MA141 – 012

Monday, October 8

* $x^2+y^2=1$ (find $\frac{dy}{dx}$) implicit diff. (#23)

* $x^2\cdot y^3 = x^3+3x\cdot y^2$ (find $\frac{dy}{dx}$)

* $x^2+x\cdot y^2=1$ (find $\frac{d^2y}{dx^2}$)

TEST #2

WED, OCT 10^{th} (first hour)

(mod 2.7)

$$V(r)=\frac{4}{3}\pi r^3$$

$$V(t)=\frac{4}{3}\pi\left(t^2+t+6\right)^3$$

$$\frac{dV}{dt}=V'(t)=\frac{4}{\cancel{3}}\pi\left[\cancel{3}\left(t^2+t+6\right)^2\cdot(2t+1)\right]\frac{ft^3}{sec}$$

$$V'(t)=4\pi\left(t^2+t+6\right)^2(2t+1)\ \frac{ft^3}{sec}$$

$$V'(3)=4\pi\left(3^2+3+6\right)^2(2\cdot3+1)\ \frac{ft^3}{sec}$$

implicit diff.

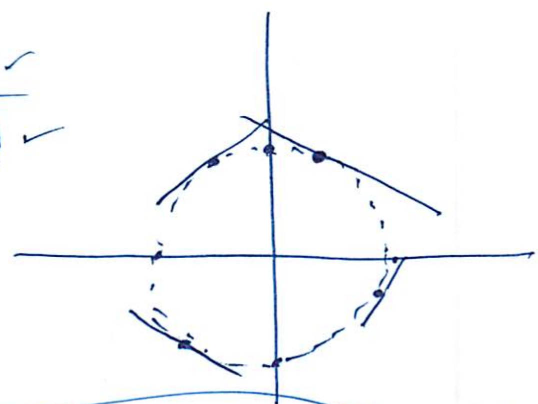
$x^2 + y^2 = 1$ (with respect to x)

find $\frac{dy}{dx}$

$$2x' + 2y' \cdot \frac{dy}{dx} = 0$$

$$\frac{\cancel{2y} \cdot \frac{dy}{dx}}{\cancel{2y}} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$



$$x^2 \cdot y^3 = x^3 + 3x \cdot y^2$$

not solving for y

find $\frac{dy}{dx}$:

$$x^2 \cdot \left[3y^2 \cdot \frac{dy}{dx}\right] + y^3 \cdot [2x] = 3x^2 + 3x\left[2y \frac{dy}{dx}\right] + y^2 \cdot [3]$$

$$3x^2y^2 \cdot \frac{dy}{dx} - 6xy \cdot \frac{dy}{dx} = 3y^2 - 2xy^3 + 3x^2$$

$$\frac{dy}{dx}\left[3x^2y^2 - 6xy\right] = 3y^2 - 2xy^3 + 3x^2$$

$$\frac{dy}{dx} = \frac{3y^2 - 2xy^3 + 3x^2}{3x^2y^2 - 6xy}$$

~~at (2, 8)~~

$x^2 + xy^2 = 1$ find $\frac{d^2y}{dx^2}$

$$2x + \left[x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right] = 0$$

$$\frac{\cancel{2xy}\frac{dy}{dx}}{\cancel{2xy}} = \frac{-2x - y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{-2x - y^2}{2xy}$$

find $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = \frac{(2xy)\left[-2 - 2y \cdot \frac{dy}{dx}\right] - (-2x - y^2)\left(2x \cdot \frac{dy}{dx} + y \cdot 2\right)}{(2xy)^2}$$

$$\frac{dy}{dx} = \boxed{\frac{-2x-y^2}{2xy}}$$

$$\frac{d^2y}{dx^2} = \frac{(2xy)\left[-2-2y\left(\frac{dy}{dx}\right)\right] - \left(-2x-y^2\right)\left(2x\cdot\frac{dy}{dx}+2y\right)}{(2xy)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(2xy)\left[-2-2y\left(\frac{-2x-y^2}{2xy}\right)\right] - \left(-2x-y^2\right)\left(2x\cdot\left(\frac{-2x-y^2}{2xy}\right)+2y\right)}{(2xy)^2}$$

DERIVATIVES OF INVERSE TRIG FUNCTIONS

$y = \sin x$

$\frac{dy}{dx} = y' = m_{TAN} = \cos x$

$\xrightarrow{\text{INU}}$ $x = \sin y$

find $\frac{dy}{dx}$:

$y = \sin(3x^2 - x)$

$y' = [\cos(3x^2-x)]\cdot(6x-1)$

$y = [\sin(8x-1)]^2$

$\frac{dy}{dx} = 2[\sin(8x-1)]^1 \cos(8x-1)\cdot 8$

$\frac{dy}{dx} = 16\sin(8x-1)\cdot\cos(8x-1)$

INVERSE OF THE SINE:

$x = \sin y$

rename:

$y = \sin^{-1} x$

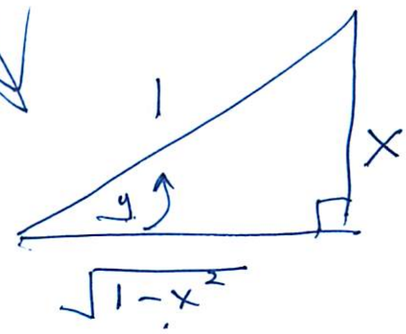
find $\frac{dy}{dx}$:

$$1 = \cos y \cdot \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{\cancel{\cos y} \cdot \frac{dy}{dx}}{\cancel{\cos y}}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$



$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\frac{\sqrt{1-x^2}}{1}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

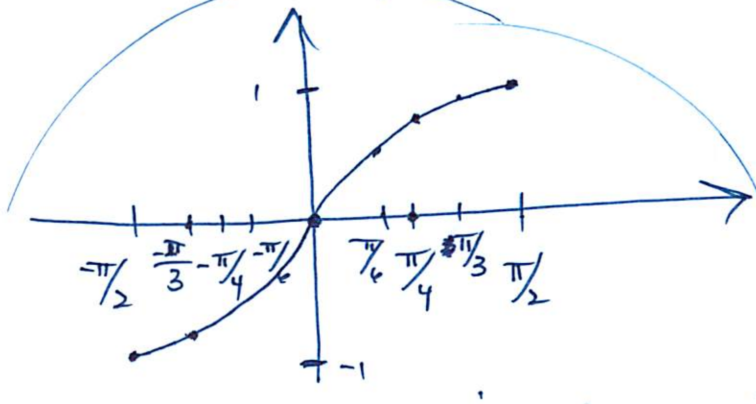
$$\begin{cases} y = \sin^{-1} x \\ \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \end{cases}$$

resume: 7:03

$y = \sin^{-1} x$

$(y = \arcsin x)$

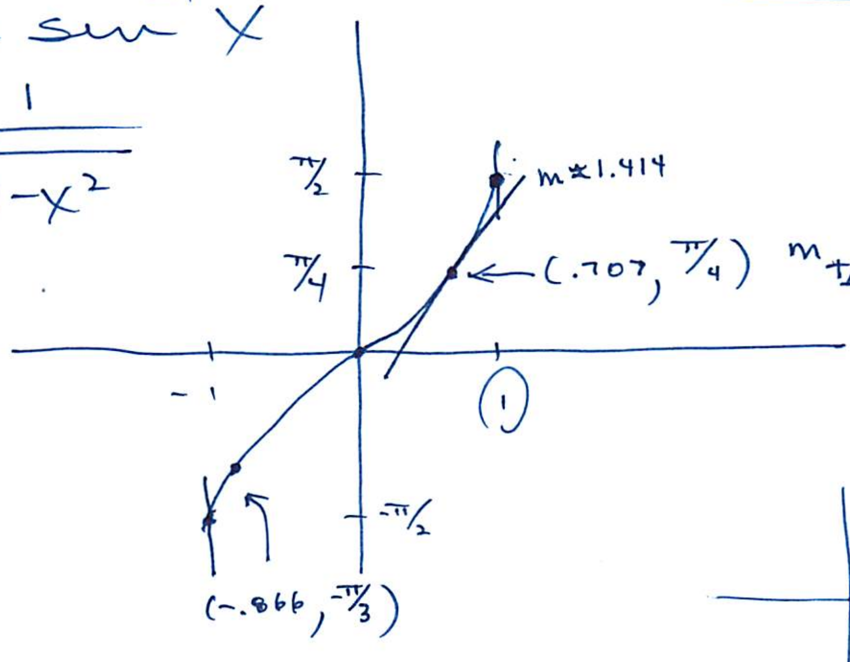
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$



$y = \sin x$

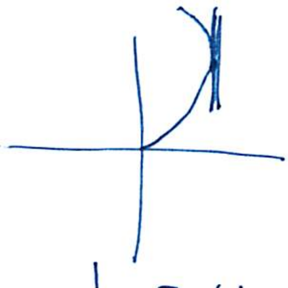
$y = \sin^{-1} x$

$$y' = \frac{1}{\sqrt{1-x^2}}$$



$$m_{tan} = \frac{1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = \frac{1}{\sqrt{1-\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{\frac{1}{2}}} = \sqrt{2}$$



y' at $x = 1$:

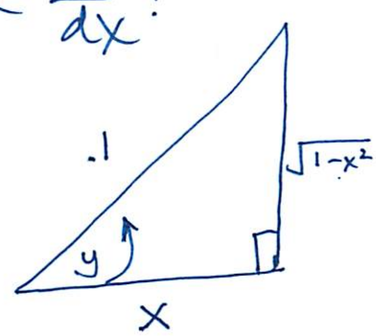
$$y' = \frac{1}{\sqrt{1-(x)^2}} = \frac{1}{\sqrt{1-1^2}} = \frac{1}{0} = \text{undef.}$$

(vertical tangent line)

$y = \cos x \quad \text{INVERSE} \rightarrow \quad x = \cos y$

$(y = \cos^{-1} x)$

find $\frac{dy}{dx}$:



$x = \cos y$

$1 = -\sin y \cdot \frac{dy}{dx}$

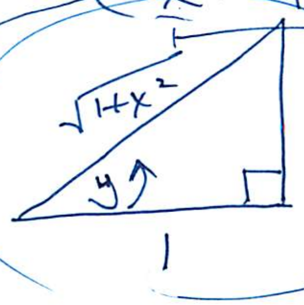
(solve for $\frac{dy}{dx}$)

$$\frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$$

$$y = \cos^{-1} x \qquad y' = \frac{-1}{\sqrt{1-x^2}}$$

$y = \tan x \quad \text{INV} \rightarrow \quad y = \tan^{-1} x$

$(x = \tan y)$



$y = \tan^{-1} x$

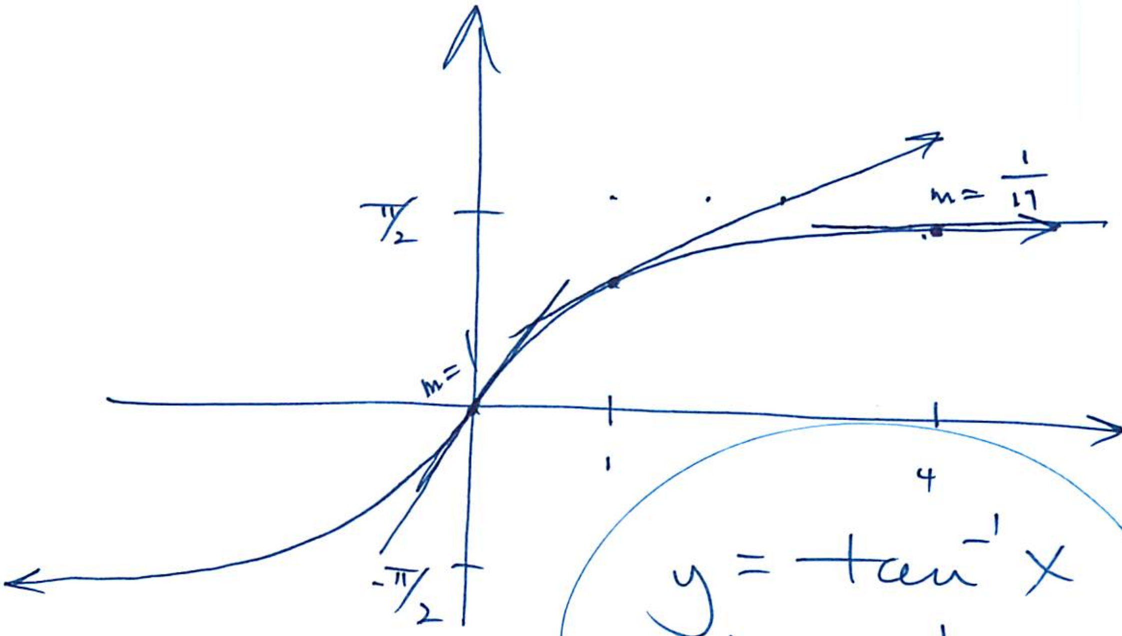
$y' = \frac{1}{1+x^2}$

$x = \tan y$

$1 = \sec^2 y \cdot \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{(\sec y)^2} = \frac{1}{\left(\frac{\sqrt{1+x^2}}{1}\right)^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$



$$y = \tan^{-1} x$$

$$y' = \frac{1}{1+x^2}$$

(1) at $x = 1$:

$$y' = m_{TAN} = \frac{1}{1+(1)^2} = \frac{1}{2}$$

(2) at $x = 4$:

$$y' = m_{TAN} = \frac{1}{1+(4)^2} = \frac{1}{17}$$

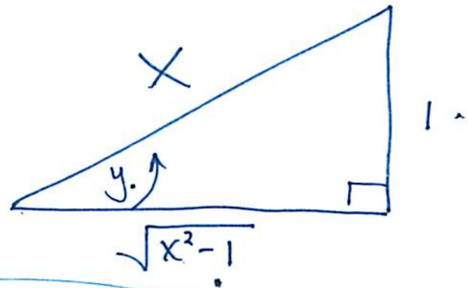
(3) at $x = 0$:

$$y' = m_{TAN} = \frac{1}{1+(0)^2} = 1$$

$y = \sec^{-1} x$

$y = \csc^{-1} x$

$y = \cot^{-1} x$



$x = \csc y$

$1 = (-\csc y \cdot \cot y) \cdot \frac{dy}{dx}$

$$\frac{-1}{\boxed{\csc y} \cdot \boxed{\cot y}} = \frac{dy}{dx}$$

$$\frac{-1}{x \cdot \sqrt{x^2-1}} = \frac{dy}{dx}$$

$y = \csc^{-1} x$

$$y' = \frac{-1}{x \cdot \sqrt{x^2-1}}$$

$\boxed{y = e^x}$ find y':

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y' = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$y' = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$\boxed{y' = e^x}$ $\boxed{\cancel{\lim_{h \to 0} \frac{e^h - 1}{h}}} \to 1$

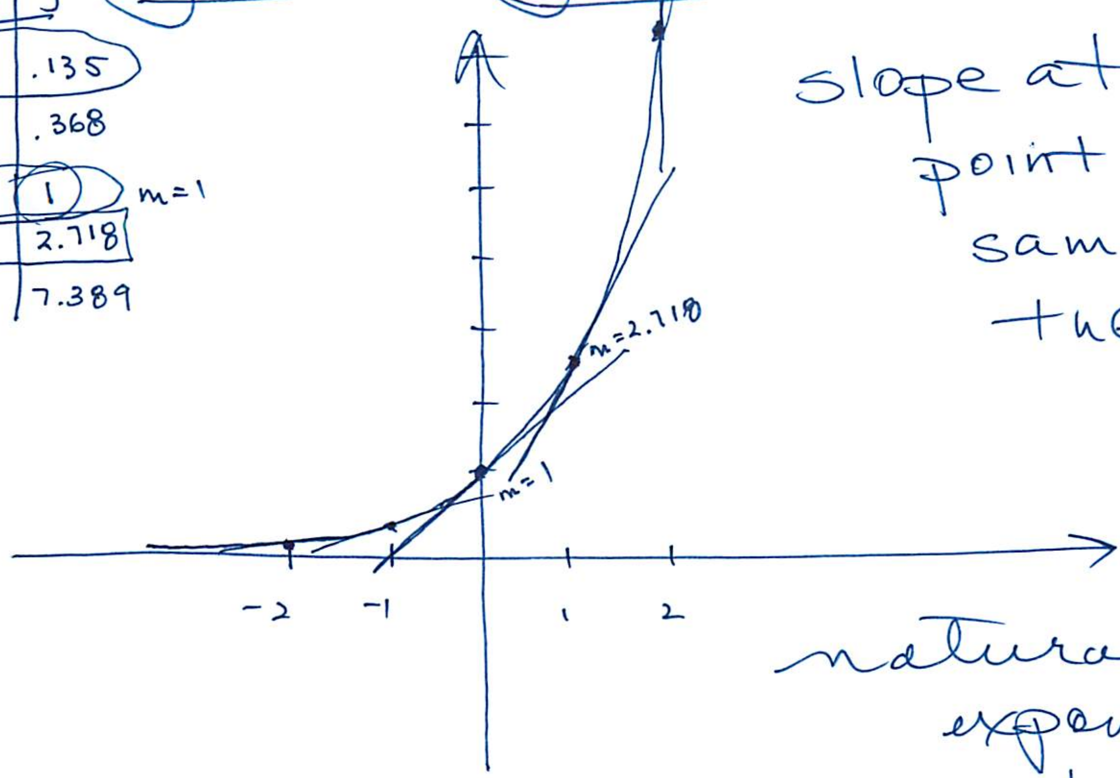
① $h = .01$ $\quad \frac{e^{.01} - 1}{.01} \approx 1.005$

② $h = .0001$ $\quad \frac{e^{.0001} - 1}{.0001} \approx 1.00005$

x	y
−2	.135
−1	.368
0	1
1	2.718
2	7.389

$m=1$

$y = e^x$ $\boxed{y' = e^x}$



slope at any point is the same as the y-value

natural exponential function

$y = 5^x$ $y = 2^x$ $y = 7^x$

$y = a^x$

$y = e^x$
$y' = e^x$

$y = e^{5x}$
$y' = e^{5x} \cdot 5 = 5 \cdot e^{5x}$

$y = e^{(x^2-2x+8)}$
$y' = \left[e^{x^2-2x+8}\right] \cdot (2x-2)$

$y = \dfrac{e^x}{x}$

$y' = \dfrac{x(e^x) - e^x(1)}{x^2}$

$y' = \dfrac{e^x(x-1)}{x^2}$

<u>2.6.2</u> : $y = e^x \quad y' = e^x$ /

MA 141-012 F18 Recitation 6, Oct 2

1. Compute the following limits:

$$a)\ \lim_{\theta\to 0}\frac{\theta^2}{\sin 5\theta} \qquad b)\ \lim_{\theta\to 0}\frac{\cos\theta+4\sin 3\theta-1}{4\theta}$$

2. Find the derivative $\frac{dy}{dx}$ of the given functions:

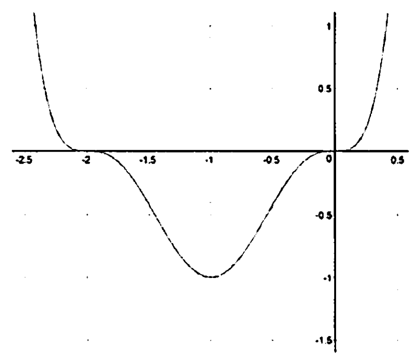
$$a)\ y=(10-5x^2)\sin x \qquad b)\ y=\frac{8\tan x}{7\sec x} \qquad c)\ y=\frac{1}{\sqrt{x}}\cos(x^2+x+1)$$

3. Suppose f and g are functions such that

$$f(1)=2, g(1)=-7, f'(1)=5, \text{ and } g'(1)=-8.$$

Determine $(\frac{f}{f-g})'(1)$.

4. The graph of the function $f(x) = (x^2+2x)^3$ is displayed below. Determine the equation of the tangent line to f at $(-1,-1)$ then sketch the tangent line. List all values of x for which the corresponding tangent is horizontal.



5. Recall that the volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$. Suppose the radius (in feet) of a sphere is a function of time t (in seconds) and is given by the following

$$r(t) = t^2+t+6.$$

Find $\frac{dV}{dt}$, the derivative of the volume (in ft^3/sec). Determine how fast the volume is changing when $t = 3$ seconds.

Relevant Info to Recall

- $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ and $\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}=0$.
- Let $f(x)$ and $g(x)$ be differentiable functions. Then,

$$\frac{d}{dx}(fg)=f'g+g'f,\quad \frac{d}{dx}\left(\frac{f}{g}\right)=\frac{f'g-g'f}{g^2},\ \text{and}\ \frac{d}{dx}(f\circ g)(x)=f'(g(x))\cdot g'(x).$$

- $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(\tan x)'=\sec^2 x$, $(\sec x)'=\sec x\tan x$