MA141-012

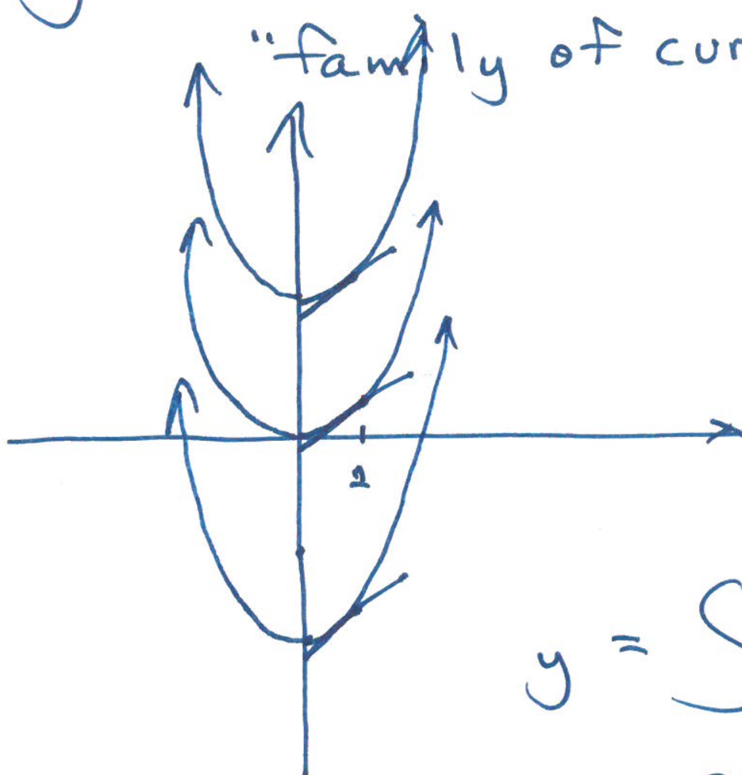
Monday, November 5

$$y = \int \underbrace{2x}_{\text{integrand}} \cdot dx$$

indef. integrals

$$y = x^2 + C$$

"family of curves"



$y = x^2+3$

$y = x^2$

$y = x^2-5$

$$y = \int \cos x \, dx$$

$$y = \sin x + C$$

$$y = \int 4 \, dx$$

$$y = 4x + C$$

$$y = \int 4 \, dt$$

$$y = 4t + C$$

$f'(x) = 3x+2$ $\quad$ $f(1) = 5$ $\quad$ $x=1$, $y=5$

find $f(x)$:

$$f(x) = \int (3x+2)\,dx$$

$$f(x) = \frac{3}{2}x^2 + \cancel{C_1} + 2x + \cancel{C_2} + C$$

$$f(x) = \frac{3}{2}x^2 + 2x + C$$

$$5 = \frac{3}{2}(1)^2 + 2(1) + C$$

$$5 = \frac{3}{2} + 2 + C$$

$$-3\tfrac{1}{2} \qquad -3\tfrac{1}{2}$$

$$1\tfrac{1}{2} = C$$

$$f(x) = \frac{3}{2}x^2 + 2x + 1\tfrac{1}{2}$$

ANTIDERIV RULES:

$y = 4x^2$

$y' = 4 \cdot 2 \cdot x^{2-1}$

power rule:

$$\int a \cdot x^n \cdot dx = \frac{a \cdot x^{n+1}}{n+1} + C$$

$$\int 3 \cdot x^4 \, dx$$

$$= 3 \cdot x^5 + C$$

check: $d(3x^5 + C) \stackrel{?}{=} 3x^4$

$3 \cdot 5 x^4$

#1 $$\boxed{\int a \cdot x^n \cdot dx = a \cdot \frac{x^{n+1}}{n+1} + C}$$ for $n \neq -1$

$$\int 3x^4 \cdot dx = 3 \cdot \frac{x^5}{5} + C$$

$$= \frac{3}{5} x^5 + C$$

$$d\left(\frac{3}{5} x^5 + C\right) = \frac{3}{\cancel{5}} \cdot \cancel{5} x^4$$

#2 $\int a \cdot x^{-1} \cdot dx = \int a \cdot \frac{1}{x} \cdot dx$

$(n=-1)$

$= a \cdot \ln|x| + C$

$\int a \cdot x^{n} \cdot dx = a \cdot \frac{x^{n+1}}{n+1} + C$

(for $n \neq -1$)

$\int \frac{2}{3} \cdot x^{-3} \cdot dx$

$= \frac{2}{3} \cdot \frac{x^{-2}}{-2} + C$

$= \frac{-1}{3} \cdot x^{-2} + C$

check: $d\left(-\frac{1}{3} \cdot x^{-2} + C\right)$

$= -\frac{1}{3} \cdot \left(-2 \cdot x^{-3}\right) = \frac{2}{3} x^{-3}$

$\int \frac{e}{\pi} \sqrt{x}\, dx = \frac{e}{\pi} \int x^{1/2} dx$

$= \frac{e}{\pi} \cdot \frac{x^{3/2}}{3/2} + C$

$= \frac{e}{\pi} \cdot \frac{2}{3} \cdot x^{3/2} + C$

#3 $\int a \cdot e^{x} \cdot dx$

$$= a \cdot e^{x} + C$$

$$\int a \cdot e^{bx} \cdot dx$$

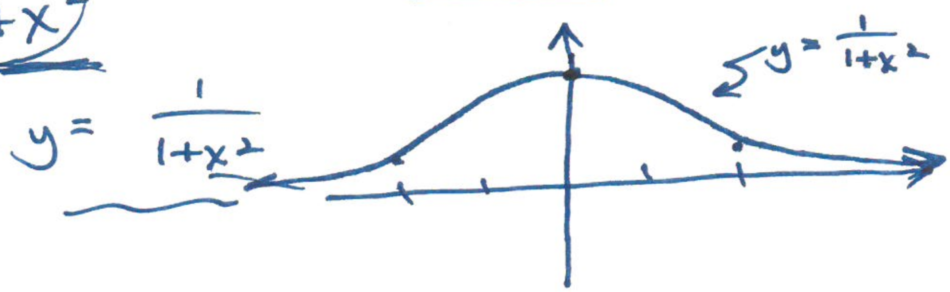
$$= a \cdot e^{bx} \cdot \frac{1}{b} + C$$

ex: $\int 17 \cdot e^{8x} \cdot dx$

$$= 17 \cdot \frac{1}{8} e^{8x} + C$$

check: $d\left(\frac{17}{8} \cdot e^{8x} + C \right)$

$$\int \frac{1}{1+x^2} \cdot dx = \tan^{-1} x + C$$



$f(x) = f(-x)$

SYM TO Y-AXIS

$$\int (3x + 2)\,dx$$

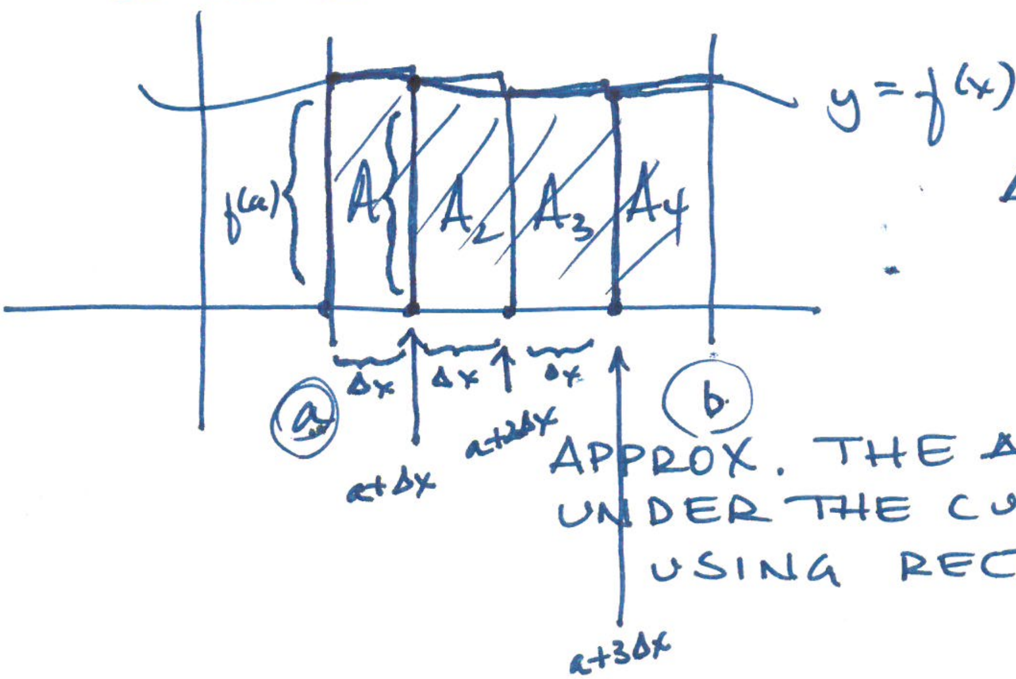
$$= 3 \cdot \frac{x^2}{2} + 2x + C$$

$$= \frac{3}{2} \cdot x^2 + 2x + C$$

resume: 7:01

CHAPTER 4:

4.1: AREAS (under a curve)



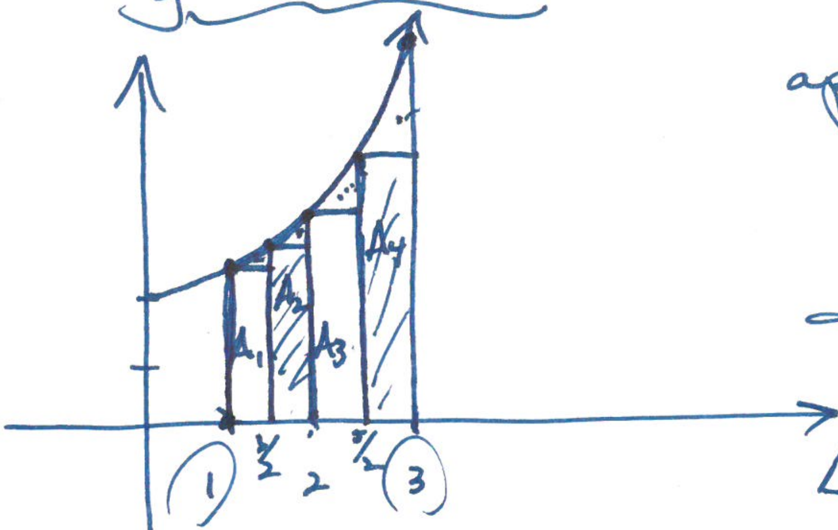
entire sub-interval

$$\Delta x = \frac{b-a}{n}$$

OF RECTANGLES

APPROX. THE AREA UNDER THE CURVE .... USING RECTANGLES

$y = x^2 + 2$



LEFT SUM

approx the area under $y = x^2 + 2$ from $x = 1$ to $x = 3$

$\Delta x = \underbrace{\text{WIDTH}}$

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4}$$

$$\Delta x = \frac{2}{4} = \frac{1}{2}$$

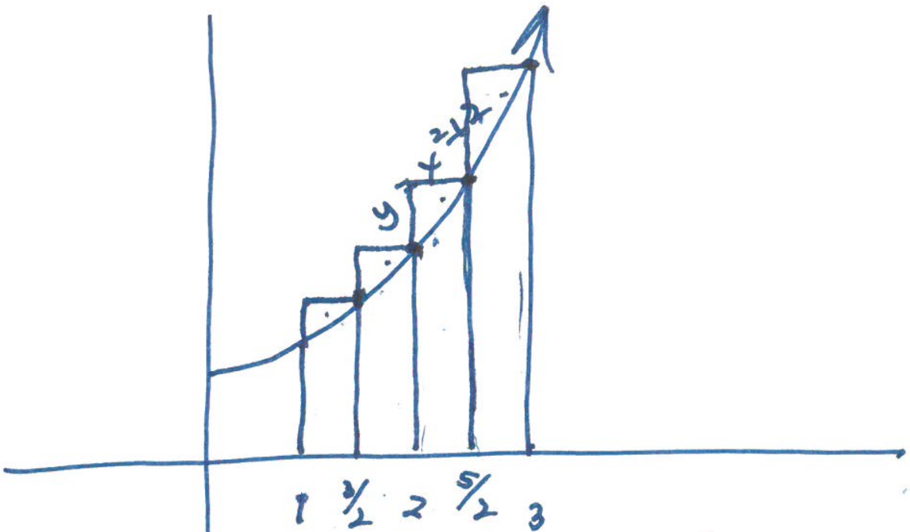
$$\left.\begin{aligned}
A_1 &\approx f(1) \cdot \Delta x = (3)\left(\tfrac{1}{2}\right) = \tfrac{3}{2} \\
A_2 &\approx f\left(\tfrac{3}{2}\right) \cdot \Delta x = \left(\tfrac{17}{4}\right)\left(\tfrac{1}{2}\right) = \tfrac{17}{8} \\
A_3 &\approx f(2) \cdot \Delta x = (6)\left(\tfrac{1}{2}\right) = 3 \\
A_4 &\approx f\left(\tfrac{5}{2}\right) \cdot \Delta x = \left(\tfrac{33}{4}\right)\left(\tfrac{1}{2}\right) = \tfrac{33}{8}
\end{aligned}\right\}$$

$$A \approx A_1 + A_2 + A_3 + A_4$$

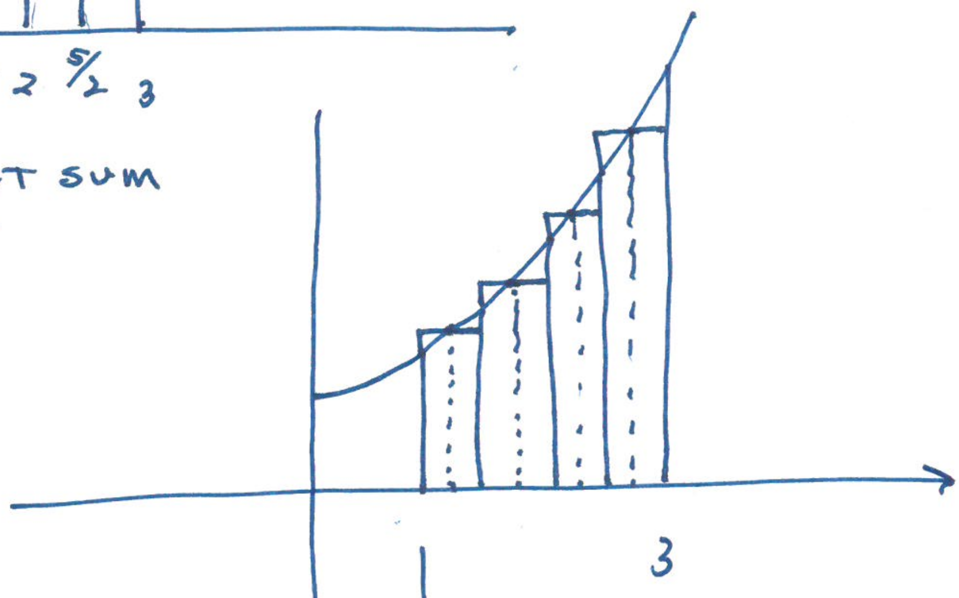
$$A \approx \frac{3}{2} + \frac{17}{8} + 3 + \frac{33}{8}$$

$$A \approx \frac{12}{8} + \frac{17}{8} + \frac{24}{8} + \frac{33}{8} = \frac{86}{8}$$

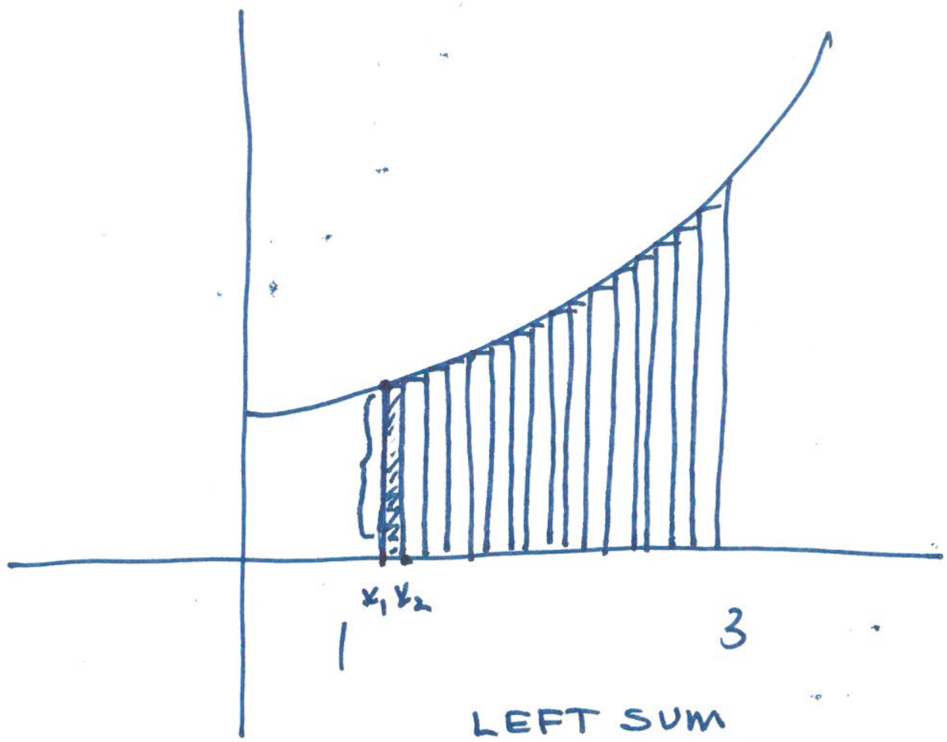
more rectangles $\longrightarrow$ higher accuracy



RIGHT SUM



MID-SUM



LEFT SUM
16 RECTANGLES

$$\text{rect } \underline{A_1} = f(\underline{x_1}) \cdot \Delta x$$
$$\text{rect } \underline{A_2} = f(\underline{x_2}) \cdot \Delta x$$
$$\vdots$$
$$\text{rect } A_{15} = f(x_{15}) \cdot \Delta x$$
$$\text{rect } A_{\underline{16}} = f(x_{16}) \cdot \Delta x$$

$$\sum_{i=1}^{16} f(\underline{x_i}) \cdot \Delta x$$

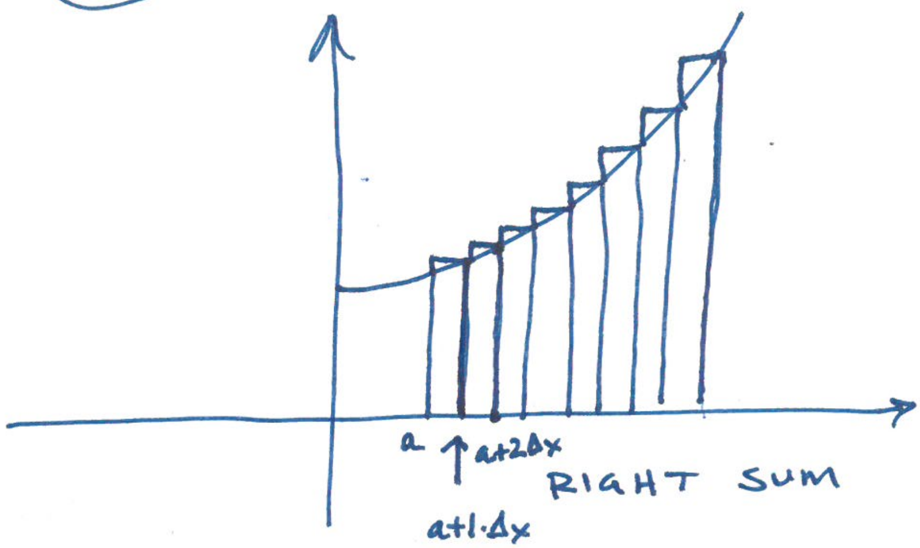
summation

$$= f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x + \cdots + f(\underline{x_{15}})\, \Delta x + f(\underline{x_{16}}) \cdot \Delta x$$

$$= f(a) \cdot \Delta x + f(a + \Delta x) \cdot \Delta x + f(a + 2\Delta x) \cdot \Delta x$$
$$+ f(a + 3\Delta x) \cdot \Delta x + \cdots + f(a + 14\Delta x) \cdot \Delta x$$
$$+ f(a + 15 \cdot \Delta x) \cdot \Delta x$$

$$\sum_{i=1}^{16} f(x_i) \cdot \Delta x \approx A$$

$x_i = a +$



$$\text{rect } A_1 = f(a + \underline{1 \cdot \Delta x}) \cdot \Delta x$$

$$\text{rect } A_2 = f(a + \underline{2 \Delta x}) \cdot \Delta x$$

$\vdots$

$$\text{rect } A_{15} = f(a + 15 \cdot \Delta x) \cdot \Delta x$$

$$\text{rect } A_{16} = f(a + \underline{16} \cdot \underline{\Delta x}) \cdot \Delta x$$

$$A \approx \sum_{i=1}^{16} \underline{f(x_i)} \cdot \underline{\Delta x}$$

$$x_i = a + \underline{i} \cdot \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$x_i = a + i\Delta x$

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x}$$

$\Delta x = \frac{b-a}{n}$

find the exact area under $f(x) = x^2+2$ from $x=1$ to $x=3$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + i\cdot\frac{2}{n}\right)\cdot\left[\frac{2}{n}\right]$$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{2i}{n}\right)^2 + 2\right]\cdot\left[\frac{2}{n}\right]$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{4i}{n} + \frac{4i^2}{n^2} + 2\right)\left(\frac{2}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)$$

$$A = \lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{6}{n} + \sum_{i=1}^{n} \frac{8i}{n^2} + \sum_{i=1}^{n} \frac{8i^2}{n^3}\right]$$

$$A = \lim_{n\to\infty} \left[\frac{6}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$\boxed{\sum_{i=1}^{n} 1 = n} \qquad \boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$