ma141 – 012

Wednesday, November 7

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{5} 1 = 1 + 1 + 1 + 1 + 1 = 5$$

$$\sum_{i=1}^{11} 1 = 11$$

$$\sum_{i=1}^{6} i = 1 + 2 + 3 + 4 + 5 + \underline{6} = \frac{6(6+1)}{2} = 21$$

$$\sum_{i=1}^{10} i = 1+2+3+ \cdots + 7+8+9+10 = \frac{10(10+1)}{2} = 55$$

$$\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = \frac{4(4+1)(2\cdot 4+1)}{6} = 30$$

$$1 + 4 + 9 + 16$$

$$= \frac{4(5)(9)}{6}$$

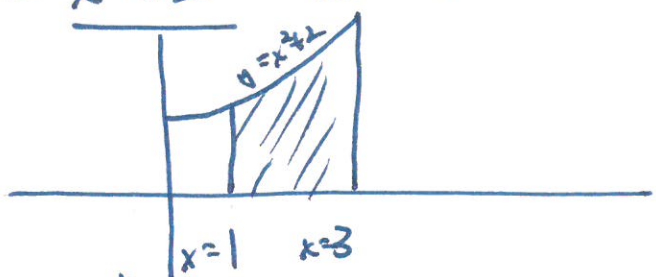
$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + \underline{n}^2 = \frac{n(n+1)(2n+1)}{6}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x$$ (2)

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$f(x) = x^2 + 2 \quad a = 1 \quad b = 3$



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{2i}{n}\right)\cdot\left[\frac{2}{n}\right]$$

$$\lim_{x\to\infty} \frac{3x^2 + x}{1x^2}$$

⋮

$$A = \lim_{n\to\infty}\left[\frac{6}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$A = \lim_{n\to\infty}\left[\frac{6}{\cancel{n}}\cdot(\cancel{n}) + \frac{8}{n^2}\cdot\left[\frac{n(n+1)}{2}\right] + \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]\right]$$

$$A = \lim_{n\to\infty}\left[6 + \frac{8}{2}\left[\frac{n^2+n}{n^2}\right] + \frac{8}{6}\cdot\left[\frac{2n^3+\cdots}{n^3}\right]\right]$$

$$A = \cancel{\lim_{n\to\infty}}\left[6 + 4(1) + \frac{4}{3}(2)\right]$$

$$A = 10 + \frac{8}{3} = 12\frac{2}{3}$$

APPROX: $10\frac{3}{4}$
(4 rectangles)

$\int_0 \quad f(x) = 2 - 3(x)^3$

$x = 0 \qquad x = 4$

$a = 0 \quad b = 4$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{4-0}{n} = \frac{4}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + i\left(\frac{4}{n}\right)\right)\cdot\left(\frac{4}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\left(\frac{4i}{n}\right)\right)\cdot\left(\frac{4}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[2 - 3\left(\frac{4i}{n}\right)^3\right]\cdot\left[\frac{4}{n}\right]$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(2 - \frac{192 i^3}{n^3}\right)\left(\frac{4}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8}{n} - \frac{768 i^3}{n^4}\right)$$

$$A = \lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{8}{n} - \sum_{i=1}^{n} \frac{768 i^3}{n^4}\right]$$

$$A = \lim_{n\to\infty} \left(\frac{8}{n}\sum_{i=1}^{n} 1 - \frac{768}{n^4}\sum_{i=1}^{n} i^3\right)$$

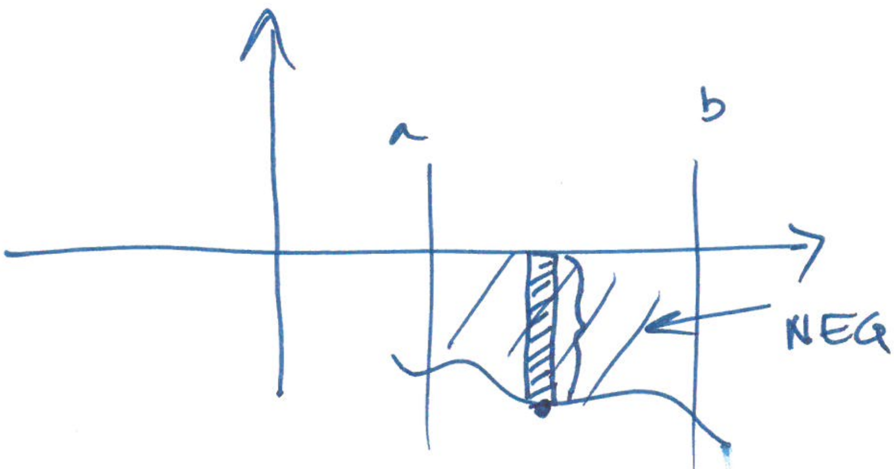
$$A = \lim_{n\to\infty} \left(\frac{8}{\cancel{n}}\cdot\cancel{n} - \frac{768}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2\right)$$

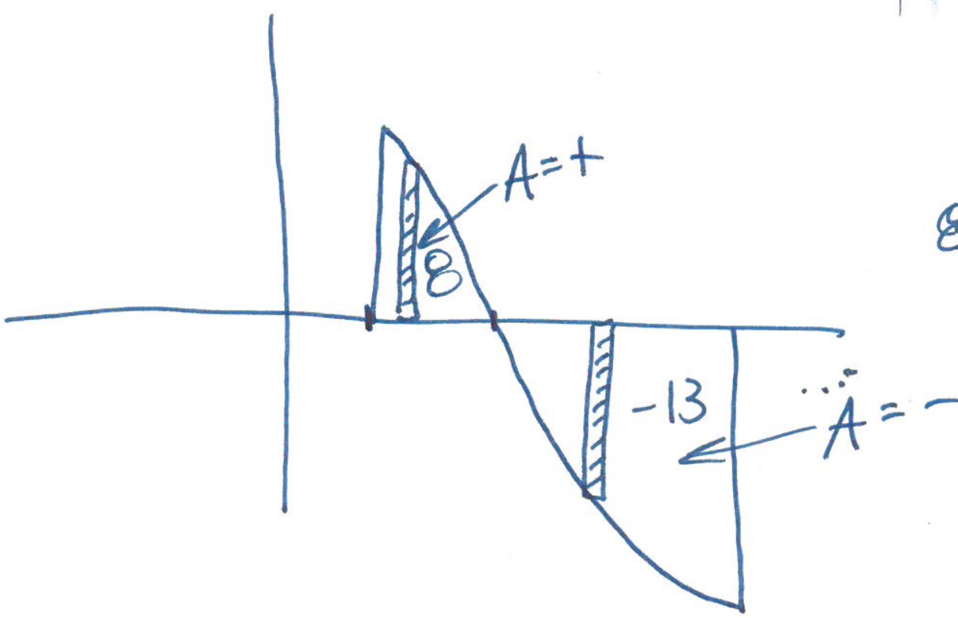
$$A = \lim_{n\to\infty} \left(8 - \frac{768}{2^2}\left(\frac{n^4 + \cdots\cdots}{n^4}\right)\right)$$

$$A = 8 - \frac{768}{4}(1) = 8 - \quad = -184$$

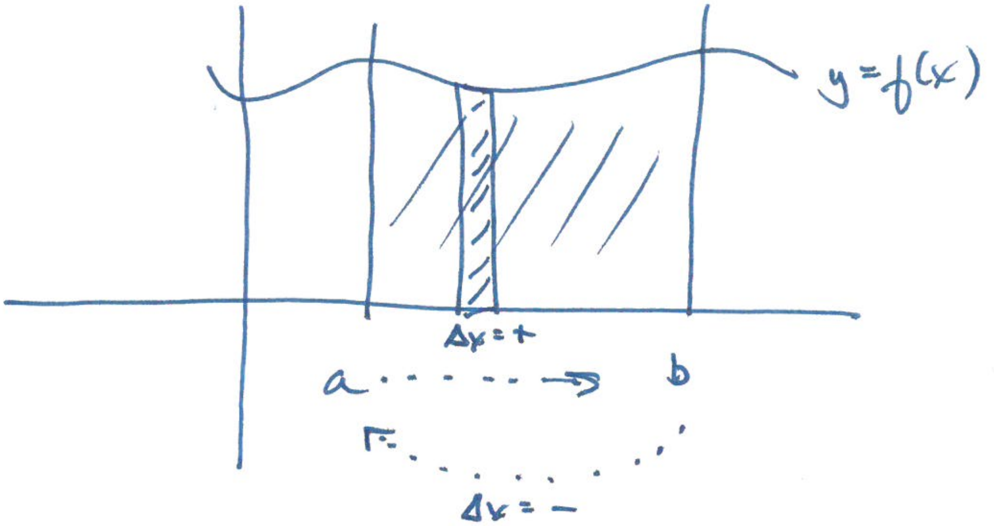
$$f(x) = 2 - 3x^3$$

$x = 0$ to $x = 4$





$8 + (-13) = -5$

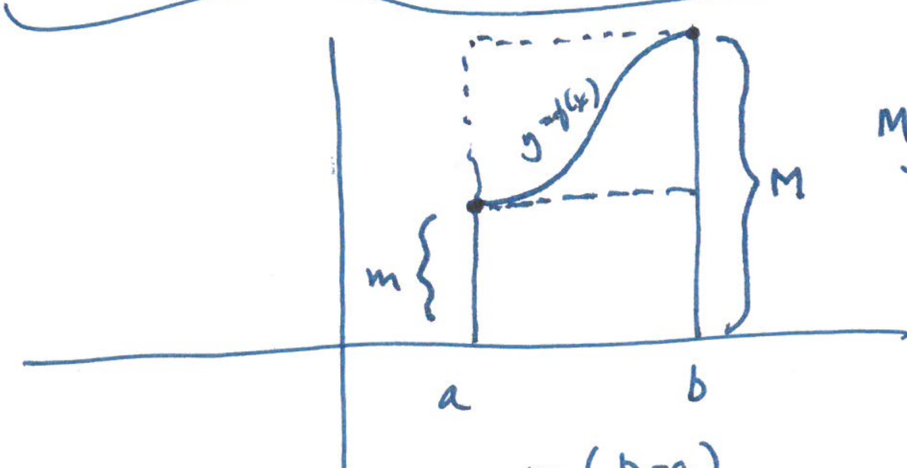


$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \overbrace{f(a+i\cdot\Delta x)}^{HT} \cdot \overbrace{\Delta x}^{WDTH} = \int_a^b f(x)\cdot dx$$

DEFINITE INTEGRAL

$$\boxed{A = \int_a^b f(x)\,dx} = F(x)\Big]_a^b = \boxed{F(b) - F(a)}$$

(where $F'(x) = f(x)$)



$M(b-a)$

$m(b-a)$

~~5~~ F.T.O.C.

{ fundamental theorem of calculus

$$\underline{A} = \underline{\int_a^b f(x)\,dx} = F(x)\Big]_a^b = \underline{F(b) - F(a)}$$

(where $F'(x) = f(x)$)

$\int a \cdot x^n dx = \frac{a \cdot x^{n+1}}{n+1}$

$f(x) = x^2 + 2$ from $x = 1$ to $x = 3$

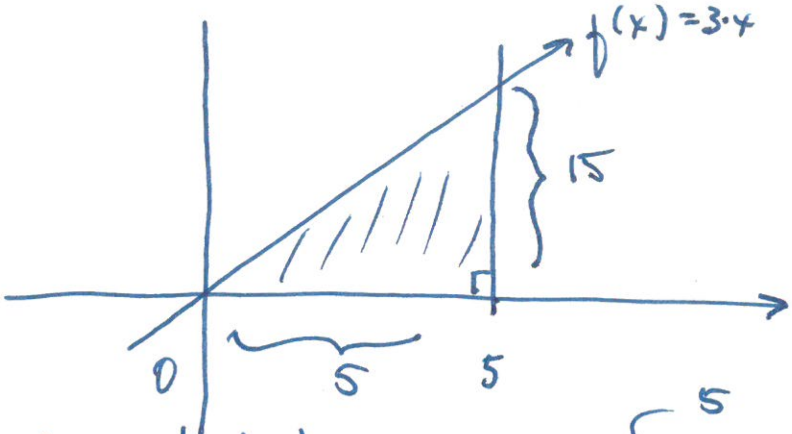
$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \underline{f(x_i)} \cdot \underline{\Delta x} = \underline{12 \tfrac{2}{3}} \checkmark$$

$$A = \int_1^3 (\underline{x^2 + 2})\,\underline{dx} = \frac{x^3}{3} + 2x \cancel{+c}\Big]_1^3$$

$$= \left(\frac{3^3}{3} + 2(3) \cancel{+c}\right) - \left(\frac{1^3}{3} + 2(1) \cancel{+c}\right)$$

$$= (9 + 6) - \left(\frac{1}{3} + 2\right)$$

$$= 15 - 2\tfrac{1}{3} = \underline{12 \tfrac{2}{3}}$$



$f(x) = 3x$

from $x = 0$ to $x = 5$

$A_\Delta = \frac{1}{2}\cdot b \cdot h$

$A = \frac{1}{2}(5)\cdot(15)$

$A = \frac{75}{2}$

$A = \int_0^5 (f(x))\cdot dx$

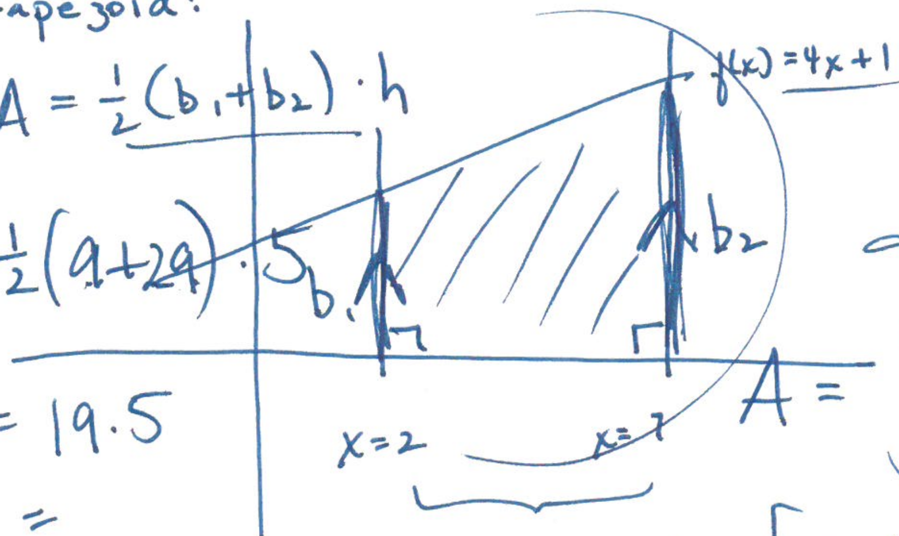
$A = \int_0^5 3(x)\cdot dx$

$A = 3\cdot\frac{x^2}{2}\Big]_0^5 = \frac{3}{2}(5)^2 - \cancel{\frac{3}{2}(0)^2}$

$A = \frac{75}{2}$

trapezoid:

$A = \frac{1}{2}(b_1 + b_2)\cdot h$



$A = \frac{1}{2}(9+29)\cdot 5$

$A = 19.5$

$=$

$f(x) = 4x+1$

from $x=2$ to $x=7$

$A = \int_2^7 (4x+1)\,dx$

$A = \left[4\cdot\frac{x^2}{2} + x\right]_2^7$

$A = \left[2x^2 + x\right]_2^7$

$\neq 95$

$A = (2(7)^2+7) - (2(2)^2+2) = (98+7) - (10)$

$$A = \int_a^b f(x)\,dx = \underline{\qquad\qquad}$$

indef. integ: $\int f(x)\,dx = \underbrace{F(x)+C}$

def. integ: $\int_a^b f(x)\,dx = \underset{(\underline{\text{area}})}{\underline{\quad \# \quad}}$.

MA 141-012 F18 Recitation 10, Nov 6 (Election Day!)

1. Let $f(x) = x^3 - 5x^2 - 6$. Determine the function $\epsilon(\Delta x)$ that vanishes as $\Delta x \to 0$. Compute the differential df of f then use it to approximate Δf, the change in f, when x changes from $x = 2$ to $x = 2.1$ Use this value of Δf to approximate $f(2.1)$. (see figure a)

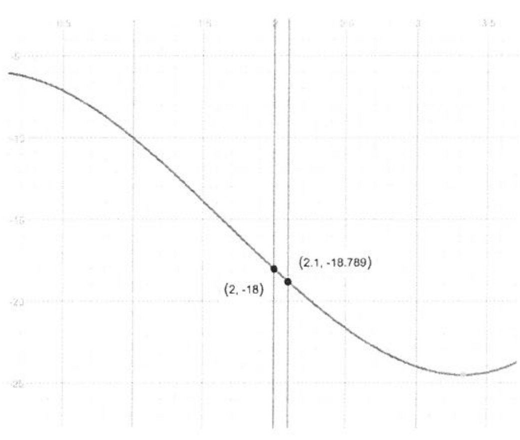
2. The end of a house has the shape of a square surmounted by an equilateral triangle. Suppose the length of the base is measured to be 39 feet, with a maximum error in measurement of 1 inch. Calculate the area of the end. Then use differentials to estimate the maximum error in the calculation of the area. (Hint: If the length of the base is x then the area is $A(x) = (1 + \frac{\sqrt{3}}{4})x^2$.)

3. Recall that the indefinite integral $\int f(x)dx$ is the family of antiderivatives $F(x) + C$ such that $\frac{d}{dx}F(x) + C = f(x)$. Evaluate $\int \left(\frac{10}{1+x^2} + x^{\frac{5}{2}}\right) dx$.

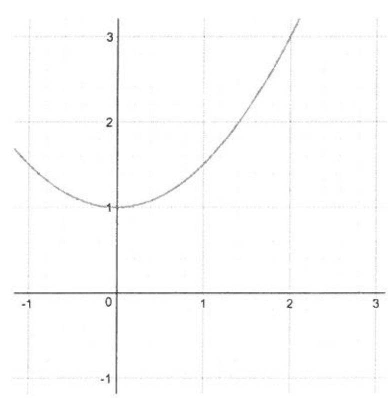
4. Find the antiderivative of $f(x) = 4e^{12x}$ that passes through the point $(0, \frac{16}{3})$.

5. Since $\frac{d}{dx}(e^{4x}\cos(x)) =$ __________________,
the indefinite integral $\int 8e^{4x}\cos(x) - 2e^{4x}\sin(x)dx =$ __________________.

6. Use Riemann Sums to approximate the area of the region under the graph of $f(x) = \frac{1}{2}x^2 + 1$ (shown below), above the x- axis, and between the lines $x = 0$ and $x = 2$. Use the left endpoints and $n = 5$ rectangles of equal width. Is this approximation for the area less than or greater than the true area? (see figure b)



a)



b)