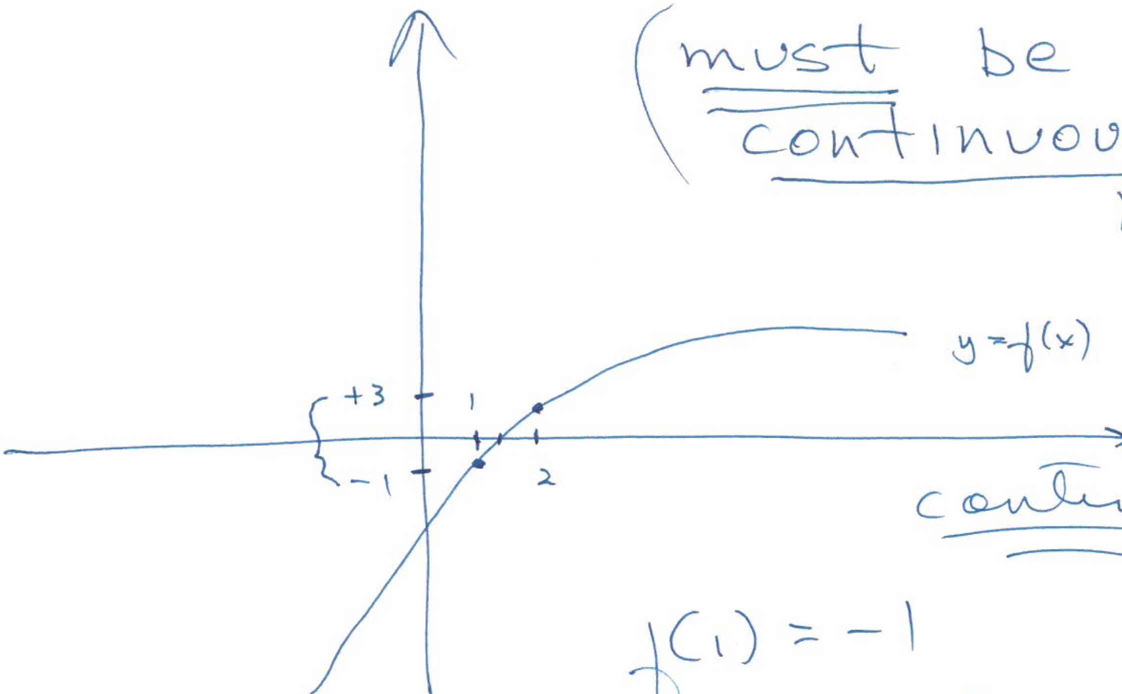
$f(c) = L$

no c in (a, b)
such that
$f(c) = L$

why not?

$f(x)$ **NOT** **CONTIN**

(<u>must</u> be <u>continuous</u> for IVT)



<u>contin</u> on $[1, 2]$

$f(1) = -1$

$f(2) = +3$

zero (root) of the function in $(1, 2)$

$1 < c < 2$

such that

$f(c) = 0$

$f(x) = 0$

$f(x) = 3x^3 - 5x^2 + 8x - 1$

contin (polynom.)

$f(3) = +$

$f(4) = -$

MA 141-012 F18 Recitation 3, Sept 11

1. Solve the trig equation $2 - 3\sin\theta = 1 - 5\sin\theta$ assuming $0 \leq \theta < 2\pi$.

2. Determine the Cartesian equation of the parametrized curve $x = \tan^2(t)$, $y = \tan(t)$.

3. Determine the Cartesian equation of the parametrized curve $x = 2\sec(\theta), y = 6\tan^2(\theta)$.

4. Using Figure 2 determine the following limits graphically: $\lim_{x\to 1} f(x), \lim_{x\to 4} f(x)$, and $\lim_{x\to\infty} f(x)$. Comment about continuity at specified points.

5. Given $\lim_{x\to 5} f(x) = 12$ and $\lim_{x\to 5} g(x) = 3$ determine the following limits:

   (a) $\lim_{x\to 5}[\frac{1}{2}f(x) + 4g(x)]$

   (b) $\lim_{x\to 5} \frac{f(x)}{g(x)}$

   (c) $\lim_{x\to 5}[f(x)]^2$

6. Let $f(x) = 4x^2 + 2$.

   (a) What is the slope of the secant line through points given by $x = -1$ and $x = 2$? What about the slope of the secant line through points given by $x = -1$ and $x = 0$?

   (b) What is another interpretations of the slope of a secant line?

   (c) What is the equation of the tangent line through the point $(-1, 6)$? What about the tangent line through the point $(0, 2)$?

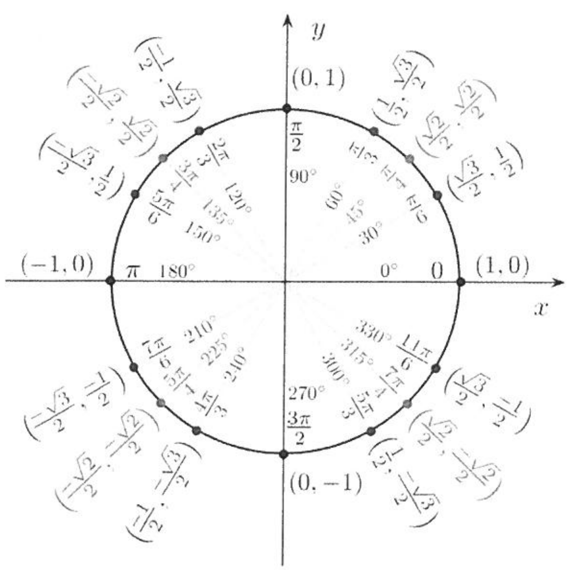


Figure 1: from Wikipedia

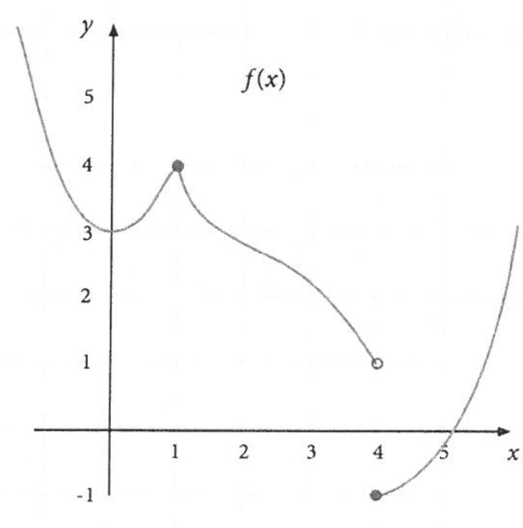


Figure 2: from Calculus for Engineers and Scientists, Volume I

MA 141-012 F18 Recitation 4, Sept 18

1. What is the equation of the tangent line to $f(x) = 4x^2 + 2$ through the point $(-1, 6)$? (Start by determining the slope of the tangent line.)

2. Determine $\lim_{x\to 2}(3x+8)^2$.

3. For $g(x) = 6x - 1$ let $\epsilon > 0$. Find $\delta > 0$ such that $0 < |x-4| < \delta$ implies $|g(x) - 23| < \epsilon$. What limit does this prove?

4. Determine the location of any vertical or horizontal asymptotes for the rational function $f(x) = \dfrac{2x-1}{x+9}$.

5. For $g(x) = \frac{x^2-16}{x^3-64} = \frac{x^2-16}{(x-4)(x^2+4x+16)}$.
   - Compute $\lim_{x\to 4} g(x)$, $\lim_{x\to\infty} g(x)$, and $\lim_{x\to -4} g(x)$.
   - Does g have any vertical or horizontal asymptotes?
   - Let $h(x) = \begin{cases} \frac{x^2-16}{x^3-64} & \text{for } x \neq 4 \\ C & \text{for } x = 4 \end{cases}$. What value C makes h continuous at $x = 4$?

6. Use the limit definition of derivative to determine $g'(x)$ for $g(x) = \frac{1}{2}x^2$.

Relevant Information to Recall

- Assume that functions f and g have limits L_1 and L_2 at the point $x = a$ so that
$$\lim_{x\to a} f(x) = L_1 \text{ and } \lim_{x\to a} g(x) = L_2. \text{ Then,}$$

$(P1)$ $\lim_{x\to a} x = a$

$(P2)$ $\lim_{x\to a} c = c$

$(P3)$ $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{L_1}{L_2}$

$(P4)$ $\lim_{x\to a}(f(x) \pm g(x)) = L_1 \pm L_2$

$(P5)$ $\lim_{x\to a} f(x)g(x) = L_1 L_2$

- The derivative $f'(x)$ of f is a function $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
- A rational function $f(x) = \frac{P(x)}{Q(x)}$, $Q(x) \neq 0$ has a **vertical asymptote** at $x = a$ if f increases or decreases without bound as $x \to a$ from either the left or the right. To determine the location of a vertical asymptote simplify the function first, if necessary, then find values $x = a$ that make the denominator equal to 0. The function has a **horizontal asymptote** at $y = b$ if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$
- Suppose $x = a$ is in the domain of $f(x)$ then f is **continuous** at $x = a$ if
$$\lim_{x\to a} f(x) = f(a).$$