Wednesday, September 26

- product rule
- quotient rule
- generalized power rule
- higher order derivatives

$\frac{d(x)}{dx}$

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from last class:

$$\boxed{f(x) = 11x^4 + 5x^2 - 8x + 13}$$

$$f'(x) = 11(4 \cdot x^3) + 5 \cdot (2 \cdot x) - 8(1) + 0$$

$$f'(x) = 44x^3 + 10x - 8$$

$$f(x) = 11x^4 + 5x^2 - 8x + 13$$

$$f'(x) = 44x^3 + 10x - 8$$

product rule:

$\boxed{f(x) = (3x+1)(2x+5)} = 6x^2 + 17x + 5$

¿ is the deriv. of a prod the prod. of the derivs ?

$f'(x) \overset{??}{=} (3)\cdot(2) = 6$

prod. rule:

$y = (2x-1)^4 \cdot (8x^2+x-11)^{11}$

$y = f(x) \cdot g(x), \quad y' = ??$

$$y' = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

$$y' = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x+h) + f(x) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

$$y' = \lim_{h \to 0} \left[\frac{g(x+h)\left[f(x+h) - f(x)\right]}{h} + \frac{f(x)\left[g(x+h) - g(x)\right]}{h} \right]$$

$$y' = \lim_{h \to 0} g(x+h) \boxed{\left[\frac{f(x+h) - f(x)}{h}\right]} + \lim_{h \to 0} f(x) \cdot \left[\frac{g(x+h) - g(x)}{h}\right]$$

$$\boxed{y' = g(x) \cdot f'(x) + f(x) \cdot g'(x)}$$ prod. rule

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$f(x) = (3x+1)(2x+5) = 6x^2 + 17x + 5$

$f'(x) = (3x+1)\cdot(2) + (2x+5)\cdot(3)$

$f'(x) = 6x + 2 + 6x + 15$

$f'(x) = 12x + 17$ ____

$f(x) = (3x^2+4)(-4x+1)$

$f'(x) = (3x^2+4)(-4) + (-4x+1)\cdot(6x)$

$f'(x) =$ ____________________

QUOTIENT RULE:

$$y = \frac{f(x)}{g(x)}$$

[ex: $y = \frac{2x+1}{5x+4}$]

$y' = ??$

$$y' = \lim_{h\to 0} \frac{\boxed{\frac{f(x+h)}{g(x+h)}} - \frac{f(x)}{g(x)}}{h}$$

$$y' = \lim_{h\to 0} \left[\frac{f(x+h)\cdot g(x)}{g(x+h)\cdot g(x)} - \frac{f(x)\cdot g(x+h)}{g(x)\cdot g(x+h)} \right] \cdot \frac{1}{h}$$

$$y' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x+h)}{g(x+h)\cdot g(x)\cdot h}$$

$$y' = \lim_{h\to 0} \left[\frac{f(x+h)\cdot g(x) - f(x)\cdot g(x) + f(x)\cdot g(x) - f(x)\cdot g(x+h)}{g(x+h)\cdot g(x)\cdot h} \right]$$

$$y' = \lim_{h\to 0} \frac{g(x)\left[f(x+h) - f(x)\right] - f(x)\left[g(x+h) - g(x)\right]}{g(x+h)\cdot g(x)\cdot h}$$

$$y' = \lim_{h\to 0} \frac{g(x)\cdot\left[\frac{f(x+h)-f(x)}{h}\right] - f(x)\left[\frac{g(x+h)-g(x)}{h}\right]}{g(x+\overset{0}{h})\cdot g(x)}$$

$$\boxed{y' = \cancel{\lim_{h\to 0}} \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}}$$

QUOT. RULE

$$y = \frac{f(x)}{g(x)}$$

$$y' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

ex: $y = \dfrac{2x+1}{5x+4}$

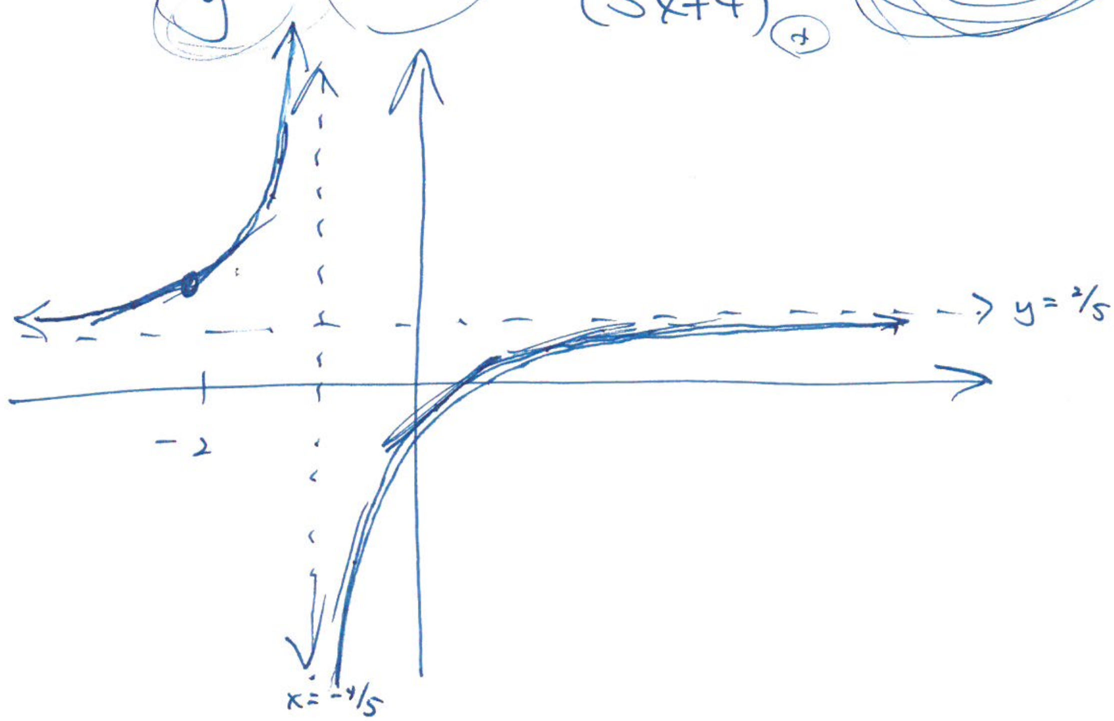
V.A.: $x = -4/5$

H.A.: $y = 2/5$

$$y' = \frac{(5x+4)(2) - (2x+1)\cdot(5)}{(5x+4)^2}$$

$$y' = \frac{10\cancel{x} + 8 - 10\cancel{x} - 5}{(5x+4)^2} = \frac{3}{(5x+4)^2}$$

$$y' = m_{TAN} = \frac{3^{(+)}}{(5x+4)^2_{(+)}} = \text{always } +$$



$$z = \frac{2r - 1}{8r^2 + 5r - 7}$$

$$z' = \frac{dz}{dr} = \frac{(8r^2+5r-7)\cdot(2) - (2r-1)(16r+5)}{(8r^2+5r-7)^2}$$

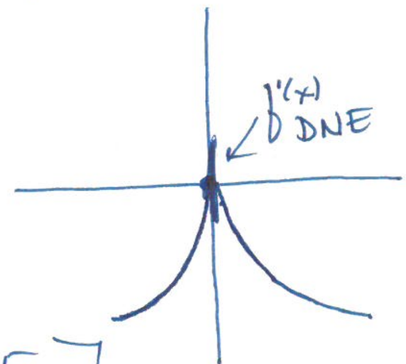
$$z' = \frac{\cancel{16r^2} + \cancel{10r} - 14 - \cancel{32r^2} + \cancel{16r} + 5 - \cancel{10r}}{(8r^2+5r-7)^2}$$

$$z' = \frac{-16r^2 + 16r - 9}{(8r^2+5r-7)^2}$$

$f(x) = 14 \cdot x^{1/3} \quad (0,0)$

$f'(x) = 14 \cdot \frac{1}{3} \cdot x^{\frac{1}{3}-1} = \frac{14}{3} x^{-2/3}$

$f'(x) = \frac{14}{3 \cdot x^{2/3}}$



[*not* DIFFERENTIABLE at $x=0$]

VERTICAL TANGENT LINE THERE

$g(x) = \underline{x^2} + \sqrt{x} \leftarrow x^{1/2}$

$g'(x) = 2x + \frac{1}{2} \cdot x^{-1/2}$

$g'(x) = 2x + \frac{1}{2 \cdot \sqrt{x}}$

$g'(2) = 2(2) + \frac{1}{2 \cdot \sqrt{2}}$

$$\boxed{g'(2) = 4 + \frac{1}{2\sqrt{2}}}$$

$$h(x) = \frac{4}{x^3} = 4 \cdot x^{-3}$$ ✓

$$h'(x) = -12 \cdot x^{-4} = \frac{-12}{x^4}$$

quotient rule:

$$h'(x) = \frac{\cancel{(x^3) \cdot (0)} - (4)(3x^2)}{[x^3]^2}$$

$$= \frac{-12x^2}{x^6}$$

HIGHER ORDER DERIV.

(free-falling object) —

$s(t) = \underline{-16t^2} + \underline{84t} + \underline{18}$

s: DIST; HT; POS (FT.)

t: TIME (sec.)

$s(0) = -16(0)^2 + 84(0) + 18$

$s(0) = 18$ FT.

$s(1) = -16(1)^2 + 84(1) + 18$

$s(1) = -16 + 84 + 18 = 86$ FT

$s(2) =$ ______

$s'(t) = \boxed{v(t) = -32t + 84}$

$v(0) = -32(0) + 84 = 84 \frac{FT}{SEC}$

initial velocity

$v(1) = -32(1) + 84 = 52 \frac{FT}{SEC}$

$v(2) = -32(2) + 84 = 20 \frac{FT}{SEC}$

$s''(t) = v'(t) = a(t) = -32 \frac{FT/SEC}{SEC} = -32 \frac{FT}{SEC^2}$

$a(0) = \underline{-32} \frac{FT/SEC}{SEC}$

$a(1) = \underline{-32}$ "

$a(??) = \underline{-32}$ "

1. Suppose the function f is defined on the interval (a,b) where $f'(c)$ exists for all $c \in (a,b)$. Is f continuous on (a,b)?

2. Let g be defined by

$$g(x) = \begin{cases} 2x-1 & \text{if } x \leq 3 \\ 8-x & \text{if } x > 3 \end{cases}$$

   (a) Is g continuous at $x = 3$?
   (b) Find the right-hand and left-hand derivatives of g at $x = 3$.
   (c) Is g differentiable at $x = 3$?

3. Let f be defined by

$$f(x) = \begin{cases} x-1 & \text{if } x \leq 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$$

   (a) Is f continuous at $x = 0$?
   (b) Is f differentiable at $x = 0$?

4. Show that the point $(-1,0)$ is on the graph of $f(x) = x^4 - 3x^2 + 2$. Find the derivative of f then determine the equation of the tangent line to f at $(-1,0)$.

5. The volume of a spherical hot air balloon $V(r) = \frac{4}{3}\pi r^3$ changes as its radius changes. The radius is a function of time given by $r(t) = 3t$. Find the average rate of change of the volume with respect to t as t changes from $t = 1$ to $t = 3$. Find the instantaneous rate of change of the volume with respect to t at $t = 1$.

Relevant Info to Recall

- Let the function f be defined on the interval (a,b) and $c \in (a,b)$. The **right-hand (left-hand) derivative of f at** c is the number $f'(c^+)$ $(f'(c^-))$ given by the following limit, provided the limit exists,

$$f'(c^+) = \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h} \quad \left(f'(c^-) = \lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h}.\right)$$

- The function f is **differentiable at** c provided the following limit exists

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}.$$

- Let $f(x)$ and $g(x)$ be differentiable functions and $c \in \mathbb{R}$ some real number. Then,

$$\frac{d}{dx}(f \pm g) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) \quad \text{and} \quad \frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x).$$

- (Power Rule) For an integer n, the derivative $\frac{d}{dx}(x^n) = nx^{n-1}$.