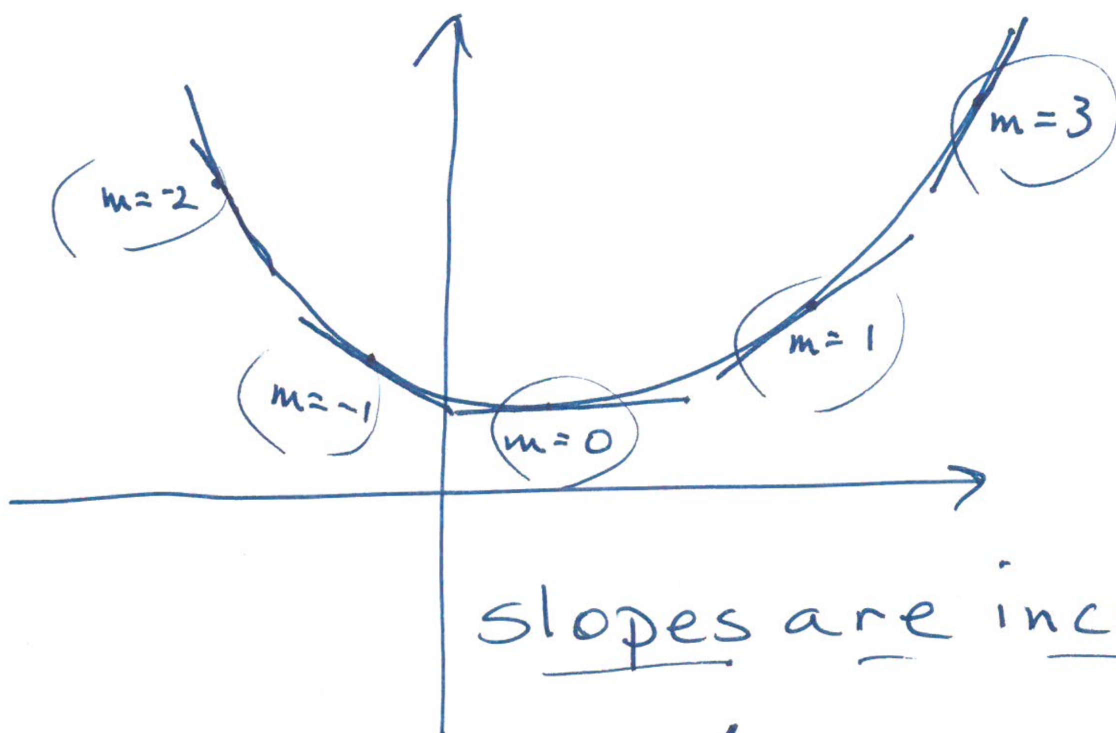
MA141-012

Wednesday, October 24

if $f''(x) =$ NEG, then $f'(x)$ is DECR.

(if $f''(x) =$ NEG, ..., then $f(x)$ is CONCAVE DOWN)



slopes are increasing

if $f''(x) =$ POS, then $f'(x)$ is INCREASING.

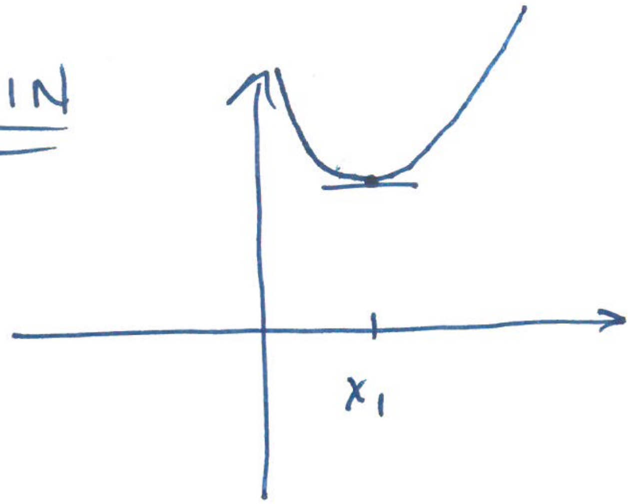
if $f''(x) =$ POS, ..., then $f(x)$ is CONCAVE UP.

(C.UP)

① $f'(x_1) = 0$

② $f''(x_1) = +$

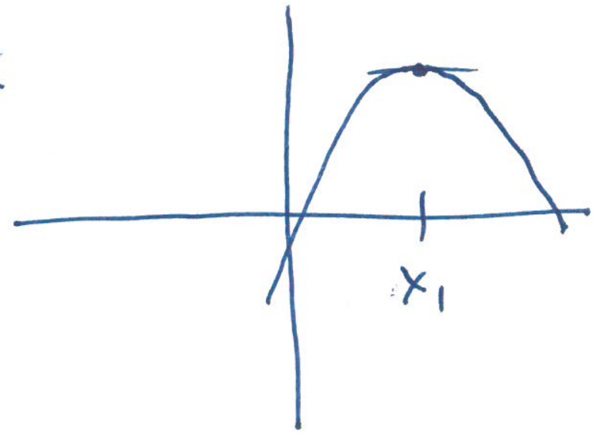
$\Big\}$ <u>MIN</u>



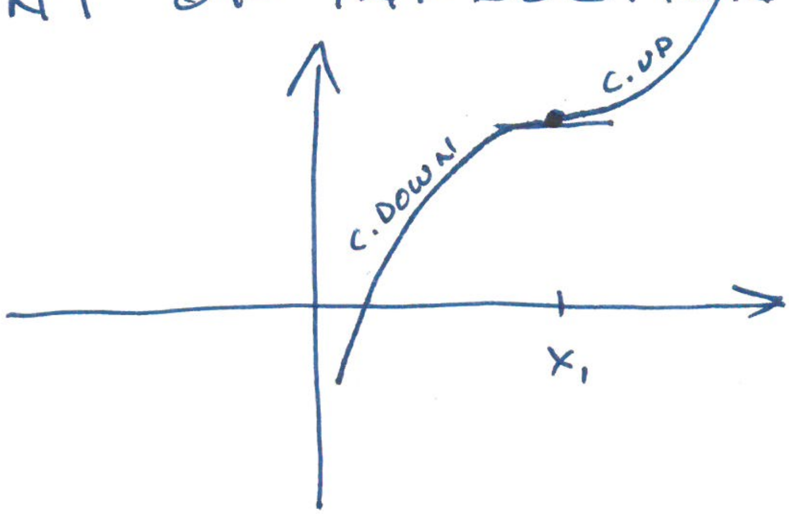
① $f'(x_1) = 0$

② $f''(x_1) = -$

$\Big\}$ <u>MAX</u>



if at a pt. on a graph the curve CHANGES concavity → then that point is a POINT OF INFLECTION.



Polynomial:

1. $f'(x)$ INFO
2. $f''(x)$ INFO

$$f(x) = \underline{\tfrac{1}{3}x^3} + \underline{x^2} - \underline{3x^1}$$

polynom: { exponents are NON-NEG integers }

$$f'(x) = x^2 + 2x - 3$$

1. $f'(x) = 0$ (HORIZ. TANGENT LINES)

$$x^2 + 2x - 3 = 0$$

$$\boxed{(x+3)(x-1)} = 0$$

$x = -3$ $\quad x = 1$

$(-3,\ 9)$ & $(1,\ -1\tfrac{2}{3})$

$f(-3)$ $\quad f(1)$

$$f(-3) = \tfrac{1}{3}(-3)^3 + (-3)^2 - 3(-3)$$

$$f(-3) = \cancel{-9} + \cancel{9} + 9$$

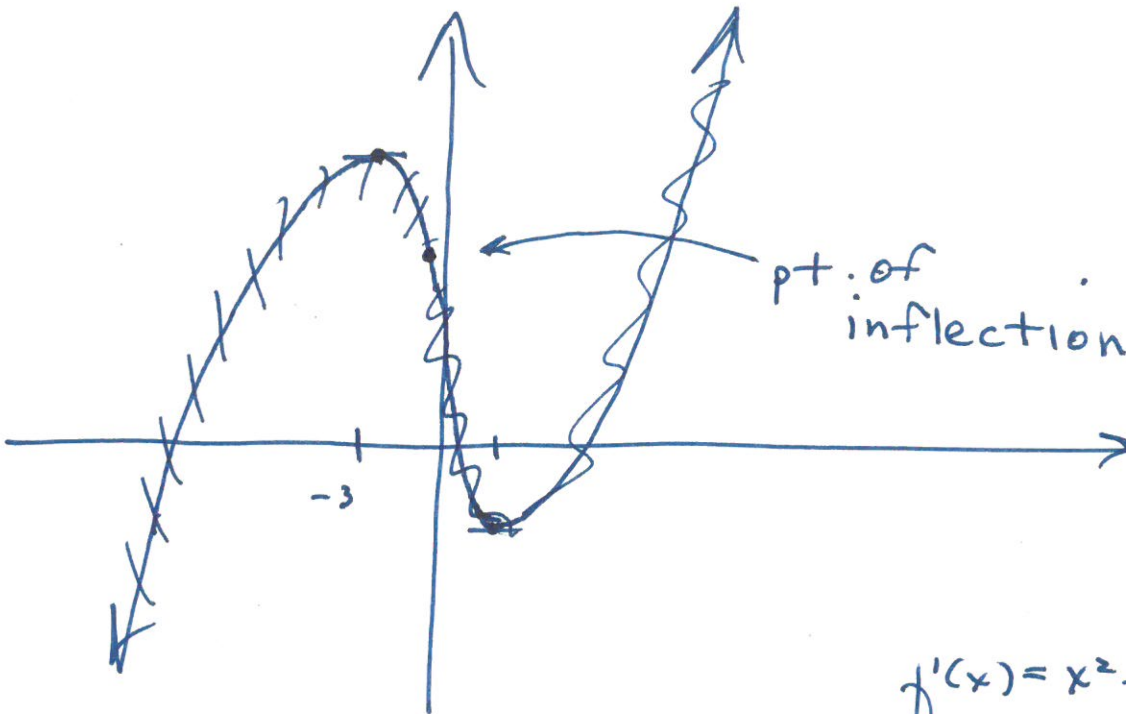
$$f(1) = \tfrac{1}{3}(1)^3 + (1)^2 - 3(1)$$

$$f(1) = \tfrac{1}{3} + 1 - 3 = -1\tfrac{2}{3}$$

$f'(x)$:

$f'(-4) = - \cdot - = +$ $\quad f'(0) = + \cdot - = -$ $\quad f'(2) = + \cdot + = +$

$f(x)$ INCR $\quad -3 \quad$ $f(x)$ DECR $\quad 1 \quad$ $f(x)$ INCR



$f'(x) = x^2 + 2x - 3$

$f''(x) = \boxed{2x+2}$

① $f''(x) = 0$

$2x + 2 = 0$

$2x = -2$

$x = -1$

② ~~$f''(x)$ undef~~

$(-1, f(-1)) = \underline{(-1, 3\tfrac{2}{3})}$

$f(-1) = \frac{1}{3}(-1)^3 + (-1)^2 - 3(-1)$

$f(-1) = -\frac{1}{3} + 1 + 3 = 3\tfrac{2}{3}$

$\underline{f''(x):}$

$f''(-2) = -$ | -1 | $f''(0) = +$

$f(x)$ conc. down | | $f(x)$ conc. up

(5)

INCREASING: $(-\infty, -3]$ and $[1, +\infty)$

DECREASING: $[-3, 1]$

CONCAVE UP: $(-1, +\infty)$

CONCAVE DOWN: $(-\infty, -1)$

$f(x) = 3(x-1)^{2/3} - x$ non-polynom.

$f'(x) = \cancel{3} \cdot \frac{2}{\cancel{3}}(x-1)^{-1/3}(1) - 1$

① $f'(x) = \dfrac{2}{\sqrt[3]{x-1}} - 1 = 0$

$\dfrac{2}{\sqrt[3]{x-1}} = \dfrac{1}{1}$

$2 = \sqrt[3]{x-1}$

$2^3 = \left(\sqrt[3]{x-1}\right)^3$

$8 = x - 1$

$9 = x$

$f(9) = ?$

$f(9) = 3(9-1)^{2/3} - 9$

$f(9) = 3$

$(9, 3)$ "FLAT"

② $f'(x) = \dfrac{2}{\sqrt[3]{x-1}} - 1$ undef

when $x = 1$

$(1, f(1))$

$f(1) = 3(1-1)^{2/3} - 1$

$f(1) = -1$

VERTICAL TANGENT LINE $(1, -1)$ "STEEP"

6

$$f'(x) = \frac{2}{\sqrt[3]{x-1}} - 1$$

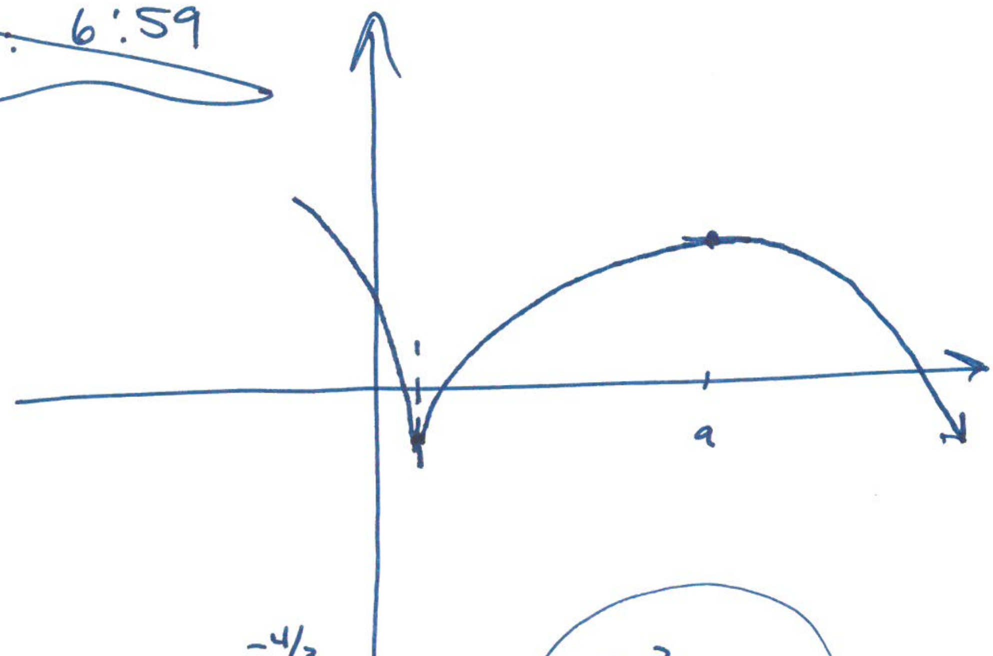
$$f'(x) = 2(x-1)^{-1/3} - 1$$

$f'(x)$:

$f'(0) = -$ | $f'(4) = +$ | $f'(10) = -$

$f(x)$ DECR. | 1 | $f(x)$ ~~DECR~~ INCR. | 9 | $f(x)$ DECR

~~resume: 6:59~~



$$f''(x) = -\frac{2}{3}(x-1)^{-4/3}(1) = \frac{-2}{3\left[\sqrt[3]{x-1}\right]^4}$$

① $f''(x) = 0$

$$\frac{-2}{3\left[\sqrt[3]{x-1}\right]^4} \neq 0$$

② $f''(x)$ undef.

$$\frac{-2}{3\left[\sqrt[3]{x-1}\right]^4} \text{ undef.}$$

at $x = 1$ $(1, -1)$

possible pt. of infl.

$f''(x)$:

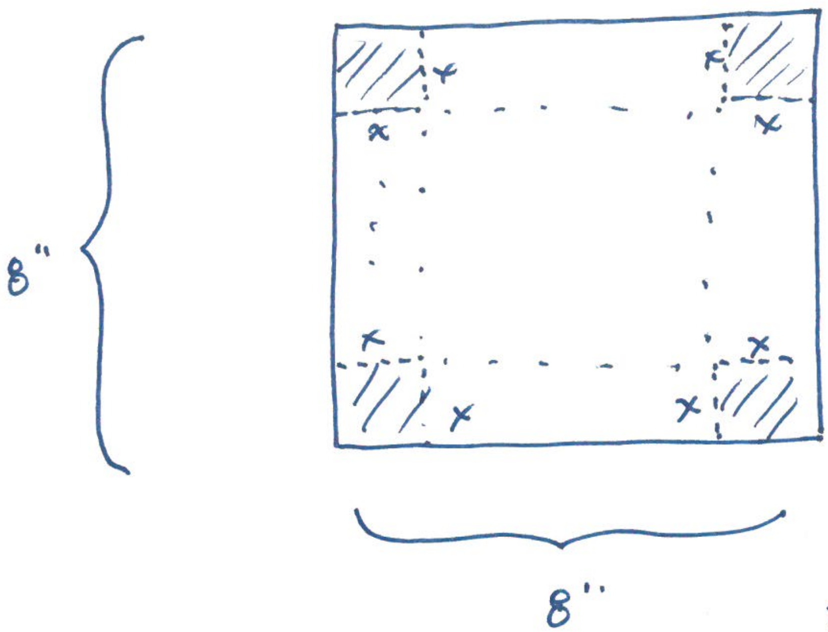
$f''(0) = -$ | 1 | $f''(2) = -$

$f(x)$ C. DOWN | $f(x)$ C. DOWN

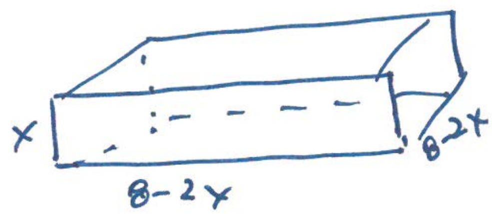
3.4:

MAX/MIN WORD PROBLEMS

(OPTIMIZATION PROB.)



form a box of max. VOL:



$0 < x < 4$

$$V(x) = x(8-2x)(8-2x)$$

$$V(x) = x(64-32x+4x^2)$$

$$V(x) = 64x - 32x^2 + 4x^3$$

$8-2(4/3)$

$8 - 8/3$

$\frac{24}{3} - \frac{8}{3} = \frac{16}{3}$

$$V'(x) = 64 - 64x + 12x^2 = 0$$

$$12x^2 - 64x + 64 = 0$$

$$4(3x^2 - 16x + 16) = 0$$

$$4(3x-4)(x-4) = 0$$

$$(3x-4) = 0 \qquad (x-4) = 0$$

$$3x = 4 \qquad \cancel{x = 4}$$

$$x = 4/3$$

$4/3$" by $4/3$"

VALIDATE:

max or min?

$$V''(x) = -64 + 24x$$

$$V''(4/3) = -64 + 24(4/3) = -$$

$\therefore$ C. DOWN $\therefore$ MAX

MAX VOL:

$$V = (4/3)(16/3)(16/3)$$

$$V = \frac{1024}{27}\ in^3$$

let x = number of times the price is decreased.

$$R(x) = \underbrace{(40 - x \cdot 1)}_{\text{ticket price}}\underbrace{(40{,}000 + x \cdot 2000)}_{\text{fans attending}}$$

$$R(x) = (40 - x)(40{,}000 + 2{,}000x)$$

$$R(x) = 1{,}600{,}000 + 80{,}000x - 40{,}000x - 2{,}000x^2$$

$$R(x) = 1{,}600{,}000 + 40{,}000x - 2{,}000x^2$$

$$R'(x) = 40{,}000 - 4{,}000x = 0$$

$$\cancel{R'(x)} = \frac{40{,}\cancel{000}}{4{,}\cancel{000}} = \frac{4\cancel{000}x}{4\cancel{000}} = 10$$

price: \$30.00

\# attending: 60,000

$(30.00)(60{,}000) = \$1{,}800{,}000.00$

$R''(x) = -4{,}000$

$\therefore$ c. down $\therefore$ max

MA 141-012 F18 Recitation 9, Oct 23

1. (a) Find the linear approximation, $L(x)$, of $f(x) = \ln(x^2)$ at $x = 1$. Then use $L(x)$ to approximate $\ln(1.44) = \ln(1.2^2)$.

   (b) Suppose that $L(x)$ is the linear approximation of $g(x)$ at some fixed point $x = c$ and $L(x) = g(x)$. What is the shape of the graph of $g(x)$?

2. Use Newton's Method (3 iterations with initial guess $x_0 = 3$) to approximate the intersection of the curves given by $y = \frac{1}{2}x^3$ and $y = 2x + 3$.

3. The function $f(x) = \frac{10x}{x^2+1}$ has as its domain $\mathbb{R}$ and $f(x)$ is continuous.

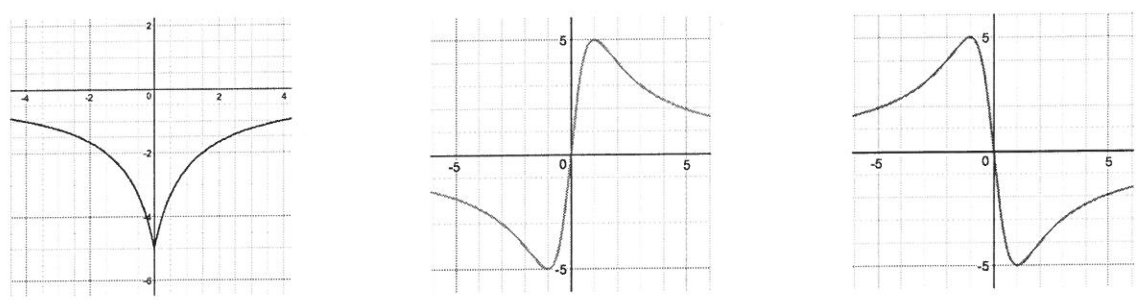
   (a) Where is $f(x)$ differentiable?

   (b) Determine the critical numbers then use the Increasing and Decreasing on an Open Interval Theorem to determine where f is increasing and where it is decreasing.

   (c) Does $f(x)$ have any local minimum or local maximum points?

   (d) Which of the following is the graph of $f(x)$?

(i) *(ii)* *(iii)*



4. The position of a particle moving along the x-axis is given by $s(t) = t^{2/3}(5 - t)$ where $t \in [0, 5]$ (in seconds). The function $s(t)$ is continuous on $\mathbb{R}$. Show that $s(t)$ is differentiable on the open interval $(0, 5)$.

   (a) By Rolle's Theorem what must be true about the movement of the particle on the interval $(0, 5)$?

   (b) Notice that $s(1) = 4$. By the Mean Value Theorem what must be true about its movement on the interval $(0, 1)$?

Theorem 2. Increasing and Decreasing on an Open Interval
Let the function f be differentiable on the open interval (a, b). Then

1. *If $f'(x) > 0$ on (a, b), then f is increasing on (a, b).*
2. *If $f'(x) < 0$ on (a, b), then f is decreasing on (a, b).*

Theorem 3. Rolle's Theorem
Let f be a function that is continuous on the closed, bounded interval $[a, b]$, differentiable on the open interval (a, b), and suppose that $f(a) = f(b)$. Then there is at least one point $c \in (a, b)$ where $f'(c) = 0$.

Theorem 4. The Mean Value Theorem
Let the function f be continuous on the closed and bounded interval $[a, b]$ and differentiable on the open interval (a, b). Then there exists at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$