MA141-012

Monday, October 29

let x = # of increases in rent

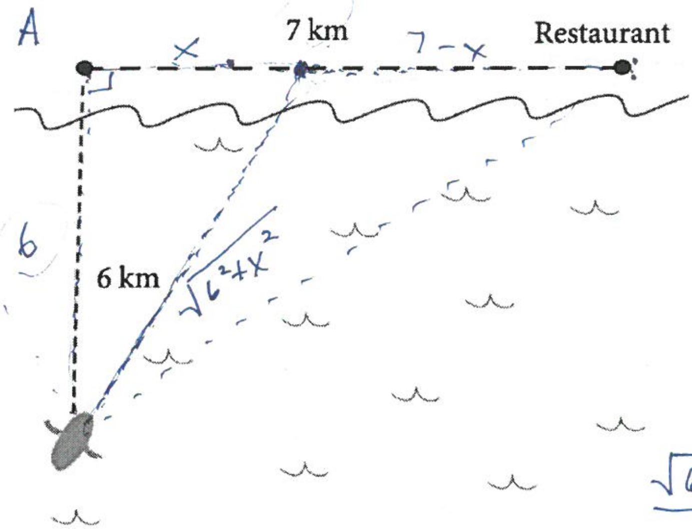
$$R(x) = \underbrace{(800^{00} + x \cdot 25)}_{\text{rent}} \underbrace{(120 - x \cdot 3)}_{\text{apts occupied}}$$

$R(x) =$

15. Traci is in the ocean in her kayak 6 km from the nearest point A on a straight beach (see Figure 80). She is really hungry and wants to get to the restaurant that is 7 km down the beach from point A as quickly as possible. If she can paddle at the rate of 4 km per hour and jog at the rate of 6 km per hour, where should she land in order to get from her current location to the restaurant in the least possible time?

$\frac{6}{4} + \frac{\cancel{8}}{6}$

$\frac{\sqrt{6^2+7^2}}{4}$



min TIME:

$D = r \cdot T$

$\frac{D}{r} = T$

$\frac{\sqrt{6^2+x^2}}{4} + \frac{7-x}{6} = T(x)$

$\frac{km}{\frac{km}{hr}}$

Figure 80

16. Norman windows have the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 18 feet, find the dimensions of the window which will allow the maximum amount of light to enter (maximum area).

17. Find two positive real numbers whose sum is 40 and whose product is a maximum.

18. A steel storage tank for propane gas has the shape of a right circular cylinder with a hemisphere at each end. If the capacity of the tank is 200 ft^3, what dimensions will require the least amount of steel?

19. A real estate company owns 120 apartments which are fully occupied when the rent is \$800 per month. Analytics indicate that for each \$25 increase in rent, 3 apartments will become unoccupied. What rent should be charged in order to obtain the largest gross income? What is the largest gross income?

20. A page of a book is to have an area of 90 in^2. Each page has a 1-inch margin at the top and the sides and a $\frac{1}{2}$-inch margin at the bottom. Find the dimensions of the page which has the largest printed area.

21. A rectangle is to be inscribed in a semicircle with radius 6 in. Find the dimensions of the rectangle with the largest area.

10. A farmer wants to enclose two rectangular areas along a river (see Figure 79), one for cattle and the other for alpacas. He has budgeted for 300 yd of fencing for the project. No fence is needed along the river. What is the largest total area that can be enclosed?

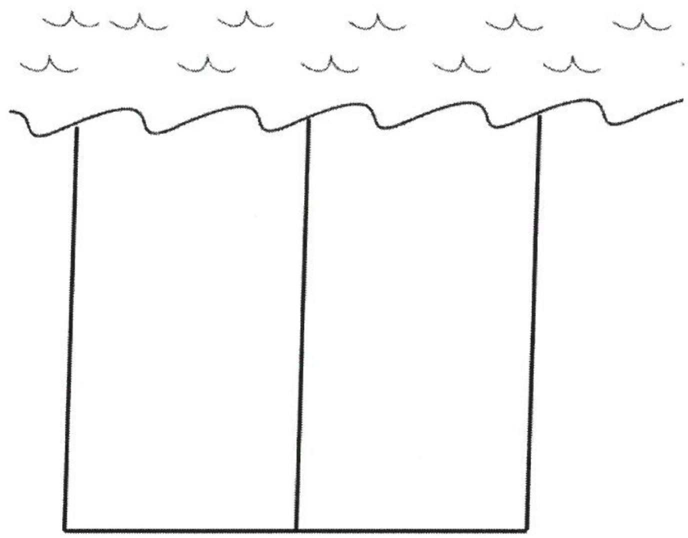


Figure 79

11. From square sheet of sheet metal that is 40 cm by 40 cm, square corners are cut out so that the sides can be folded up to make a box. What dimensions will result in a box of maximum volume? What is the maximum volume?

12. (Challenging) An 9 ft tall fence is located 6 ft from a building. What is the minimum length of a ladder that will reach from the ground over the fence and rest up against the building?

13. A piece of wire 12 ft long is cut into two pieces. One piece is used to form a square, the remaining piece is used to form a circle. Where should the wire be cut so that the combined area of the two figures is a maximum? Where should the wire be cut so that the combined area of the two figures is a minimum?

14. Find the dimensions of the right-circular cylinder of maximum volume that can be inscribed in a right-circular cone that has a radius of 5cm and a height of 12 cm.

$\pi\left(\frac{x}{2\pi}\right)^2 + \left(3-\frac{x}{4}\right)^2 = A(x)$

x $\quad$ $12-x$

radius

$A=\pi\left(\frac{x}{2\pi}\right)^2$

$C=x$

$C=\boxed{2\pi r = x}$

$3-\frac{x}{4}$

$3-\frac{x}{4}$

$P=12-x$

$\frac{1}{4}(12-x)$

$A=\left(3-\frac{x}{4}\right)^2$

Last update: May 9, 2017

endpoints: $x=0$ & $x=12$

~~0~~ circle $+3$ 3 $A_\square = 9$

$12 = 2\pi r$ $\quad r = \frac{12}{2\pi} = \frac{6}{\pi}$

$A_o = \pi\left(\frac{6}{\pi}\right)^2$

3.5 :

INDETERMINATE FORMS :

① $\frac{0}{0}$ or $\frac{\infty}{\infty}$ ("standard indet. forms")

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

L'HOPITAL'S RULE :

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 4} \frac{(x-4)}{x^2-16} = \lim_{x \to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} =$$

$$\lim_{x \to 4} \frac{1}{(x+4)} = \frac{1}{8}$$

$$\lim_{x \to 4} \frac{(x-4)}{x^2-16} = \frac{0}{0} \xrightarrow{\text{L'HOP}}$$

$$\lim_{x \to 4} \frac{1}{2x} = \frac{1}{8}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0} \frac{\overset{\to 0}{16x^2}}{\underset{\to 0}{5\cos x - 5}} \Rightarrow \text{L'HOP}$$

$$= \lim_{x\to 0} \frac{\overset{\to 0}{32x}}{\underset{\to 0}{-5\sin x}} \Rightarrow \text{L'HOP}$$

$$= \lim_{x\to 0} \frac{32}{-5\cos x} = \frac{32}{-5}$$

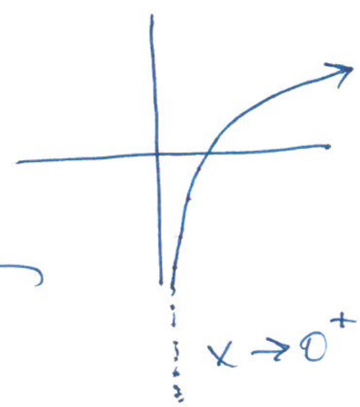
OTHER INDET. FORMS:

$0 \cdot \infty$ and $\infty - \infty$

$$\lim_{x\to 0} \underset{\downarrow 0}{x} \cdot \underset{\downarrow \infty}{\frac{1}{x}} = \lim_{x\to 0} 1 = 1$$

$$x^3 - x^2$$

$$x \to \infty$$



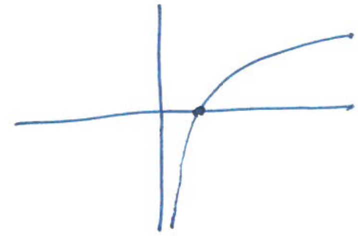
$$\lim_{x\to 0^+} (\overset{0}{x} \cdot \overset{-(\infty)}{\ln x})$$

from the right

* [convert to "standard" indet. form ($\frac{0}{0}$ or $\frac{\infty}{\infty}$)]

$$\lim_{x\to 0^+} \frac{\overset{-\infty}{\ln x}}{\underset{+\infty}{\frac{1}{x}}} \xrightarrow{\text{L'HOP}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$$

x^{-1}



$$\lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^2}{-1} = \lim_{x\to 0^+} (-1)(x) = 0$$

$\infty - \infty$:

$$\lim_{x\to 1^+} \underbrace{\left(\frac{x}{x-1}\right)}_{\infty} - \underbrace{\left(\frac{1}{\ln x}\right)}_{\infty}$$

[convert to "standard" indet. form : $\frac{0}{0}$ or $\frac{\infty}{\infty}$]

$$\lim_{x\to 1^+} \frac{x\cdot \ln x}{(x-1)\cdot \ln x} - \frac{1\,(x-1)}{(\ln x)(x-1)}$$

$$\lim_{x\to 1^+} \frac{\overset{1\cdot 0}{\boxed{x\cdot \ln x}} \overset{-1}{-\,x} \overset{+1}{+\,1}}{(x-1)\cdot \ln x} \quad \nearrow 0 \quad \searrow 0$$

$$\xrightarrow{\text{L'HOP}} \lim_{x\to 1^+} \frac{\overset{x\cdot\ln x\ -\ \ln x}{\cancel{x\cdot\frac{1}{x}} + (\ln x)(1) - \cancel{1}}}{\cancel{x\cdot\frac{1}{x}} + (\ln x)(1) - \cancel{\frac{1}{x}}}$$

$$\lim_{x\to 1^+} \frac{\ln x}{\ln x} \xrightarrow{\text{L'HOP}} \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{x\to 1} \frac{1}{\cancel{x}}\cdot\frac{\cancel{x}}{1}$$

$$\lim = 1$$

other indet forms:

1^{∞}; ∞^{0}; 0^{0}

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$\lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^{x}}$$

$$\lim_{x\to\infty} e^{\boxed{x\cdot\ln\left(1+\frac{1}{x}\right)}}$$

$$e^{\lim_{x\to\infty} x\cdot\ln\left(1+\frac{1}{x}\right)}$$

$$e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}}$$

L'HOP

$$e^{\lim_{x\to\infty}\frac{\left(\frac{1}{1+\frac{1}{x}}\right)\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}}}$$

$$e^{\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}}$$

$$e^{1}=e$$

$$\ln 2^{3}=3\cdot\ln 2$$

π^{a}

$$\lim_{x\to 0^+} x^{x} \quad \text{(rewrite)}$$

$$\boxed{e^{\ln u}=u}$$

$$\ln e^{u}=u$$

$$\lim_{x\to 0^+} e^{x\cdot\ln x}$$

$$\left[e^{\ln x^{x}}=x^{x}\right]$$

$$\lim_{x\to 0^+} e^{\boxed{x\cdot\ln x}} = \boxed{e}^{\lim_{x\to 0^+}[x\cdot\ln x]} \longrightarrow e^{0}=1$$

$$\Rightarrow \lim_{x\to 0^+} x\cdot\ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \quad x^{-1}$$

$$\overset{\text{L'HOP}}{\Longrightarrow} \lim_{x\to 0^+}\frac{\left(\frac{1}{x}\right)}{\left(\frac{-1}{x^2}\right)} = \lim_{x\to 0^+}\frac{1}{x}\cdot\frac{x^2}{-1}$$

$$=\lim_{x\to 0^+}(-1\cdot x)=0$$

$$\lim_{x \to \infty} \frac{e^{2x}}{x^2} \xrightarrow{\text{L'HOP}} \lim_{x \to \infty} \frac{2 \cdot e^{2x}}{2x} \xrightarrow{\text{L'HOP}}$$

$$\lim_{x \to \infty} \frac{4 \cdot e^{2x}}{2} = \infty \quad (\text{D.N.E.})$$