MA141-012

Monday, October 22

10/22 : 3.2; 3.3
10/24 : 3.3; begin 3.4

10/29 : 3.4; 3.5
10/31 : TEST #3; 3.6 (after test)

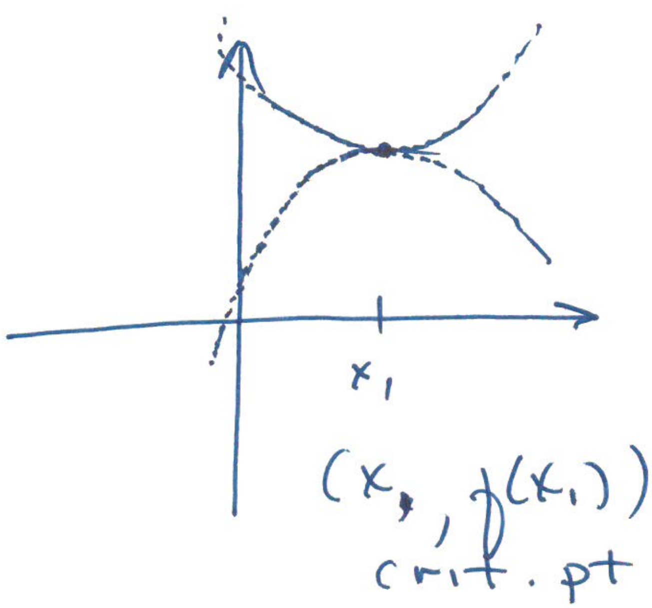
↑

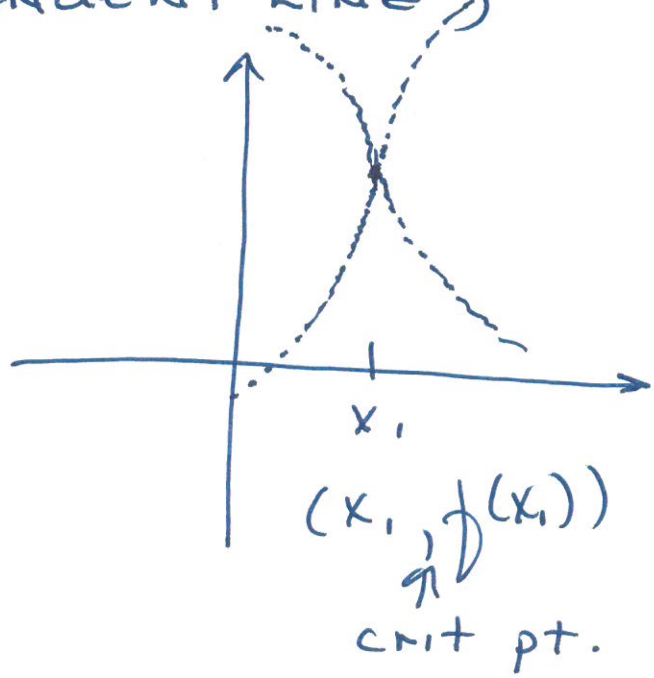
(2.7; 3.1; 3.2; 3.3; 3.4; 3.5)

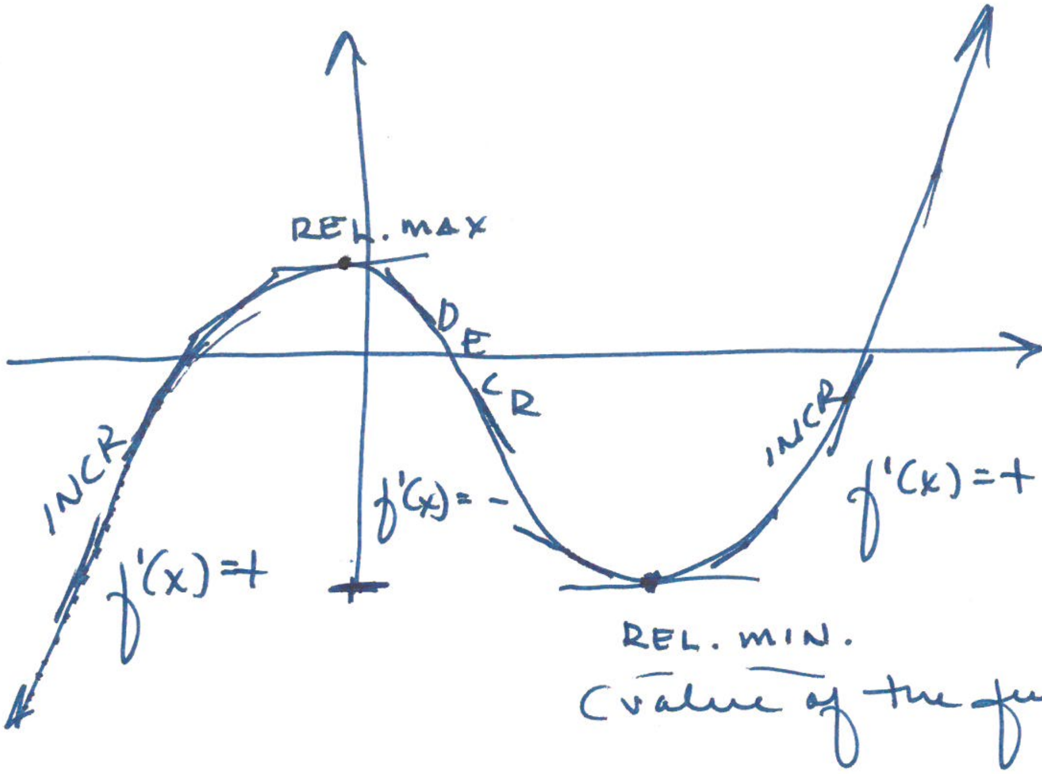
CRITICAL VALUES:

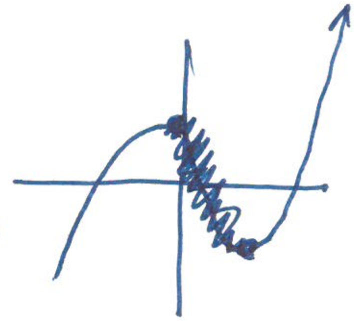
① $f'(x) = 0$ "FLAT" (HORIZONTAL TANGENT LINE)

② $f'(x)$ undef (D.N.E.) "STEEP" (VERTICAL TANGENT LINE)









Polynomial:

$f(x) = x^3 - 3x - 3$

$f'(x) = \boxed{3x^2 - 3 = 0}$

~~$3x^2 = -3$~~

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1$ and $x = -1$

$(1, -5) = (1, f(1))$ $\quad$ $(-1, f(-1)) = (-1, -1)$

$f(1) = 1^3 - 3(1) - 3$

$f(1) = 1 - 3 - 3$

$f(1) = -5$

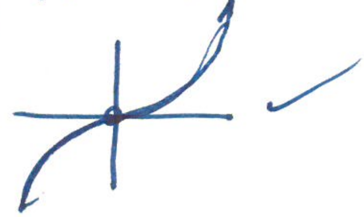
$f(-1) = (-1)^3 - (3)(-1) - 3$

$f(-1) = -1 + 3 - 3$

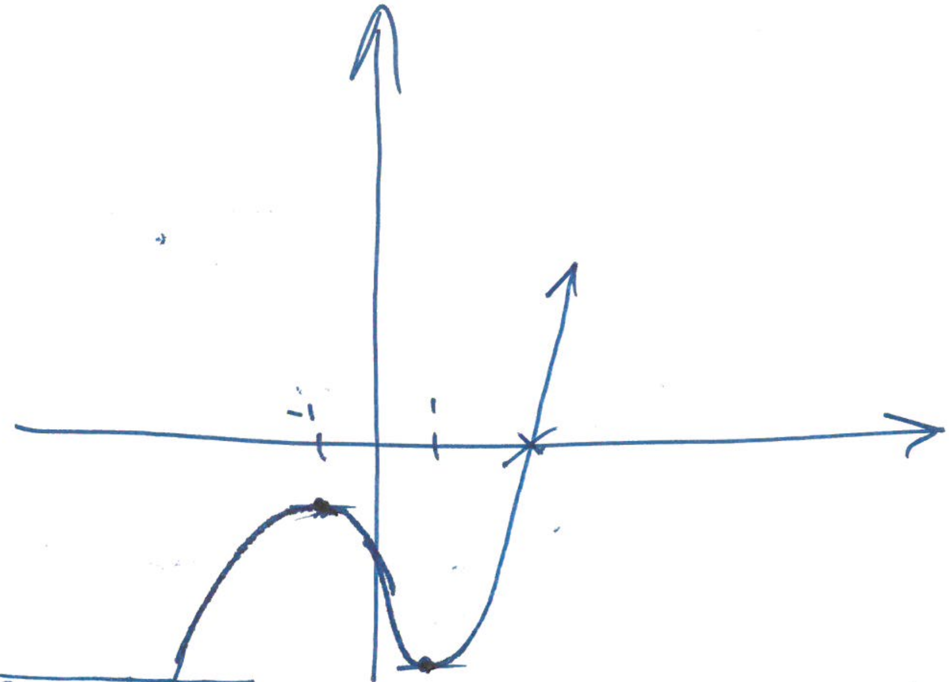
$f(-1) = -1$

$f(x) = x^3$

$f'(x) = 3x^2 = 0$



~~$f'(x)$ UNDEF (NEVER)~~

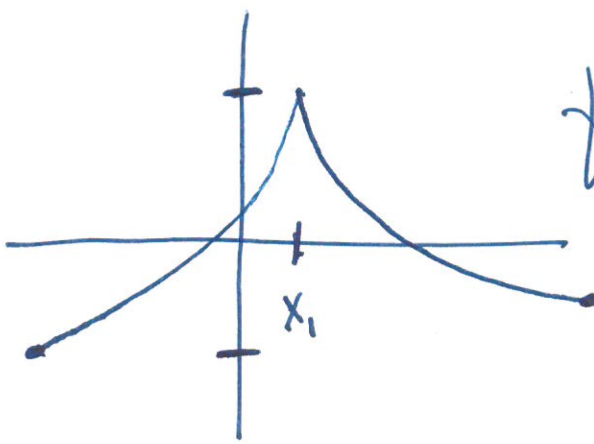


$f'(x) = 3(x-1)(x+1)$

$f'(x)$:

$f'(-2) = (+)(-)(-) = +$ | -1 | $f'(0) = + \cdot - \cdot + = -$ | 1 | $f'(2) = + \cdot + \cdot + = +$

$f(x)$ INCR, $f(x)$ DECR, $f(x)$ INCR.



$f'(x_1)$ D.N.E.

$$f(x) = \frac{6x}{1+9x^2}$$ not a polynom.

V.A.: none

H.A.: $\lim_{x\to\infty} \frac{6x}{1+9x^2} = 0$ $(y=0)$

$$f'(x) = \frac{(1+9x^2)\cdot(6) - (6x)(18x)}{(1+9x^2)^2}$$

$$f'(x) = \frac{6 + 54x^2 - 108x^2}{(1+9x^2)^2}$$

$$f'(x) = \frac{6 - 54x^2}{(1+9x^2)^2} = \frac{6(1-9x^2)}{(1+9x^2)^2}$$

(1) $f'(x) = 0$

$f'(x) = 0 = \frac{6(1-9x^2)}{(1+9x^2)^2}$

$6(1-9x^2) = 0$

$6(1-3x)(1+3x) = 0$

$x = 1/3$ and $x = -1/3$

$(1/3, 1)$ "FLAT"

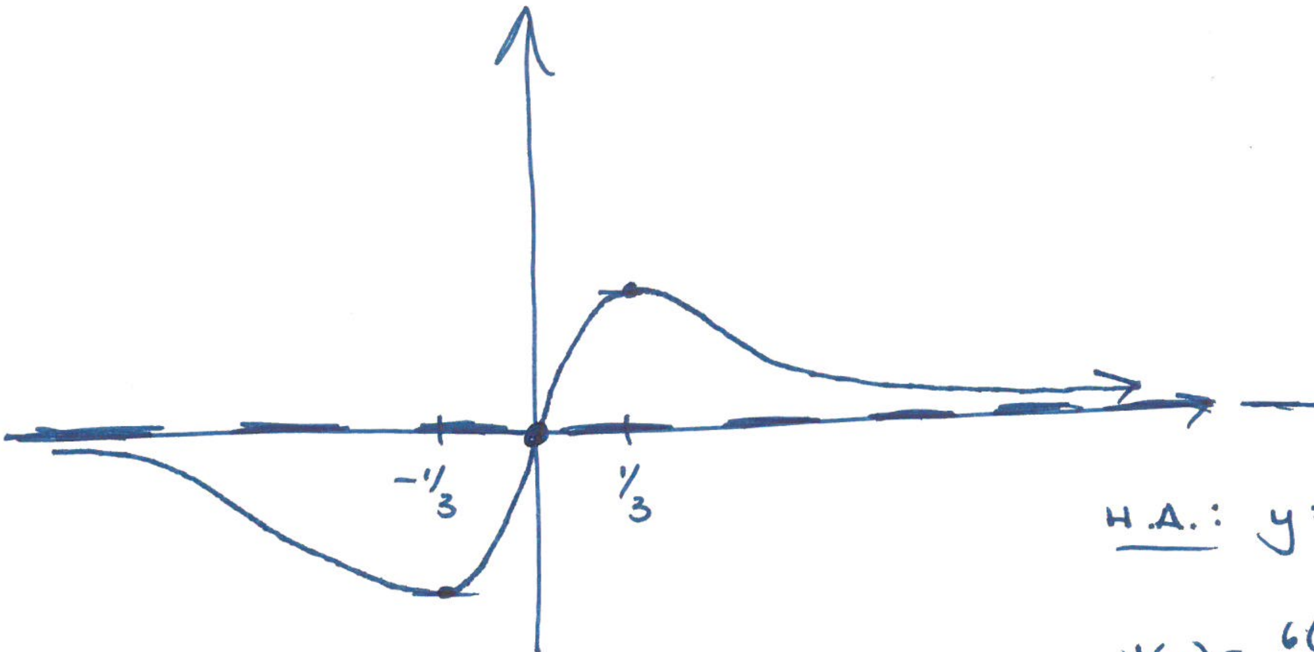
$$f(1/3) = \frac{6(1/3)}{1+9(1/3)^2} = \frac{2}{2} = 1$$

$$f(-1/3) = \frac{6(-1/3)}{1+9(-1/3)^2} = \frac{-2}{2} = -1$$

(2) $f'(x)$ undef. (NEVER)

$\frac{6(1-9x^2)}{(1+9x^2)^2}$ undef when $(1+9x^2)^2 \neq 0$

$(-1/3, -1)$ "FLAT"



H.A.: $y = 0$

$$f'(x) = \frac{6(1-9x^2)}{(1+9x^2)^2}$$

$f'(x)$:

$f'(-1) = \frac{+ \cdot -}{+} = -$ | $f'(0) = \frac{+ \cdot +}{+} = +$ | $f'(1) = \frac{+ \cdot -}{+} = -$

$f(x)$ DECR $\;-1/3\;$ $f(x)$ INCR $\;1/3\;$ $f(x)$ DECR.

$$f(x) = \frac{6x}{1+9x^2}$$

resume: 7:00

INT: $(0, 0)$

$$f(x) = x^{2/3}(4 - x^2)$$

$$f'(x) = x^{2/3}(-2x) + (4 - x^2) \cdot \frac{2}{3} x^{-1/3}$$

$$f'(x) = \frac{-2 \cdot (3 \cdot x^{1/3}) x^{5/3}}{3x^{1/3}} + \frac{2(4 - x^2)}{3x^{1/3}}$$

$$f'(x) = \frac{-6x^2}{3x^{1/3}} + \frac{2(4-x^2)}{3x^{1/3}} = \frac{-6x^2 + 2(4-x^2)}{3x^{1/3}}$$

$$f(x) = x^{2/3}(4-x^2)$$ ⑥

$$-6x^2 + 8 - 2x^2$$

$$f'(x) = \frac{-6x^2 + 2(4-x^2)}{3x^{1/3}}$$

$$f'(x) = \frac{8 - 8x^2}{3x^{1/3}} = \frac{8(1-x^2)}{3x^{1/3}}$$

(1) $f'(x) = 0$

$$\frac{8-8x^2}{3x^{1/3}} = 0$$

$$8 - 8x^2 = 0$$

$$8(1-x^2) = 0$$

$$8(1-x)(1+x) = 0$$

$x = 1$ and $x = -1$

$(1, f(1))$ & $(-1, f(-1))$

$(1, 3)$ & $(-1, 3)$

HORIZ. TANGENT LINES

(2) $f'(x)$ undef.

$\frac{8(1-x^2)}{3x^{1/3}}$ undef.

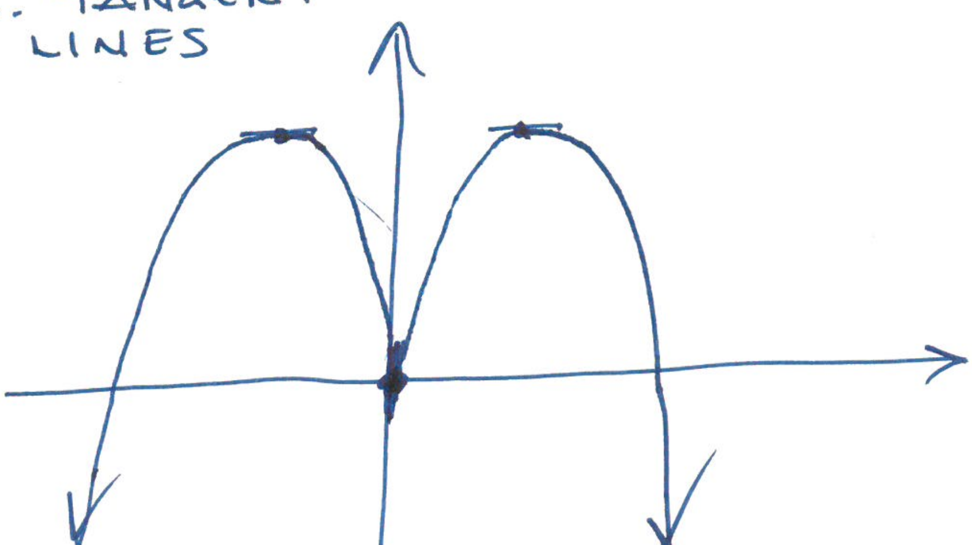
$$3x^{1/3} = 0$$

$$x = 0$$

$(0, f(0))$

$(0, 0)$

VERT TANGENT LINE

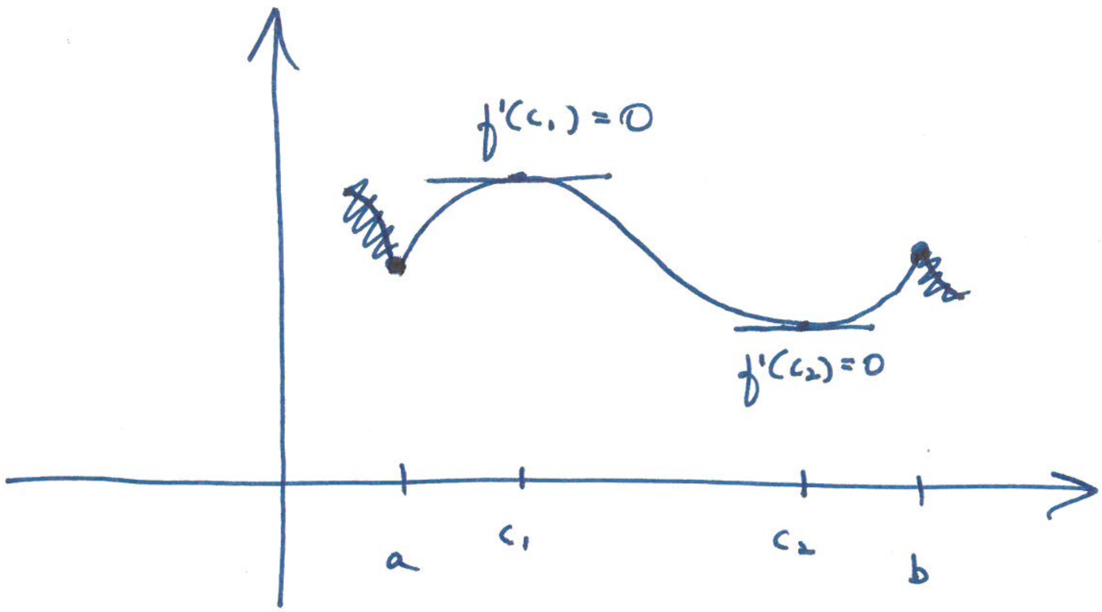


$f'(x)$:

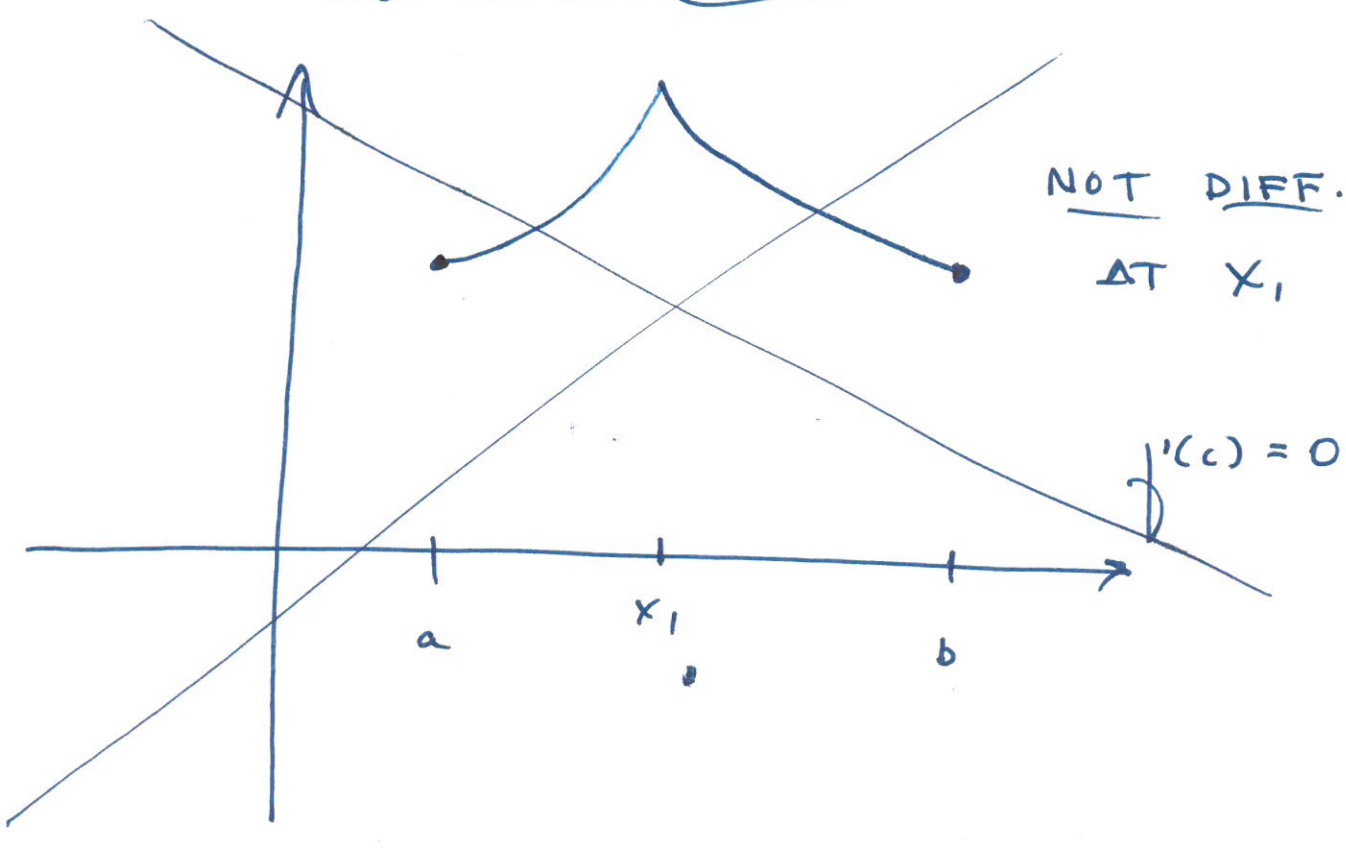
$f'(-8) = \frac{+ \cdot -}{-} = +$ | $f'(-\tfrac{1}{2}) = \frac{+ \cdot +}{-}$ | $f'(\tfrac{1}{2}) = +$ | $f'(8) = -$

$f(x)$ INCR (−1) $f(x)$ DECR (0) $f(x)$ INCR (1) $f(x)$ DECR

ROLLE'S THEOREM:



$f(a) = f(b)$. contin on $[a, b]$;
differentiable on (a, b)



(1) verify the hypotheses of Rolle's Thm:

(1) $f(a) = f(b)$

(2) contin on $[a,b]$

(3) differentiable on (a,b)

$f(x) = x^3 - 4x$ VERIFY on $[-2,2]$

(1) $f(-2) \stackrel{?}{=} f(2)$ ✓

$0 \stackrel{?}{=} 0$

(2) contin ?? polynomial

(3) diff ?? $f'(x) = 3x^2 - 4$ DEFINED on $[-2,2]$

find c in $[a,b]$ such that

$f'(c) = 0$

find c in $[-2,2]$ such that

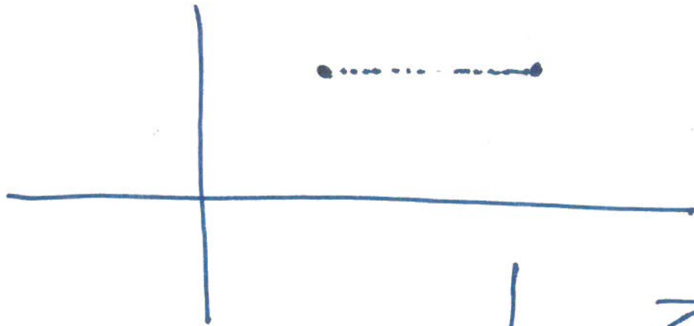
$3x^2 - 4 = 0$

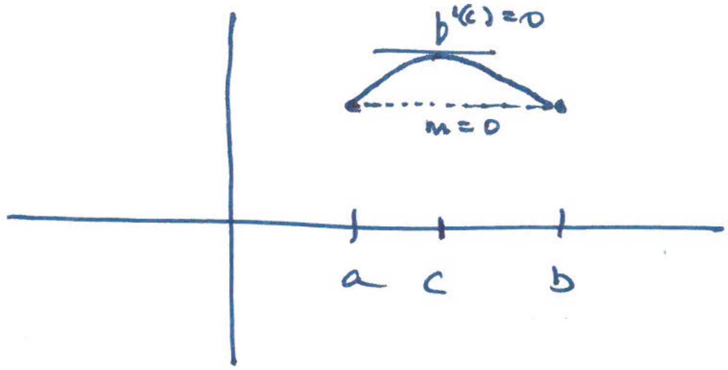
$3x^2 = 4$

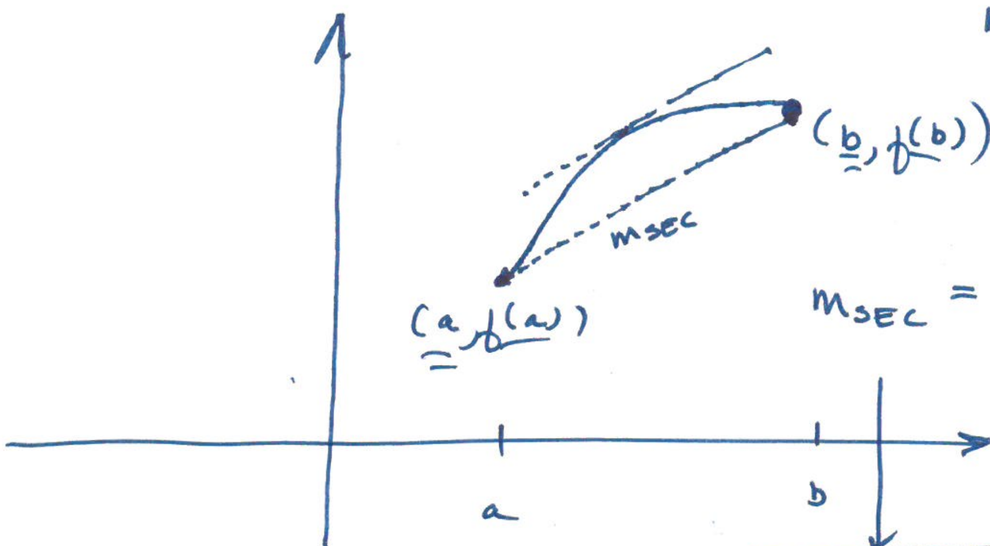
$x^2 = 4/3 \quad x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.15$

$c_1 \approx -1.15$

$c_2 \approx +1.15$







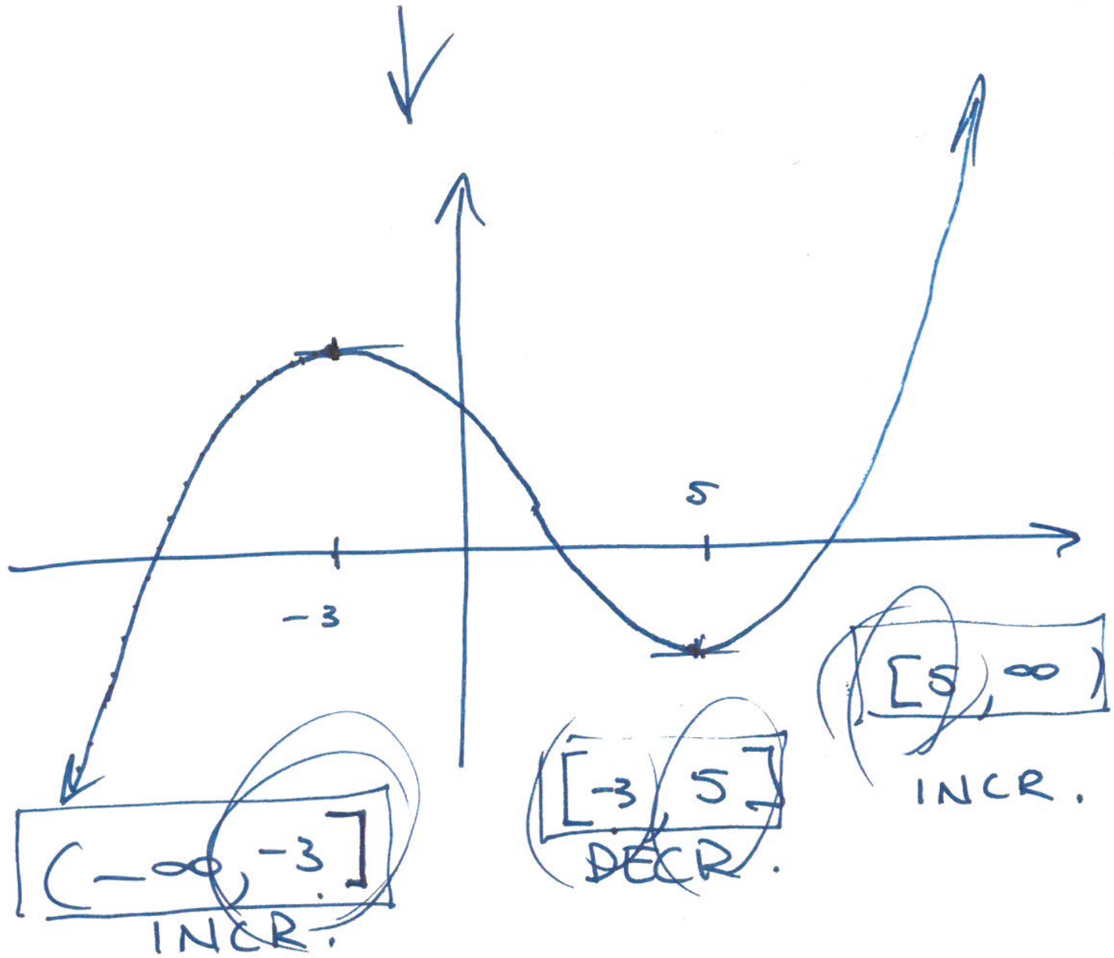
MEAN VALUE THEOREM

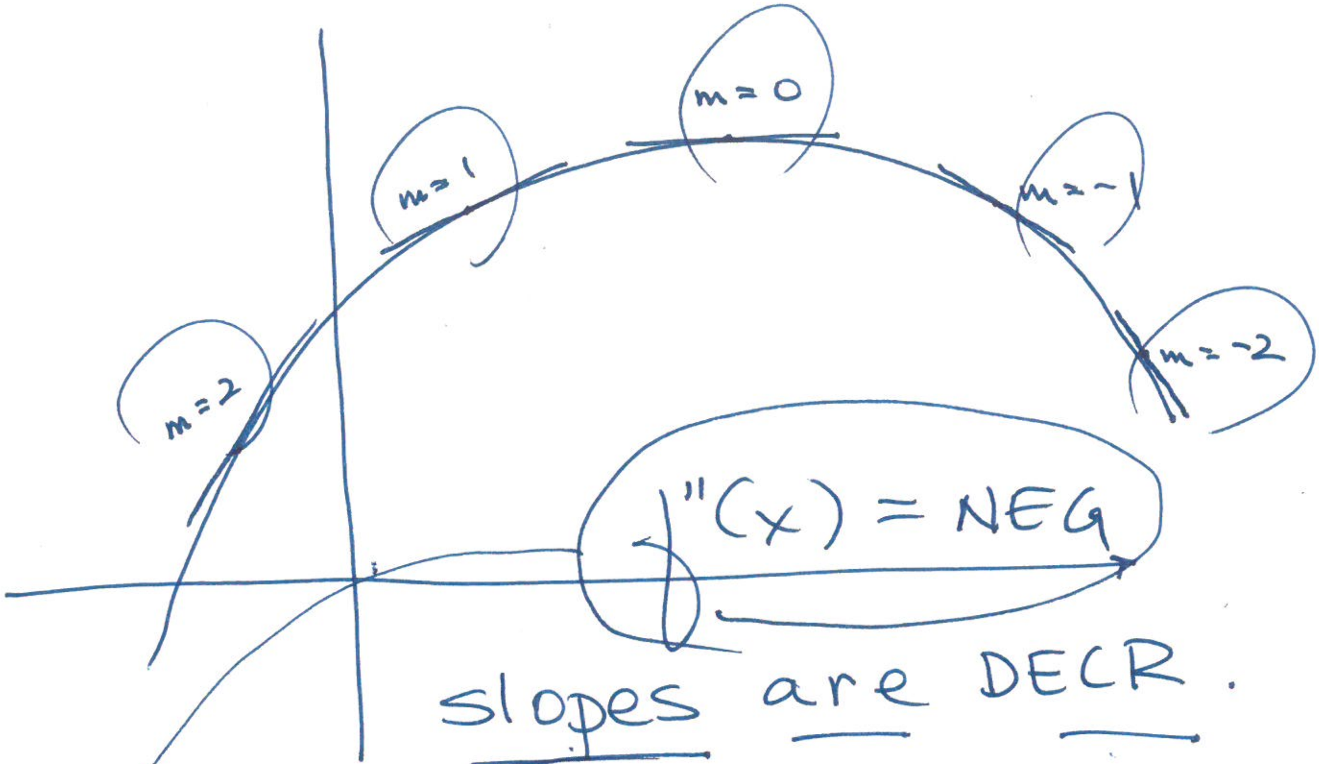
$m_{SEC} = m_{TAN}$

$$\boxed{\frac{f(b) - f(a)}{b - a} = f'(c)}$$

~~contin~~

increasing & decreasing





$f''(x) = \text{NEG}$

slopes are DECR.

$f'(x) = +$ ⟷ $f(x)$ INCR

$f''(x) = -$ ⟷ $f'(x)$ DECR

→ CONCAVE DOWN