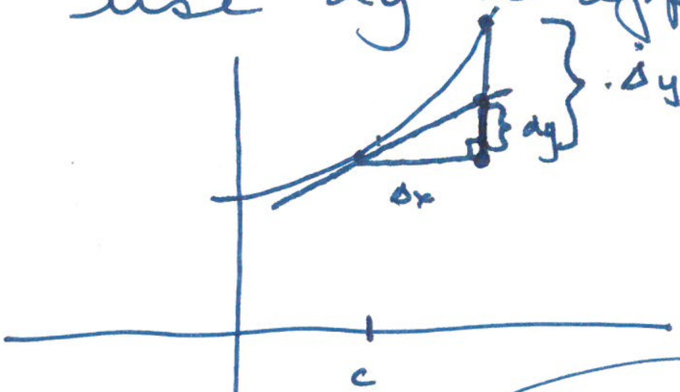
MA 141-012

Wednesday, October 31

1st hour: TEST #3

2nd hour: 3.6:

use dy to approx Δy



$$dy = f'(c) \cdot \Delta x$$

$f'(c)$ = slope of tangent line at $(c, f(c))$

$$f'(c) = \frac{dy}{\Delta x}$$

The term $f'(c)\Delta x$ in Equation (13) is called the **principal part** of Δy, since this is the part that is determined by the derivative $f'(c)$, and the term $\varepsilon\Delta x$ is called the **error part** of Δy. The two parts of Δy are shown in the following figure.

if Δx is "small" then $dy \approx \Delta y$

$\varepsilon\Delta x = \varepsilon(\Delta x)\cdot\Delta x$ smaller! smaller

$\varepsilon(\Delta x)$ = error term

$\varepsilon\cdot\Delta x$

$dy \approx \Delta y$

$\boxed{dy = f'(c)\cdot\Delta x}$

(exact) Δy = change in y on the curve

dy = change in y along the tangent to the curve.

Δx = change in x along the curve & along the tangent to the curve.

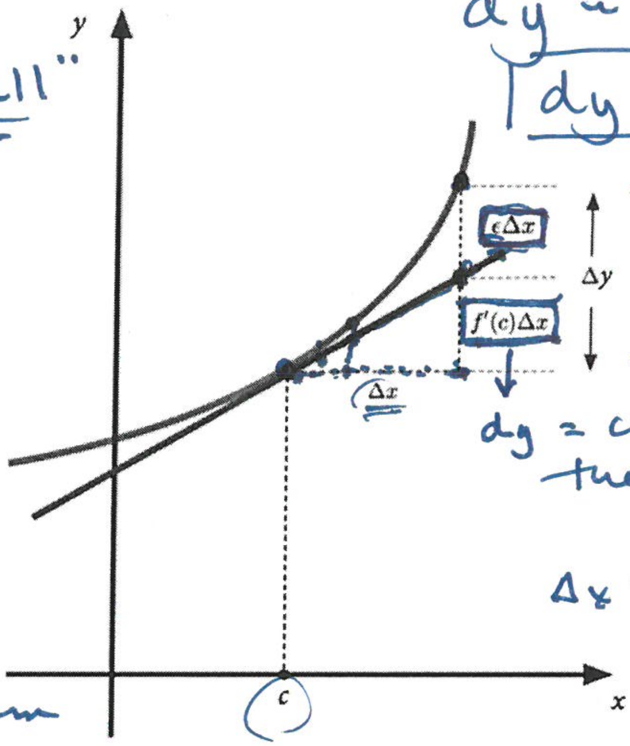


Figure 82: $\Delta y = f'(c)\Delta x + \varepsilon\Delta x$

REMARK: There are two notations involving ε and Δx which look very similar but have quite different meanings. This first comes from the definition of ε as a *function* of Δx. In this case we write $\varepsilon = \varepsilon(\Delta x)$. The second is the product of ε and Δx, $\varepsilon \cdot \Delta x = \varepsilon\Delta x$. This product is the *error* in using $dy = f'(c)\Delta x$ to approximate Δy.

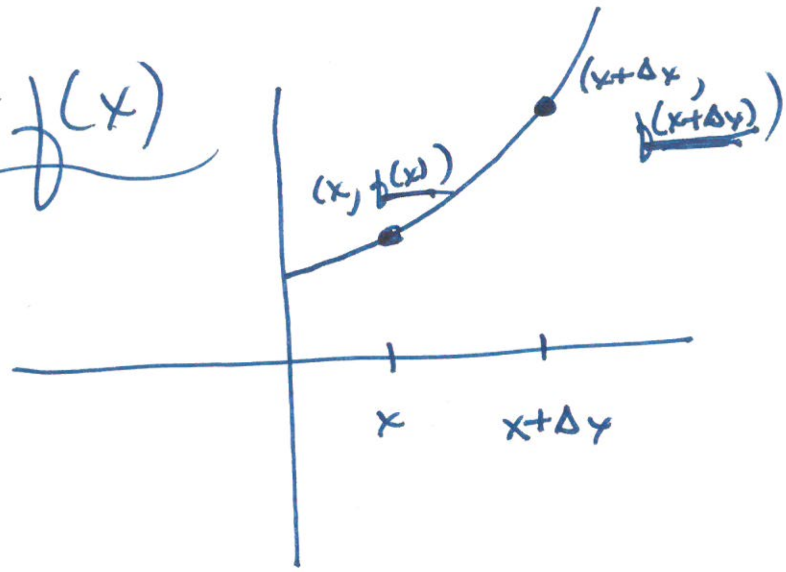
Let's look at two examples to see what ε looks like in specific cases.

Example 34. *Let $f(x) = x^2$ with $f'(x) = 2x$. Find the function ε that appears in the Formula (13) above.*

Last update: May 9, 2017

$$f(x) = x^3$$

$$\Delta y = f(x+\Delta x) - f(x)$$



$$\Delta y = (x+\Delta x)^3 - x^3$$

$$\Delta y = \cancel{x^3} + 3x^2 \cdot \Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \cancel{x^3}$$

$$\Delta y = \left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right]\Delta x$$

$$\Delta y = \left[f'(x) + 3x(\Delta x) + (\Delta x)^2\right]\Delta x$$

if $\Delta x \to 0$

$$\varepsilon(\Delta x) = 3x(\Delta x) + (\Delta x)^2$$

$$\Delta y \approx f'(x) \cdot \Delta x$$

$$\boxed{\Delta y \approx dy}$$

$$dy = f'(x) \cdot \Delta x$$

$$\Delta y \approx 3x^2 \Delta x$$

change in y along the tangent to the curve

exact ~~ ~~ change in y on the CURVE

Example 38. *A spherical water tank of radius 5 meters is to be painted. The thickness of the paint on the sphere will be 3 mm. Use differentials to approximate the amount of paint in m^3 that it will take to paint the water tank. How many gallons of paint are required?*

Solution: The volume of the spherical tank is

$$V(r) = \frac{4}{3}\pi r^3$$

$$dV = V'(r)\,dr = (4\pi r^2)\,dr = (4\pi r^2)\,\Delta r$$

Using $r = 5$ m and $\Delta r = 3$ mm $= 0.003$ m yields

$$dV = (4\pi \cdot 5^2)\,0.003 \approx 0.9425\,\mathrm{m}^3$$

There are 264 gallons in 1 cubic meter. Hence it will take approximately 249 gallons of paint to cover the water tank.

Example 39. *If* $f(x) = \tan x$, *compute its differential* df *and then use it to approximate the change* Δf *in the function when* x *changes from* $x = \frac{\pi}{4}$ *to* $x = \frac{\pi}{4} + 0.1$. *Find an approximation for the relative change in* $\tan x$ *as* x *changes from* $\frac{\pi}{4}$ *to* $\frac{\pi}{4} + 0.1$, *i.e. find* $\dfrac{df|_{x=\frac{\pi}{4},\,dx=0.1}}{f(\frac{\pi}{4})}$.

Solution:

$$\begin{aligned} df &= f'(x)dx \\ &= (\sec^2 x)dx \end{aligned}$$

To approximate the change in f we take $x = \frac{\pi}{4}$ and $\Delta x = dx = (\frac{\pi}{4} + 0.1) - \frac{\pi}{4} = 0.1$. Then

$$\Delta f \approx df|_{x=\frac{\pi}{4},\,dx=0.1} = \sec^2\left(\frac{\pi}{4}\right)(0.1) = (\sqrt{2})^2(0.1) = 0.2$$

Note that $\tan\dfrac{\pi}{4} = 1$. Thus the relative error in using $\tan\frac{\pi}{4}$ to approximate $\tan(\frac{\pi}{4} + 0.1)$ is

$$\frac{\Delta f}{f(\frac{\pi}{4})} \approx \frac{df}{f(\frac{\pi}{4})} = \frac{0.2}{1} = 0.2 = 20\%$$

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dy is differential in y

$\Delta y \approx dy = f'(x) \cdot \Delta x$

gallons of paint

$V = \frac{4}{3}\pi r^3$

$V' = \frac{4}{\cancel{3}}\pi \cancel{3} r^2$

$V' = 4\pi r^2$

$V(5.003) - V(5) = \Delta V$

$\boxed{\frac{4}{3}\pi(5.003)^3 - \frac{4}{3}\pi(5)^3} \approx .943$

is Δr "small"?
$\Delta r = .003$

$\Delta V \approx dV = V'(r) \cdot \Delta r$

$dV = (4\pi r^2) \cdot \Delta r$

$dV = [4\pi (5)^2] \cdot (.003)$

$dV = 100\pi (.003) \approx \cancel{249.442}$

$\boxed{.942 \text{ m}^3}$

264 gallons = 1 m^3

$(.942)(264) \approx 249$ gallons

ANTI DERIVATIVES:

$$dy = f'(x) \cdot \Delta x$$

$$dy = \frac{dy}{dx} dx$$

$$dy = f'(x) \cdot dx$$

$$\int dy = \int f'(x) \cdot dx$$

$$y = \int \underbrace{f'(x)\,dx}_{\text{integrand}}$$

$$y = f(x) + C$$

"family of functions"

indefinite integral

$f(x) = x^2 + 8$ $\qquad f'(x) = 2x$

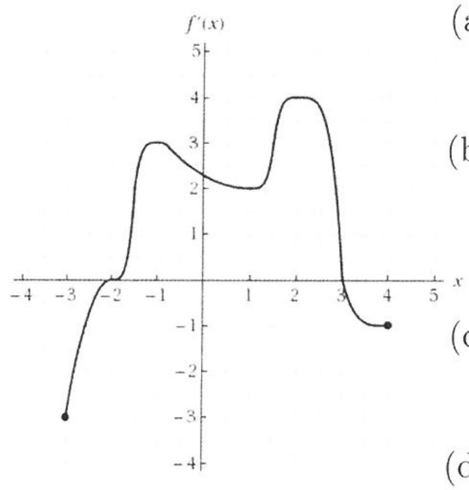
$$f(x) = \int f'(x) \cdot dx$$

$$= \int (2x) \cdot dx$$

$$f(x) = x^2 + C$$

MA 141-012 F18 Recitation 10, Oct 30

1. Suppose **the derivative** of some function $f(x)$ is graphed below. The domain of the function $f(x)$ is the interval $[-3, 4]$.



(a) Find the intervals on which $f(x)$ is increasing and the intervals on which it is decreasing.

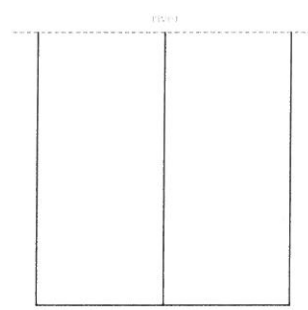
(b) Find the critical numbers of $f(x)$ and determine, for each, if it is the location of a local maximum or minimum (or neither).

(c) Find the intervals on which $f(x)$ is concave up and the intervals on which it is concave down.

(d) Find the points of inflection of $f(x)$.

2. Waste Management Company wants to construct a dumpster in the shape of a rectangular solid with no top whose length is twice its width. Its volume is fixed at 36 yd^3. Find the **dimensions** of the dumpster that will **minimize its surface area**.

3. A farmer wants to enclose two rectangular areas along a river, one for cattle and the other for alpacas. (Assume the rectangular areas have the same dimensions and share the fence in the middle.) He has budgeted for 180 yd of fencing for the project. No fence is needed along the river. What is the **largest total area** that can be enclosed?



4. Evaluate the following limits.

$$a) \quad \lim_{x\to\infty} \frac{\sin\frac{5}{x}}{\tan^{-1}\left(\frac{1}{x}\right)} \qquad\qquad b) \quad \lim_{x\to 0}(1+3x)^{\frac{1}{8x}}$$

**images taken directly from Webassign

Theorem 1. **Increasing and Decreasing on a Closed Interval**

Let the function f be continuous on the closed, bounded interval $[a, b]$ and differentiable on the open interval (a, b). Then

1. *If $f'(x) > 0$ on (a, b), then f is increasing on $[a, b]$.*
2. *If $f'(x) < 0$ on (a, b), then f is decreasing on $[a, b]$.*

Definition 8. **Concavity**

Let the function f be differentiable on the interval (a, b).

1. *If f' is increasing on (a, b), then the graph of f is* concave up *on (a, b).*
2. *If f' is decreasing on (a, b), then the graph of f is* concave down *on (a, b).*

Theorem 5. **Test for Concavity**

Let f be defined and twice differentiable on an open interval (a, b). Then:

1. *If $f''(x) > 0$ on (a, b), then the graph of f is concave up on (a, b).*
2. *If $f''(x) < 0$ on (a, b), then the graph of f is concave down on (a, b).*

Theorem 6. **Second Derivative Test for Local Extrema**

Let $c \in (a, b)$ be a critical number of the function f, and assume that f'' is continuous on the open interval (a, b).

If $f''(c) > 0$, then $f(c)$ is a local minimum of f
If $f''(c) < 0$, then $f(c)$ is a local maximum of f
If $f''(c) = 0$, then this test gives no information about $f(c)$