MA 141 – 012

Monday, September 24

first hour: TEST #1

second hour: begin Chapter 2

2.1:

DERIVATIVE:

m_{TAN} = INSTANTANEOUS RATE OF CHANGE $= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

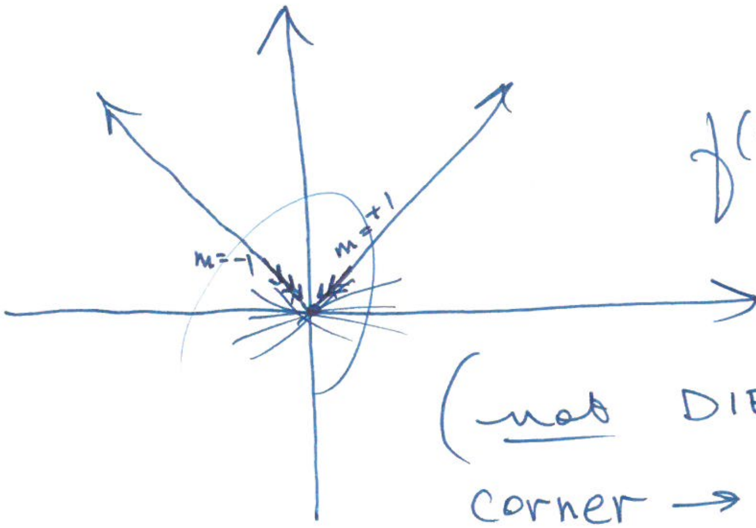
differentiable (has a deriv.)

* $f'(x)$; * $\frac{dy}{dx}$; y'; $D_x y$

DERIV. OF Y WITH RESPECT TO X

($p \to q$)
THM: If a function is DIFFERENTIABLE then that function is CONTINUOUS.

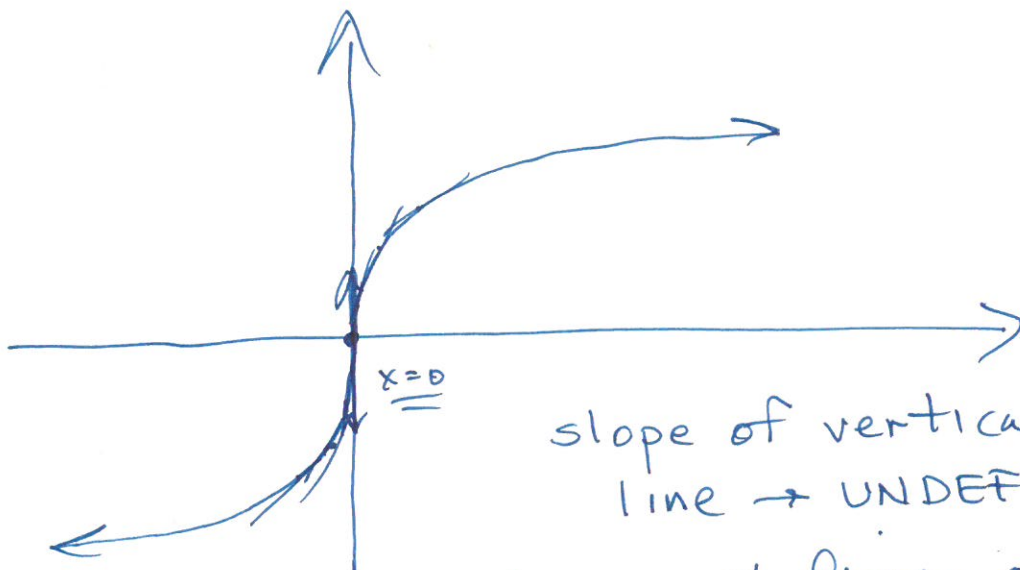
($q \to p$)
converse: If a function is CONTIN, then it is DIFF. ??? (NOT TRUE)



$f(x) = |x|$

(not DIFF. at $x=0$)

corner $\rightarrow$ NOT DIFF.



slope of vertical line $\rightarrow$ UNDEF.

tangent line at the origin is VERTICAL

$(f'(x) = m_{TAN} = \text{undef.})$

$$\boxed{f(x) = x^{1/3}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

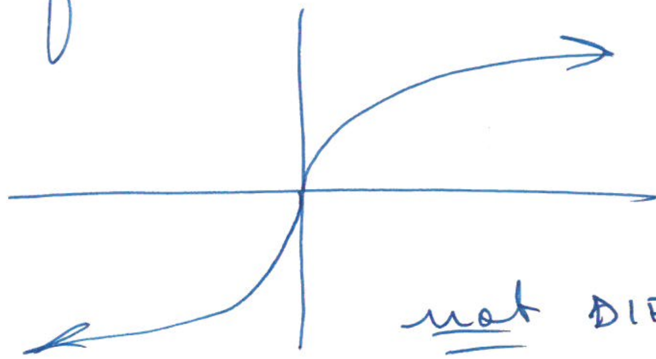
$$f'(0) = \lim_{h \to 0} \frac{\boxed{f(0+h)} - \boxed{f(0)}}{h}$$

$$= \lim_{h \to 0} \frac{(0+h)^{1/3} - (0)^{1/3}}{h}$$

$$= \lim_{h \to 0} \frac{h^{1/3}}{h^1} = \lim_{h \to 0} h^{-2/3}$$

$$= \lim_{h \to 0} \frac{1}{h^{2/3}} \to \text{undef.}$$

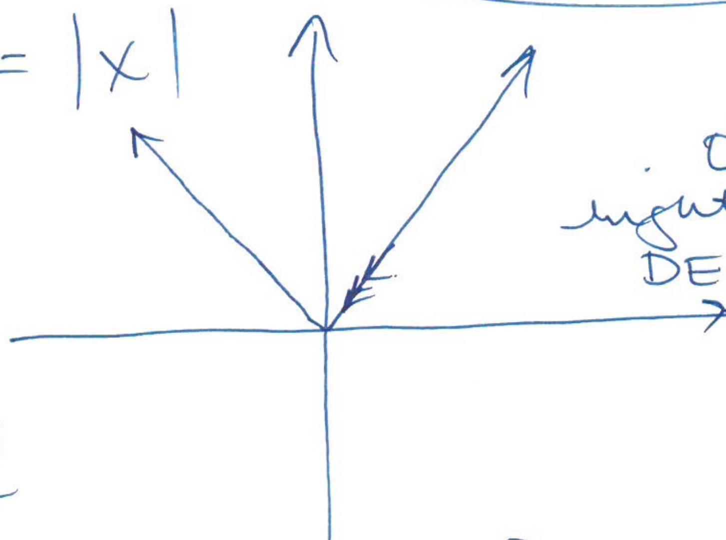
$$f(x) = \sqrt[3]{x} = x^{1/3}$$



<u>not</u> DIFF. at $x = 0$

$(0, 0)$

$$f(x) = |x|$$



0^-
left hand
DERIV:

$m_{TAN} = -1$

0^+
right hand
DERIV:

$m_{TAN} = +1$

DERIV D.N.E.
AT $x = 0$

$\cancel{f(x) =}$

$$y = \underline{f(x) + g(x)}$$

$\underline{\text{ex:}}$ $y = \underbrace{8x^2}_{} + \underbrace{14x}_{}$

$y' = ?$ (DERIV. OF A SUM ....) IS THE SUM OF THE DERIV.

$$y' = \lim_{h \to 0} \cancel{f(x+h) - f(x)}$$

$$= \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right]$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

$y = \underline{k} \cdot f(x)$

$$y' = \lim_{h \to 0} \frac{k \cdot f(x+h) - k \cdot f(x)}{h}$$

$$= k \cdot \underline{\boxed{\lim_{h \to 0} \left(\sim\sim\sim \right)}}$$

$$= \underline{k} \cdot f'(x)$$

$\boxed{f(x) = x^n}$ (n = non-neg. integer)

polynomial

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \left(\cancel{x^n} + n \cdot x^{n-1} \cdot h + \frac{n(n-1)}{2} x^{n-2} \cdot h^2 + \text{larger powers of } h \right. \left. \cancel{- x^n} \right)$$

0 0

$(h \neq 0)$
$$= \lim_{h \to 0} \frac{\cancel{h}\left(n \cdot x^{n-1} + \boxed{\cancel{\frac{n(n-1)}{2} x^{n-2} h}} + \boxed{\cancel{\text{still have an "h" in them}}} \right)}{\cancel{h}\ 1}$$

$$\boxed{f'(x) = n \cdot x^{n-1}}$$

ex: $f(x) = x^7 \quad f'(x) = 7 \cdot x^6$

$f(x) = \underline{11} x^4 \quad f'(x) = 11(4 \cdot x^3)$

$f(x) = \underline{5x^2} - \underline{8x} \quad f'(x) = 5(2x^1) - 8(1)$