Monday, November 12

- Fundamental Theorem of Calculus (4.3)
- Integration using SUBSTITUTION (4.4)

Part I: F.T.O.C.

$$G(x) = \int_a^x f(t)\,dt$$ ✓

$G(x)$ is an antideriv. of f on $[a,b]$

find $G'(x)$: $G'(x) = f(x)$

ex: $$G(x) = \int_2^x (t^2 + t + 3)\,dt$$

find $G'(x)$:

$$G(x) = \frac{t^3}{3} + \frac{t^2}{2} + 3t \Big]_2^x$$

$$G(x) = \left(\frac{1 \cdot x^3}{3} + \frac{1 x^2}{2} + 3x\right) - \left(\frac{2^3}{3} + \frac{2^2}{2} + 3 \cdot 2\right)$$

$$G'(x) = x^2 + x + 3$$

$$G(x) = \int_1^x \frac{1}{t^3+1}\,dt$$

find $G'(x)$:

$$G'(x) = \frac{1}{x^3+1}$$

$$G(x) = \int_x^1 \frac{1}{t^3+1}\,dt$$

find $G'(x)$:

$$G'(x) = \frac{-1}{x^3+1}$$

$$G(x) = \int_2^{x^4} 5\ln t \cdot dt \qquad u = x^4$$

find $G'(x)$:

$$G'(x) = d\left[\int_2^{u} 5\ln t\,dt\right] \cdot \frac{du}{dx}$$

$$= 5\ln u \cdot \frac{d(x^4)}{dx}$$

$$= 5\ln x^4 \cdot 4x^3$$

$$= 20x^3 \cdot \ln x^4$$

$$G(x) = \int_5^{\boxed{3x^2+2x+1}} \frac{1}{t^2+t+8}\,dt$$

find $G'(x)$:

$$G'(x) = \frac{1}{(3x^2+2x+1)^2+(3x^2+2x+1)+8} \cdot (6x+2)$$

chain rule

$$\boxed{\int_0^3 \sqrt{9-x^2}\,dx} \leftarrow \text{AREA}$$

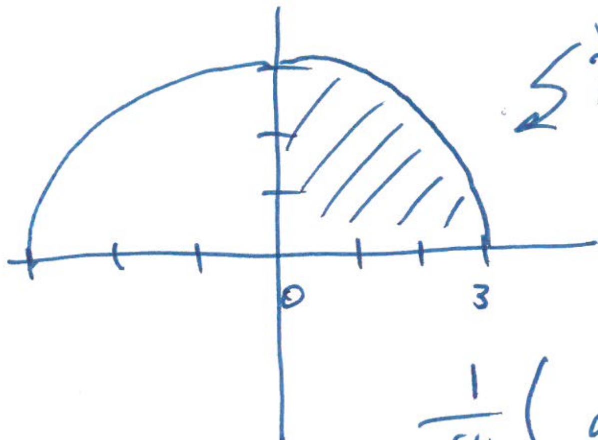
$$d(???) = (9-x^2)^{1/2}$$

$$\boxed{f(x) = \sqrt{9-x^2}}$$

$$y = \sqrt{9-x^2}$$

$$y^2 = 9 - x^2$$

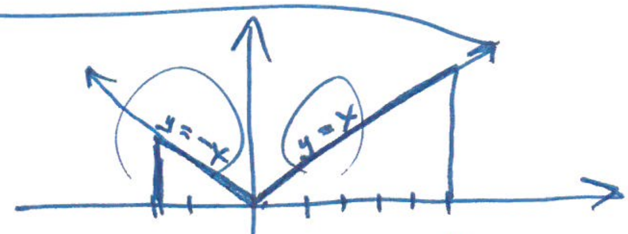
$$x^2 + y^2 = 9$$



$$\frac{1}{4}(\text{area of circle})$$

$$\frac{1}{4}(\pi \cdot 3^2) = \frac{9\pi}{4}$$

$$\int_{-2}^{5} |x|\,dx$$



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$A_1 = \int_{-2}^{0} (-x)\,dx \qquad A_2 = \int_0^5 x\,dx$$

$$\int_1^{16} \frac{u+1}{\sqrt[4]{u}}\,du$$

$$d(???) = \frac{u+1}{\sqrt[4]{u}}$$

$$\frac{u+1}{\sqrt[4]{u}} = \frac{u^1}{\sqrt[4]{u}} + \frac{1}{\sqrt[4]{u}}$$

$$= u^{3/4} + u^{-1/4}$$

$$\int_1^{16} \left(u^{3/4} + u^{-1/4}\right) du$$

$$\int_1^{16} u^{3/4}\,du + \int_1^{16} u^{-1/4}\,du$$

$$\left.\frac{u^{7/4}}{7/4} + \frac{u^{3/4}}{3/4}\right]_1^{16} \qquad \left(16^{7/4}\right)^7$$

$$\left[\frac{4}{7}u^{7/4} + \frac{4}{3}u^{3/4}\right]_1^{16}$$

$$= \left(\frac{4}{7}16^{7/4} + \frac{4}{3}16^{3/4}\right) - \left(\frac{4}{7}\cdot 1^{7/4} + \frac{4}{3}\cdot 1^{3/4}\right)$$

$$= \left[\left(\frac{4}{7}\cdot 2^7\right) + \frac{4}{3}(8)\right] - \left(\frac{4}{7}\cdot 1 + \frac{4}{3}\cdot 1\right)$$

$$\frac{4}{7}\cdot(128) + \frac{4}{3}(8) - \frac{4}{7} - \frac{4}{3}$$

$$\frac{3\cdot 512}{3\cdot 7} + \frac{32\cdot 7}{3\cdot 7} - \frac{4\cdot 3}{7\cdot 3} - \frac{4\cdot 7}{3\cdot 7}$$

$$\frac{3(512) + 32(7) - 12 - 28}{21} = \frac{\sim}{21}$$

resume 7:00 :

INTEGRATION USING SUBSTITUTION

(CHANGE-OF-VARIABLE METHOD)

(U-SUBST.)

$$\int x \cdot \sqrt{9-x^2} \cdot dx$$

$$-\frac{1}{2}\int (9-x^2)^{1/2} \cdot x \cdot dx \,(-2)$$

let $\boxed{u = 9-x^2}$

$\cancel{dx}\ \frac{du}{dx} = -2x \cdot dx$ $\qquad dx =$

$\boxed{du = -2x\, dx}$

$$-\frac{1}{2}\boxed{\int u^{1/2}\, du} = -\frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] + C$$

$$= -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}(9-x^2)^{3/2} + C$$

$$= -\frac{1}{3}(9-x^2)^{3/2} + C$$

$$d\left(-\frac{1}{3}(9-x^2)^{3/2} + C\right) \overset{??}{=} x \cdot (9-x^2)^{1/2}$$

$$-\frac{1}{3} \cdot \frac{3}{2}(9-x^2)^{1/2}(-2x) \quad ??$$

$$\int x^2 \cdot \sec^2(x^3+1)\,dx$$

$d(\tan u) = \sec^2 u$

$$\frac{1}{3}\int \boxed{\sec^2(x^3+1)} \cdot x^2 \cdot dx \cdot 3$$

let $\boxed{u = x^3+1}$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = 3x^2 \cdot dx$$

$$\boxed{du = 3x^2 \cdot dx}$$

$$\frac{1}{3}\boxed{\int \sec^2 u \cdot du} = \frac{1}{3}\tan u + C$$

$$= \frac{1}{3}\cdot\tan(x^3+1) + C$$

$\int \frac{1}{u}\,du = \ln|u| + C$

$\int \frac{du}{u}$

$$\int \tan x\,dx$$

$\tan x = \frac{\sin x}{\cos x}$

$$-1\int \frac{\sin x\,(-1)\,dx}{\cos x}$$

$\underline{d(???) = \tan x}$

let $\boxed{u = \cos x}$

$du = -\sin x \cdot dx$

$$-1\int \frac{du}{u} = -1 \cdot \ln|u| + C$$

$$= -1 \cdot \ln|\cos x| + C$$

$$= (-1) \cdot \ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$\boxed{\int \tan x\,dx = \ln|\sec x| + C}$$

$x \neq 0$ $$\int \left(\frac{1}{x}\right) dx = \ln|x| + C$$

$$d(\ln(x)) = \frac{1}{x} \cdot dx$$

$\ln(\text{NEG}) = ??$

area under

$$f(x) = \frac{3}{\sqrt{5x-1}}$$

from $x = 2$ to $x = 10$

$$\int_{x=2}^{x=10} \frac{3}{\sqrt{5x-1}}\,dx$$

$x=2 \xrightarrow{u=5x-1} u=9$

$x=10 \longrightarrow u=49$

$$\frac{1}{5} \cdot 3 \int_2^{10} \cancel{3} \cdot (5x-1)^{-1/2} \cdot dx \cdot 5$$

let $u = 5x-1$

$du = 5\,dx$

$$\frac{3}{5} \int_{9}^{49} u^{-1/2} \cdot du$$

2

$$\frac{3}{5}\left[\frac{u^{1/2}}{1/2}\right]_9^{49} = \frac{3}{5}\cdot\frac{2}{1}\left[\sqrt{u}\right]_9^{49}$$

$$= \frac{6}{5}\left(\sqrt{49}-\sqrt{9}\right)$$

$$= \frac{6}{5}\left(7-3\right) = \frac{6}{5}(4) = \frac{24}{5}$$

OR $\quad \frac{3}{5}\int_{\checkmark}^{\checkmark} u^{-1/2}\,du = \frac{3}{5}\cdot\frac{u^{1/2}}{1/2}\Big]_{\checkmark}^{\checkmark}$

$$= \frac{3}{5}\cdot\frac{2}{1}\,u^{1/2}\Big]_{\checkmark}^{\checkmark} = \frac{6}{5}(5x-1)^{1/2}\Big]_2^{10}$$

$$= \frac{6}{5}\left[49^{1/2}-9^{1/2}\right] = \frac{6}{5}\left[7-3\right] = \frac{24}{5}$$

$$\int x\sqrt{x+3}\,dx = \int \sqrt{x+3}\cdot x\cdot dx$$

1. let $u = x+3 \Rightarrow u-3 = x$

$du = 1\cdot dx$

$$\int (u-3)\cdot u^{1/2}\cdot du$$

$$\int \left(u^{3/2} - 3u^{1/2}\right) du$$

$$= \frac{u^{5/2}}{5/2} - 3\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5}(x+3)^{5/2} - \cancel{3}\cdot\frac{2}{\cancel{3}}(x+3)^{3/2} + C$$

$$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

2. let $u = \sqrt{x+3} = (x+3)^{1/2}$

$du = \frac{1}{2}(x+3)^{-1/2}(1)\,dx$

$u^2 = x+3$

$\boxed{u^2 - 3 = x} \xrightarrow[\text{DERIV}]{\text{take}} 2u\cdot\frac{du}{dx} = 1$

$2u\cdot du = 1\cdot dx$

$$\int x \cdot \sqrt{x+3}\, dx$$

$$\int (u^2-3)\cdot u \cdot 2u \cdot du$$

$$2\int u^2(u^2-3)\, du$$

$$2\int (u^4-3u^2)\, du$$

$$2\left[\frac{u^5}{5} - \cancel{3}\cdot\frac{u^3}{\cancel{3}}\right] + C$$

$$\frac{2}{5}\left(\sqrt{x+3}\right)^5 - 2\left(\sqrt{x+3}\right)^3 + C$$

MA141-012

TEST #3 RESULTS (+8)

A's	19	(22.4%)	55.3%
B's	28	(32.9%)	
C's	15	(17.6%)	
D's	13	(15.3%)	27.1%
F's	10	(11.8%)	

AVE: 76.73