MA141-012

Monday, October 1

- 2 limit theorems (TRIG)
- DERIV of sin, cos, tan, ...
- chain Rule
- RETURN TEST #1

DERIVATIVES OF TRIG FUNCTIONS

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = ?$$

$\sin\theta = AC$
$\cos\theta = OA$
$\tan\theta = BD$

from circle:

$$\cancel{\tfrac{1}{2}}(OA)(AC) < \cancel{\tfrac{1}{2}}\theta < \cancel{\tfrac{1}{2}}(\overset{1}{OB})(BD)$$

$$(OA)(AC) < \theta < (BD)$$

$$(\cos\theta)(\sin\theta) < \theta < (\tan\theta)$$

$$\frac{(\cos\theta)(\cancel{\sin\theta})}{\cancel{\sin\theta}} < \frac{\theta}{\sin\theta} < \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}}$$

$$\cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$\frac{2}{3} > \frac{1}{3}$

$\frac{3}{2} < \frac{3}{1}$

$$\frac{1}{\cos\theta} > \frac{\sin\theta}{\theta} > \cos\theta$$

$\theta \to 0$

$$1. > \overset{??}{\frac{\sin\theta}{\theta}} > 1$$

②

$$\lim_{\boxed{\theta}\to 0} \frac{\sin \boxed{\theta}}{\boxed{\theta}} = 1$$

$$\lim_{\theta \to 0} \frac{\boxed{\sin 5\theta}}{8\theta \cdot 5} \cdot 5 = \frac{5}{8}$$

$\boxed{5\theta \to 0}$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

$$\lim_{\theta \to 0} \frac{(\cos\theta - 1)(\cos\theta + 1)}{\theta \quad (\cos\theta + 1)}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$\lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta \cdot (\cos\theta + 1)}$$

$$\lim_{\theta \to 0} \frac{-\sin^2\theta}{\theta \cdot (\cos\theta + 1)}$$

$$\lim_{\theta \to 0} \boxed{\frac{(\sin\theta)}{\theta}} \cdot \frac{(-\sin\theta)}{(\cos\theta + 1)}$$

$$\boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} \cdot \lim_{\theta \to 0} \frac{-\sin\theta \to 0}{\underset{1}{\cos\theta + 1}} = \frac{0}{2} = 0$$

$$1 \cdot 0$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

cosh ←
sinh ←

$\sin(\alpha+\beta)$

$f(\theta) = \sin\theta \qquad f'(\theta) = ?$

by DEF. OF DERIV.

$\sin(A+B)$
$=$
$\sin A\cos B + \cos A \sin B$

$$f'(\theta) = \lim_{h \to 0} \frac{\boxed{\sin(\theta+h)} - \sin\theta}{h}$$

$$= \lim_{h\to 0} \frac{\sin\theta \cdot \cos h + \cos\theta \cdot \sin h - \sin\theta}{h}$$

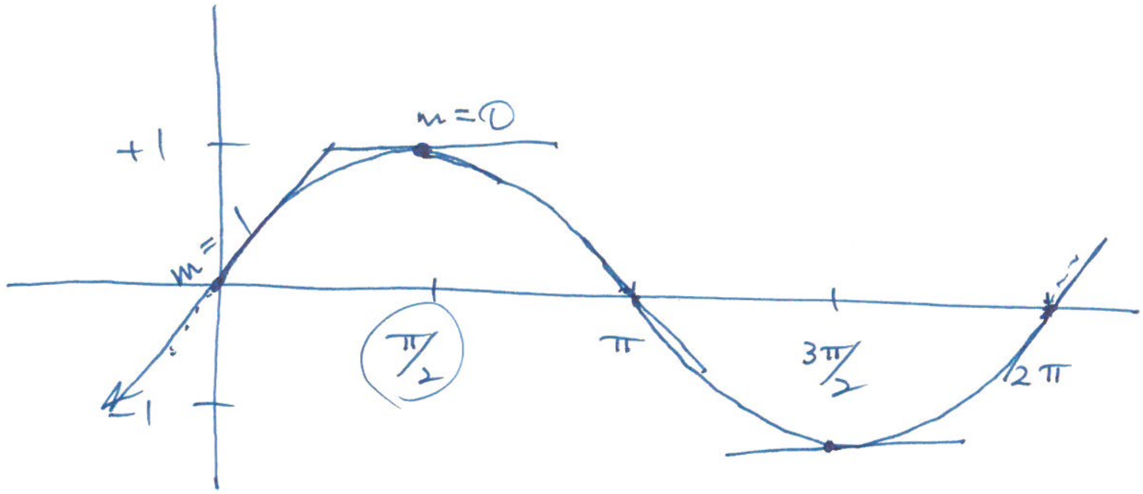
$$= \lim_{h\to 0} \frac{\sin\theta \cdot \cos h - \sin\theta}{h} + \lim_{h\to 0} \frac{\cos\theta \cdot \sin h}{h}$$

$$= \lim_{h\to 0} \sin\theta \left[\frac{\cos h - 1}{h}\right]^{\to 0} + \lim_{h\to 0} \cos\theta \cdot \frac{\sin h}{h}^{\to 1}$$

$$= \sin\theta\,(0) + \cos\theta\,(1)$$

$$= \cos\theta$$

$$\boxed{\begin{aligned} f(\theta) &= \sin\theta \\ f'(\theta) &= \cos\theta \end{aligned}}$$



$$f(\theta) = \sin\theta \qquad f'(\theta) = \cos\theta$$

$$0 = m_{TAN} = f'(\pi/2) = \cos \pi/2 = 0$$

$$\cos 3\pi/2 = 0$$

$$f(\theta) = \sin\theta \qquad f'(\theta) = \cos\theta$$

m_{TAN} when $\theta = 0$:

$$f'(0) = \cos\theta = 1$$

$\cos(\alpha+\beta)$

$\cos(A+B)$

$$f(x) = \cos x \qquad f'(x) = ??$$

$$f'(x) = \lim_{h\to 0} \frac{\boxed{\cos(x+h)} - \cos x}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cos x\cdot\cos h - \sin x\cdot\sin h - \cos x}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cos x\cdot\cos h - \cos x}{h} - \frac{\sin x\cdot\sin h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cos x(\cos h - 1)}{h} \cdot \lim_{h\to 0} \frac{-\sin x\cdot\sin h}{h}$$

0 1

$$f'(x) = -\sin x$$

5

$$\cos\left(\frac{2\pi}{2\cdot 6}+\frac{3\pi}{3\,4}\right) = ??\qquad \cos\frac{5\pi}{12}$$

$$= \cos\frac{\pi}{6}\cdot\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\cdot\sin\frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4} \approx .2588$$

$$\cos\left(\frac{5\pi}{12}\right) \cong \cos(75^\circ)$$

$\textcircled{6}$

$f(x) = \sin x \quad f'(x) = \cos x$

$f(x) = \cos x \quad f'(x) = -\sin x$

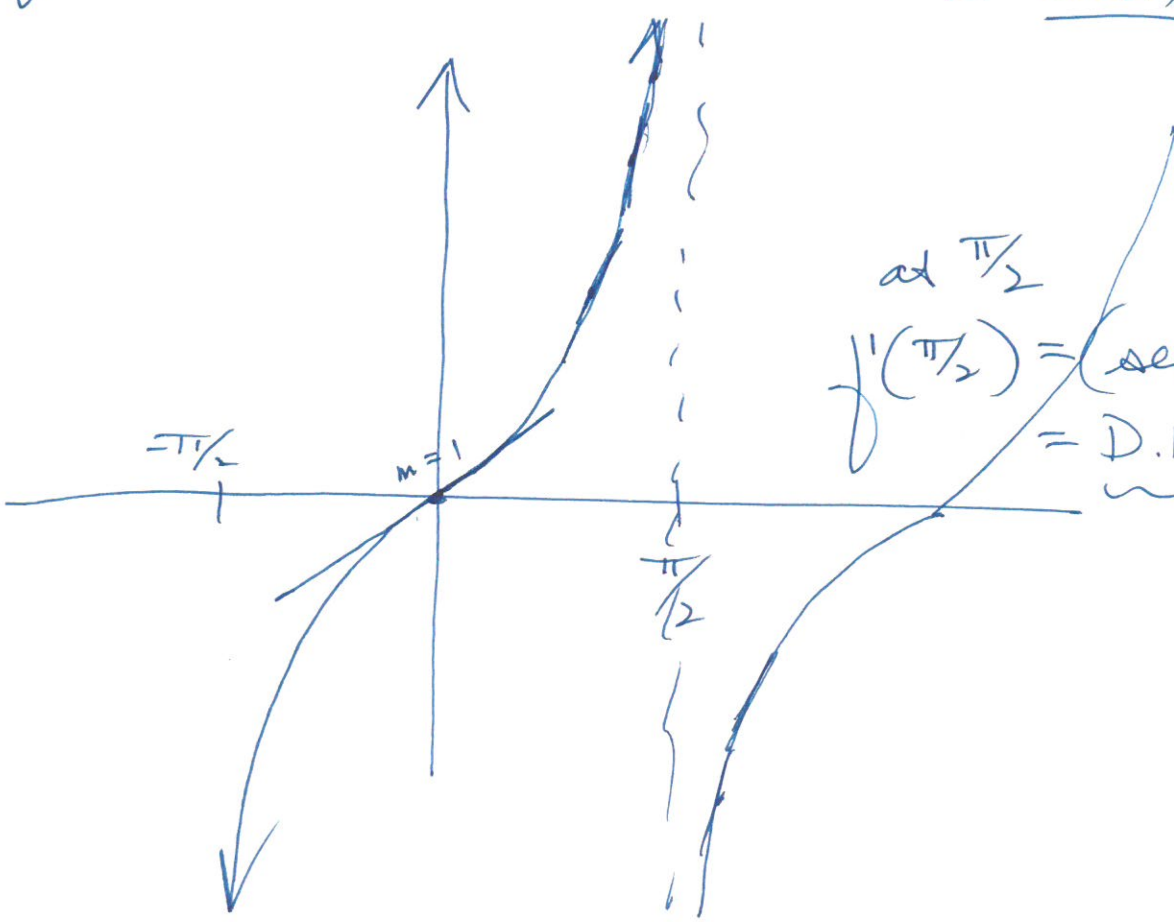
$\boxed{f(x) = \tan x} = \dfrac{\sin x}{\cos x} \quad \boxed{f'(x) = \sec^2 x}$

(use quotient rule)

$$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$f'(x) = \frac{1}{(\cos x)^2} = \left(\frac{1}{\cos x}\right)^2 = (\sec x)^2$$

$$= \underline{\sec^2 x}$$



at $\pi/2$

$f'(\pi/2) = (\sec \pi/2)^2$

$= \underline{\text{D.N.E.}}$

$$\boxed{f(x) = \sec x} = \frac{1}{\cos x}$$

$$f'(x) = \frac{\cancel{\cos x(0)} - (1)(-\sin x)}{(\cos x)^2}$$

$$f'(x) = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\boxed{f'(x) = \sec x \cdot \tan x}$$

$$\boxed{\begin{array}{ll} f(x) = \sin x & f(x) = \cos x \\ f'(x) = \cos x & f'(x) = -\sin x \end{array}}$$

$$\boxed{\begin{array}{ll} f(x) = \tan x & f(x) = \cot x \\ f'(x) = \sec^2 x & f'(x) = -\csc^2 x \end{array}}$$

$$\boxed{\begin{array}{ll} f(x) = \sec x & f(x) = \csc x \\ f'(x) = \sec x \cdot \tan x & f'(x) = -\csc x \cdot \cot x \end{array}}$$

Chain Rule:

$$f(x) = 3x^5 + 14x^2 - 11$$

$$f'(x) = 3(5 \cdot x^4) + 14(2 \cdot x^1) - 0$$

power rule

(1) EXTENDED POWER RULE:

$$f(x) = (3x-1)^2 = 9x^2 - 6x + 1$$

power rule: $f'(x) = 2(3x-1)^1 \cdot 3$ $\quad f'(x) = 18x - 6$

$$f'(x) = (6x-2)3 =$$

extended power rule:

$$y = [f(x)]^k$$

$$y' = k \cdot [f(x)]^{k-1} \cdot f'(x)$$

$$y = (3x-1)^2 \quad y' = 2(3x-1)^1 \cdot 3 = 6(3x-1)$$

$$y = (3x-1)^{14}$$

$$y' = 14 \cdot (3x-1)^{13} \cdot 3 \qquad y' \text{ at } x = 1$$

$$y = (\sin x)^5$$

$$y' = 5 \cdot (\sin x)^4 \cdot \cos x$$

COMPOSITION OF FUNCTIONS
APPROACH

$$y = (f \circ g)(x) = f(g(x))$$

inside function

$$f(x) = x^5 \qquad g(x) = \sin x$$

$$y' = f'(g(x)) \cdot g'(x)$$

$$y' = 5(\sin x)^4 \cdot \cos x$$

$\{$ y in terms of u

$$y = 8 + u^3$$

$\{$ u in terms of x

$$u = 4x^2 - x + 5$$

$$\rightarrow y = 8 + (4x^2 - x + 5)^3$$

$$y' = 3(4x^2 - x + 5)^2 (8x - 1)$$

find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = 3u^2 \cdot (8x - 1)$$

$$\frac{dy}{dx} = 3(4x^2 - x + 5)^2 (8x - 1)$$

$$\frac{dy}{du} \cdot \frac{du}{dr} \cdot \frac{dr}{dw} \cdot \frac{dw}{dx} = \frac{dy}{dx}$$

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TEST #1 RESULTS

A's:	23	(26.4%)	57.5%
B's:	27	(31.1%)	
C's:	12	(13.8%)	
D's:	9	(10.3%)	28.7%
F's:	16	(18.4%)	

AVE: 77.471