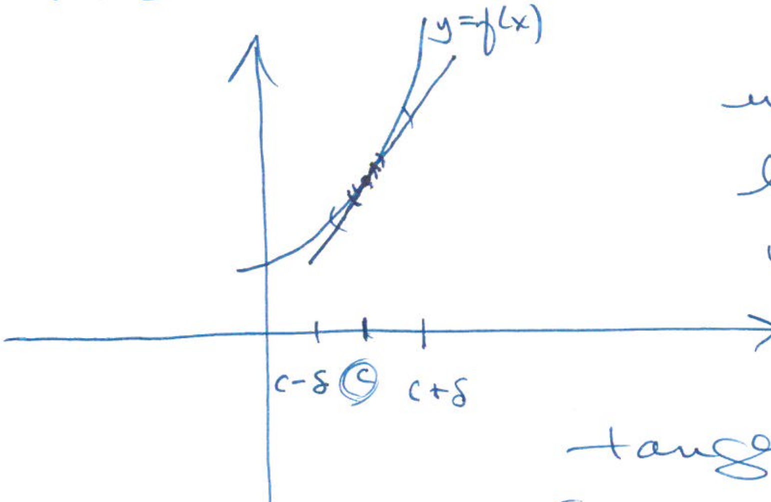
MA 141-012

Wednesday, October 17

- test #2 returned today

CH 3:

APPLICATIONS OF THE DERIV.



use the tangent line to approx. what is happening on the curve

tangent line approx
(LINEARIZATION)

if δ is "small", then the line is reasonably close to the curve.

equation of the tangent line to the curve $y = f(x)$ at the point $(c, f(c))$

$$y - y_1 = m_{TAN}(x - x_1)$$

$$y - f(c) = f'(c)(x - c)$$

solve for y:

LINEARIZATION

$$\boxed{y = f(c) + f'(c)(x-c)}$$

on tangent line

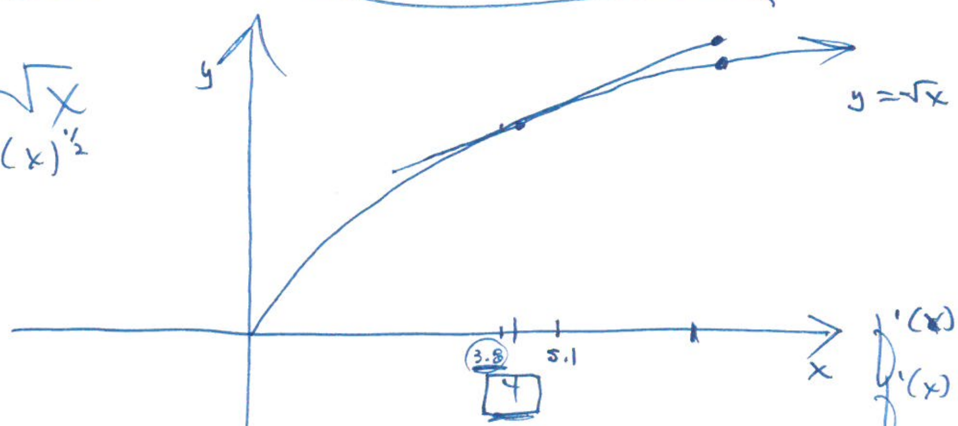
$$y \approx f(c) + f'(c)(x-c)$$

y-value on tangent line

$$L_x(c) \approx \underbrace{f(c) + f'(c)(x-c)}$$

y-value on the CURVE

ex: $y = \sqrt{x}$ $(x)^{1/2}$



Goal: $\sqrt{3.8}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$$L_x(c) \approx \underbrace{\sqrt{4} + \frac{1}{4}(x-4)}$$

$$\approx 2 + \frac{1}{4}x - 1$$

$$\sqrt{x} \approx 1 + \frac{1}{4}x \quad \text{(for } x \text{ "near" 4)}$$

$$\sqrt{3.8} \approx 1 + \frac{1}{4}(3.8)$$

$$\begin{array}{r} .95 \\ 4\overline{)3.8} \\ \underline{36} \\ 20 \end{array}$$

$$\sqrt{3.8} \approx \underline{1.95}$$

from CALCULATOR

(1.949358)

$\sqrt{x} \simeq 1 + \frac{1}{4}x$ (for x "near" 4)

$\sqrt{5.1} \simeq \underline{1 + \frac{1}{4}(5.)}$

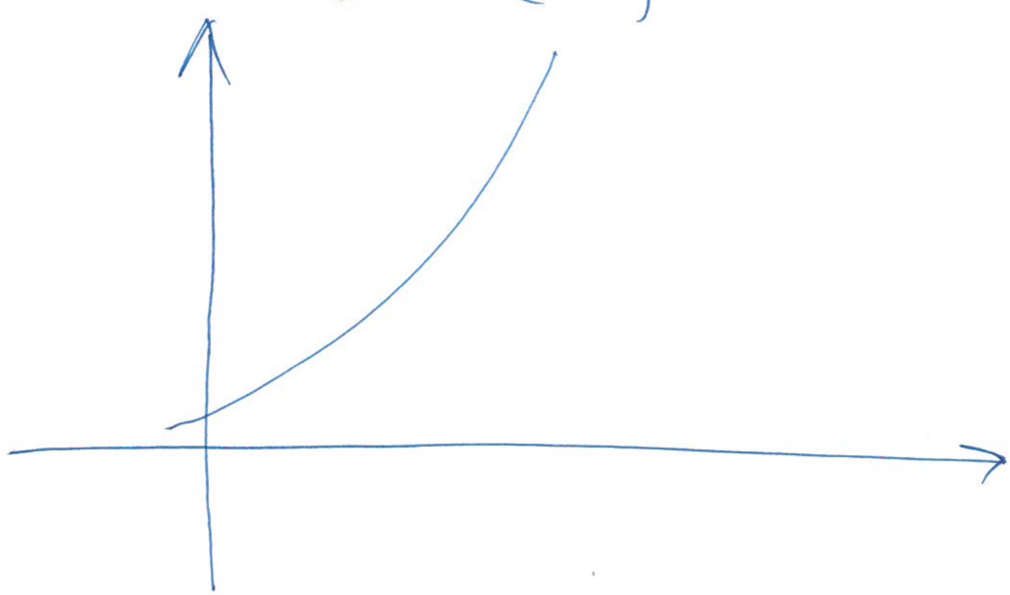
from calculator: $\underset{\sim\sim\sim}{2.2583}$

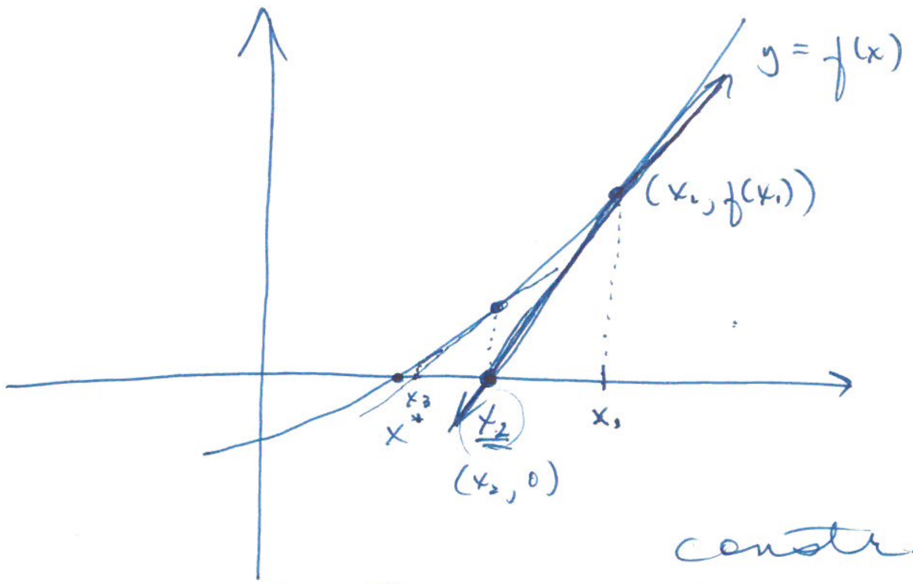
$\underline{2.25}$

NEWTON'S METHOD:

(finding the <u>root</u> (or zero) of a function)

(?x?, 0)





initial guess: x_1

root is x^*

$(x^*, 0)$

construct the tangent line to the curve at

$(\underline{x_1}, \underline{f(x_1)})$

$$\left\{ \begin{array}{l} \underline{y} - f(x_1) = f'(x_1)(\underline{x} - x_1) \\ (\underline{x_2}, \underline{0}) \text{ is ON this line} \end{array} \right.$$

$$\boxed{0 - f(x_1) = f'(x_1)(x_2 - x_1)}$$

solve this for $\underline{x_2}$:

$$\frac{-f(x_1)}{f'(x_1)} = \frac{\cancel{f'(x_1)}(x_2 - x_1)}{\cancel{f'(x_1)}}$$

$$-\frac{f(x_1)}{f'(x_1)} = x_2 - \cancel{x_1}$$

$$+x_1 \qquad\qquad +\cancel{x_1}$$

$$x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

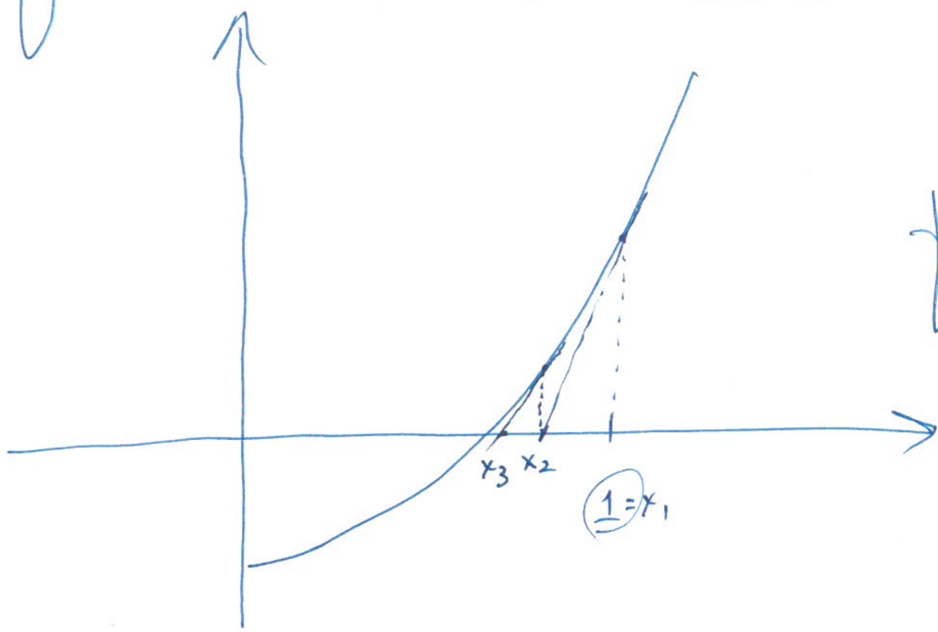
$n, n+1$

$\vdots$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

NEWTON'S METHOD:

$f(x) = x^2 - \frac{1}{2}$ $\qquad$ $x_1 = 1$ (guess)



$f'(x) = 2x$

using x_1, find x_2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\underline{x_2} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{\frac{1}{2}}{2} = 1 - \frac{1}{4} = \underline{3/4}$$

$x_1 = 1$
$x_2 = 3/4$
$x_3 = .70833$
$x_4 = .707107$

(resume: 7:02)

using x_2, find x_3:

$f(x) = x^2 - \frac{1}{2}$
$f'(x) = 2x$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 3/4 - \frac{\boxed{f(3/4)}}{f'(3/4)} = 3/4 - \frac{.0625}{1.5} = .70833$$

$$x_4 = .70833 - \frac{f(.70833)}{f'(.70833)} \approx .70833 - \frac{(.70833)^2 - \frac{1}{2}}{2(.70833)}$$

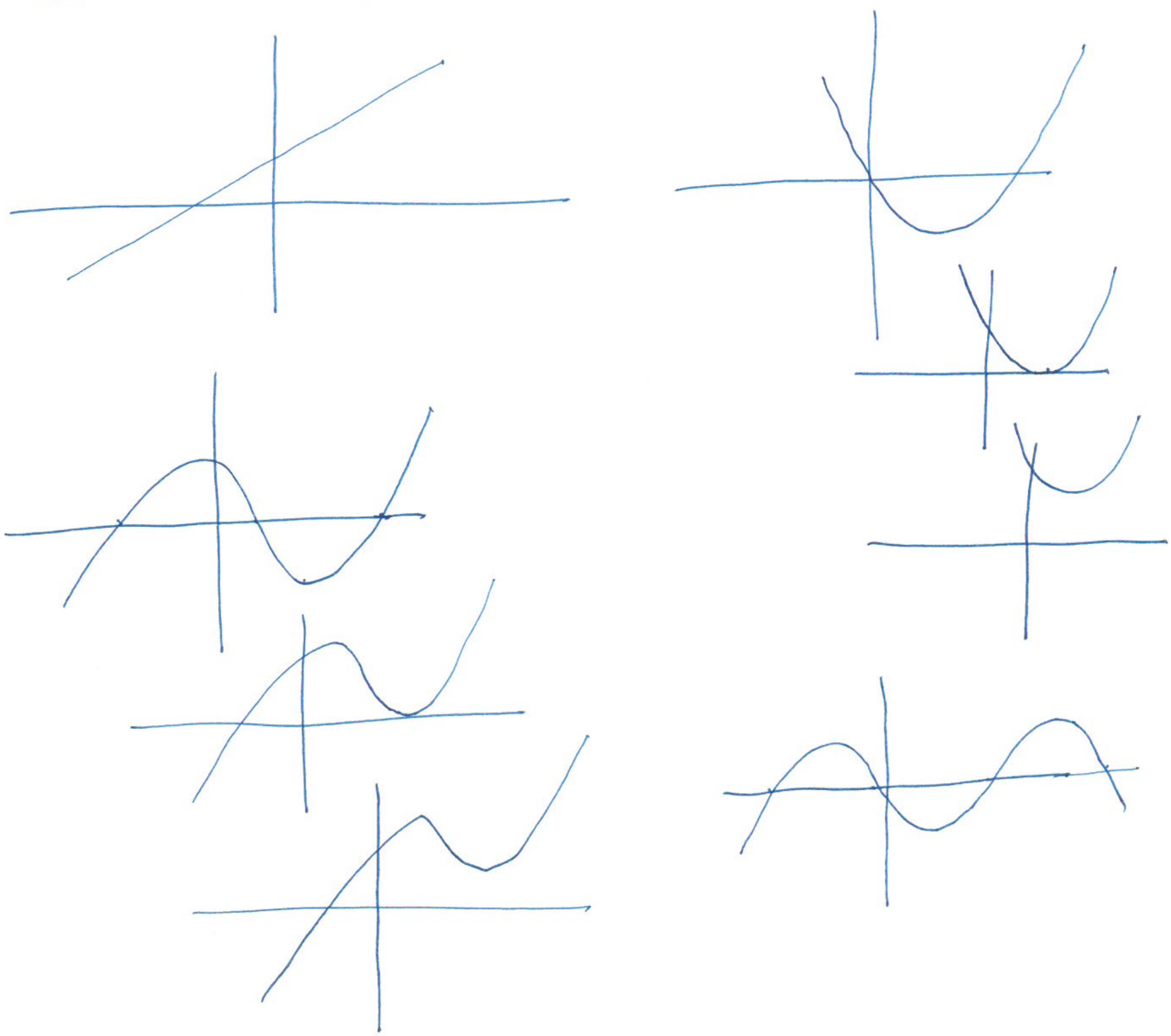
$$\approx \underline{.707107}$$

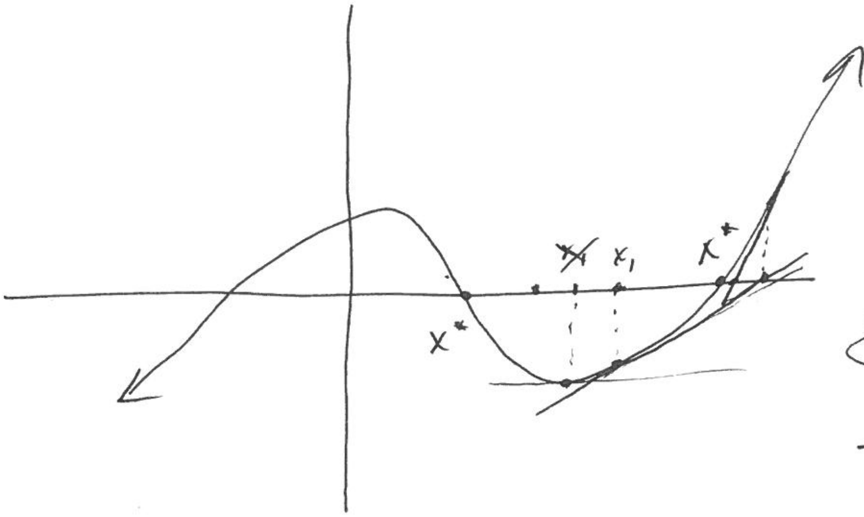
$$x_5 \approx .707107 - \frac{f(.707107)}{f'(.707107)}$$

$$x_5 \approx .707107 - \frac{\left[(.707107)^2 - \frac{1}{2}\right]}{2(.707107)}$$

$$x_5 \approx \underline{.707106}$$

$$\vdots$$



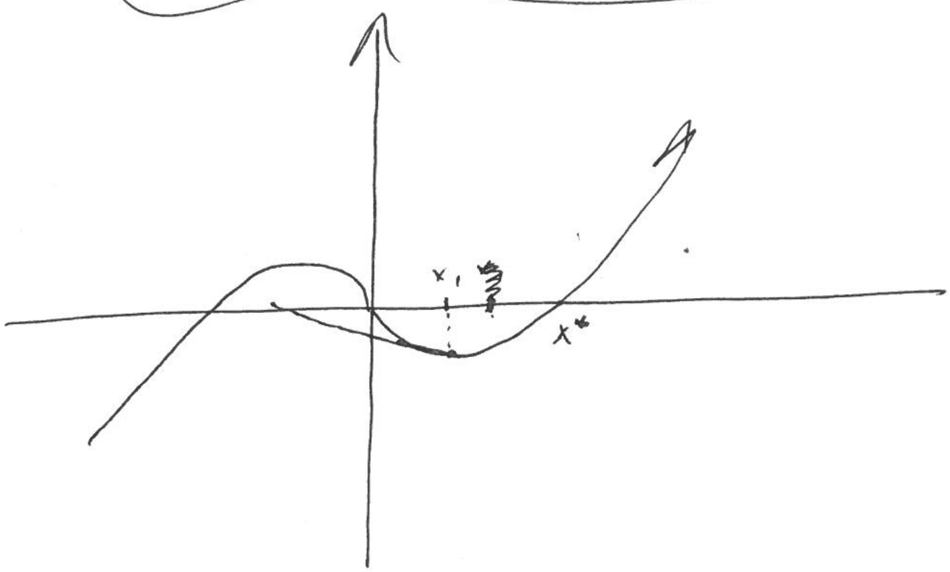


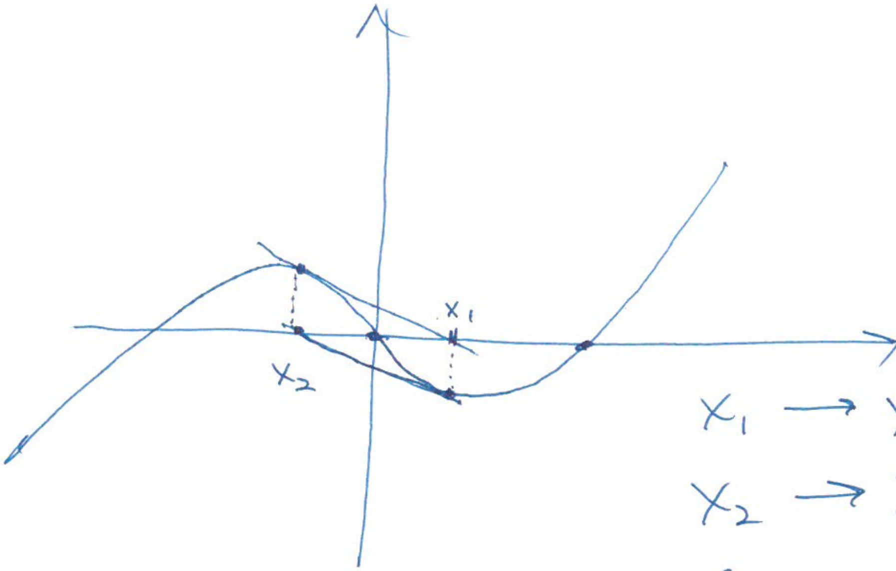
FAILS

guess x_1

$f'(x_1) = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$





FAILS:

$x_1 \rightarrow x_2$

$x_2 \rightarrow x_1$

loop

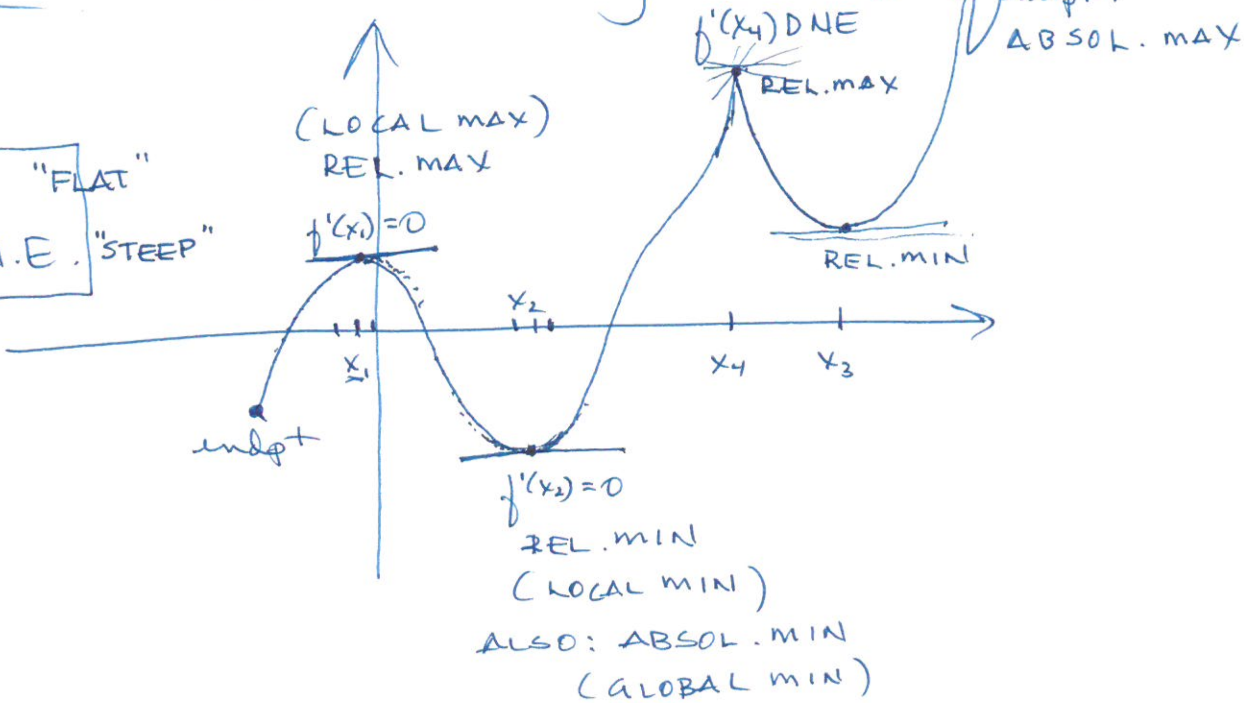
3.1.2 Simple Pendulum (Challenging)

no!! (not now)

3.2: Curve sketching (using $f'(x)$)

① endpts

$\boxed{\begin{array}{l} ② \; f'(x) = 0 \quad \text{"FLAT"} \\ ③ \; f'(x) \text{ D.N.E.} \quad \text{"STEEP"} \end{array}}$



CRITICAL VALUES:

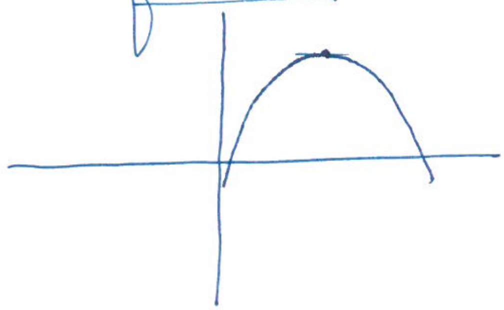
① $f'(x) = 0$
HORIZONTAL TANGENT LINE

$(x_1, f(x_1))$

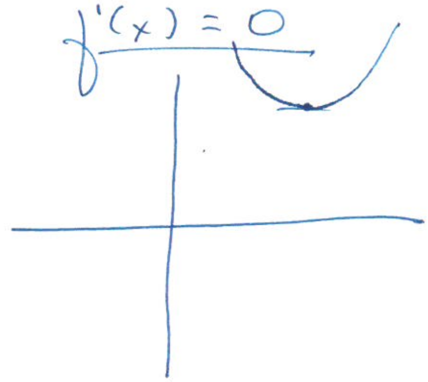
CRITICAL POINTS

② $f'(x)$ D.N.E.
($f'(x)$ undef)
VERTICAL TANGENT LINE

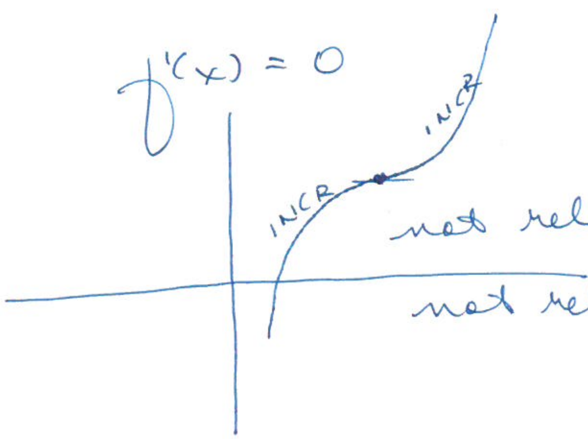
$f'(x) = 0$



$f'(x) = 0$

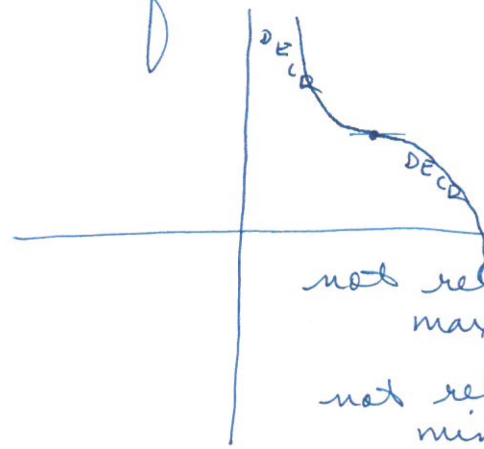


$f'(x) = 0$



not rel max,
not rel min

$f'(x) = 0$



not rel max
not rel min

$f(x) = \sqrt{x}$ critical values

$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

① $f'(x) = 0$

$\frac{1}{2\sqrt{x}} \neq 0$ no "FLAT" places

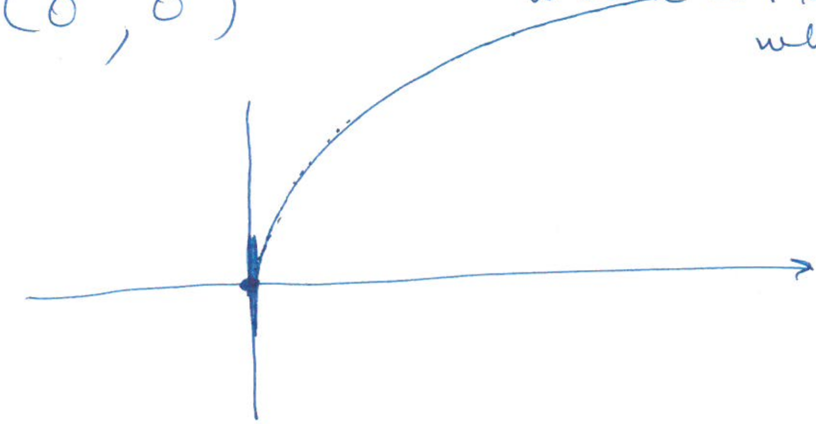
② $f'(x)$ undef

$\frac{1}{2\sqrt{x}}$ undef.

when $2\sqrt{x} = 0$

when $x = 0$

$(0, 0)$



MA 141-012 **F18** **Recitation 8, Oct 16**

1. Use logarithmic differentiation to determine $\frac{dy}{dx}$ for the given functions.

$$a)\ y = x^{\cos(3x)} \qquad b)\ y = \frac{2^x \sin(2x+1)}{\cos(x)\sqrt[3]{x^2+1}}$$

2. Claire starts at point P and runs east at a rate of 11 ft/sec. One minute later, Anna starts at P and runs north at a rate of 9 ft/sec. At what rate (in feet per second) is the distance between them changing after another minute?

3. As sand leaks out of a hole in a container, it forms a conical pile whose altitude is always the same as its radius. If the height of the pile is increasing at a rate of 7 in/min, find the rate (in cubic inches per minute) at which the sand is leaking out when the height is 12 in.

4. Water is flowing at a rate of 6 ft^3/min into a tank that is the shape of a right circular cylinder whose base radius is 2 ft. How fast (in feet per minute) is the water level rising?

Relevant Info to Recall

1. Volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is radius and h is height.
2. Volume of a circular cylinder is $V = \pi r^2 h$, where r is radius and h is height.

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TEST #2 RESULTS

A's	40	(46.5%)	73.2%
B's	23	(26.7%)	
C's	13	(15.1%)	
D's	5	(5.8%)	11.6%
F's	5	(5.8%)	

AVE: 84.547