MA141-012

Wednesday, November 14

- finish 4.4 (integr. using SUBST.)
- begin 4.5 (integr. BY PARTS)

$$\int (2t-3)^2\,dt$$

① $\int (4t^2 - 12t + 9)\,dt$

$$\frac{4t^3}{3} - 12\frac{t^2}{2} + 9t + C$$

$$\boxed{\frac{4}{3}t^3 - 6t^2 + 9t + C_1}$$

② $\frac{1}{2}\int (2t-3)^2\,dt\cdot 2$

let $u = \boxed{2t-3}$

$du = 2\cdot dt$

$$\frac{1}{2}\int u^2\cdot du = \frac{1}{2}\frac{u^3}{3} + C$$

$(a+b)^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

$$= \frac{1}{6}(2t-3)^3 + C$$

$$= \frac{1}{6}\left[(2t)^3 + 3(2t)^2(-3) + 3(2t)(-3)^2 + (-3)^3\right] + C$$

$$= \frac{1}{6}\left[8t^3 - 36t^2 + 54t - 27\right] + C$$

$$= \boxed{\frac{4}{3}t^3 - 6t^2 + 9t \boxed{-\boxed{\frac{9}{2} + C_2}}}$$

8.) $\int \frac{8x^3}{e^{x^4}} dx$

$\int \frac{du}{u}$

$u = e^{x^4}$

$du = e^{x^4} \cdot (4x^3)\, dx$

$-\frac{1}{4} \cdot 8 \int e^{-x^4} \cancel{8} \cdot \boxed{x^3 \cdot dx\, (-4)}$

$\int e^{u} \cdot du$

let $u = -x^4$

$du = -4 x^3 \cdot dx$

$= -2 \int e^{u}\, du = -2 e^{u} + C$

6.) $\int \frac{\cos \sqrt{t+1}}{\sqrt{t+1}} dt$

$2 \int \cos \sqrt{t+1} \cdot \boxed{\frac{1}{\sqrt{t+1}} dt \cdot \frac{1}{2}}$

let $u = \sqrt{t+1} = (t+1)^{1/2}$

$du = \frac{1}{2} (t+1)^{-1/2} (1) \cdot dt$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{t+1}} dt$

$2 \int \cos u \cdot du = 2 \sin(u) + C$

③

15.) $\int \tan x \, dx = \ln|\sec x| + C$

$$-1 \int \frac{-1 \sin x}{\cos x} \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -1 \int \frac{du}{u} = -1 \int \frac{1}{u} \cdot du$$

$$= -1 \cdot \ln|u| + C$$

$$= (-1) \cdot \ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

(4)

14.) $\frac{1}{5}\int \boxed{\cos^3(5\theta)}\ \sin(5\theta)\,d\theta\,(-5)$

let $u = \cos(5\theta)$

$du = -\sin(5\theta)(-5)\,d\theta$

$-\frac{1}{5}\int u^3 \cdot du$

$-\frac{1}{5}\,\frac{u^4}{4} + C$

$-\frac{1}{5}\cdot\frac{1}{4}\left(\cos(5\theta)\right)^4 + C$

$-\frac{1}{20}\left(\cos(5\theta)\right)^4 + C$

$\int e^u du$

12.) $\int \boxed{e^{\tan\theta}} \cdot \boxed{\sec^2\theta \cdot d\theta}$

$u = \tan\theta$

$du = \sec^2\theta \cdot d\theta$

$\int e^u \cdot du = e^u + C$

$= e^{\tan\theta} + C$

check: $d\left(e^{\tan\theta} + C\right) \overset{??}{=} e^{\tan\theta}\cdot\sec^2\theta$

resume 7:00

4.5: INTEGRATION "BY PARTS"

$$d(u \cdot v) = u \cdot dv + v \cdot du \quad \text{prod. rule}$$

$$\int x \cdot \cos x \, dx$$

~~let $u = \cos x$~~

~~$du = -\sin x \cdot dx$~~

$$\int \left[d(u \cdot v) - v\,du \right] = \int u \cdot dv$$

$$\cancel{\int} \cancel{d}(uv) - \int v\,du = \int u \cdot dv$$

$$\boxed{uv - \int v\,du = \int u \cdot dv}$$ integr. by parts

$$\int u\,dv = u \cdot v - \int v \cdot du$$

???. integrable

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$ * (6)

ex: $\int x \cdot \cos x \, dx$

let $u = \cos x$ $v =$

$du =$ $dv = x\,dx$

$u = x$ $v = \sin x$

$du = 1 \cdot dx$ $dv = \cos x \cdot dx$

$$\int x \cos x \, dx = (x)(\sin x) - \int \sin x \cdot dx$$

$$\int x \cos x \, dx = \boxed{x \cdot \sin x + \cos x + C}$$

check:

$d'(x \cdot \sin x + \cos x + C) = x \cdot (\cos x) +$

$\cancel{(\sin x)(1)} - \cancel{\sin x} + \cancel{0}$

$=$

$$\int x \cdot \cos x \, dx$$

$$\cancel{u = \cos x \qquad v = \frac{x^2}{2}}$$

$$\cancel{du = -\sin x \qquad dv = x \cdot dx}$$

$$\int x \cos x \, dx = (\cos x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cdot (-\sin x) dx$$

???

$$\int u \, dv = u \cdot v - \int v \, du$$

$$\int \boxed{x^3 \cdot \ln x \cdot} dx$$

$$u = \ln x \qquad v = \frac{x^4}{4}$$

$$du = \frac{1}{x} \cdot dx \qquad dv = \cancel{\ln x \, dx} \; x^3 dx$$

$$d\cancel{(???) = \ln x}$$

$$\int x^3 \cdot \ln x \cdot dx = (\ln x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= (\ln x)\left(\frac{x^4}{4}\right) - \int \frac{1}{4} \cdot x^3 dx$$

$$= (\ln x)\left(\frac{x^4}{4}\right) - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$\int t^2 \cdot e^{5t} \cdot dt$$

$u = e^{5t}$ $\quad v = \frac{t^3}{3}$

$du = 5 \cdot e^{5t}$ $\quad dv = t^2 \cdot dt$

$$= (e^{5t})\left(\frac{t^3}{3}\right) - \int \frac{t^3}{3} \cdot 5 \cdot e^{5t} \cdot dt$$

$$\int \boxed{t^2 \cdot e^{5t}} \cdot dt$$

$u = t^2 \cdot$ $\quad v = \frac{1}{5} e^{5t}$

$du = 2t \cdot dt$ $\quad dv = e^{5t} dt$

$$\int t^2 \cdot e^{5t} \cdot dt = (t^2)\left(\frac{1}{5} e^{5t}\right) - \int \frac{1}{5} e^{5t} \cdot 2 t \cdot dt$$

$$\int t^2 e^{5t} dt = \frac{1}{5} t^2 e^{5t} - \frac{2}{5} \boxed{\int t \cdot e^{5t} dt}$$

$U = t$ $\quad V = \frac{1}{5} e^{5t}$

$dU = 1 \cdot dt$ $\quad dV = e^{5t} dt$

$$= \frac{1}{5} t^2 \cdot e^{5t} - \frac{2}{5}\left[(t)\left(\frac{1}{5} e^{5t}\right) - \int \frac{1}{5} e^{5t} \cdot 1 \cdot dt\right]$$

$$= \frac{1}{5} t^2 \cdot e^{5t} - \frac{2}{5}(t)\left(\frac{1}{5} e^{5t}\right) + \frac{2}{5}\left[\frac{1}{5} \cdot \frac{1}{5} e^{5t}\right] + C$$

$$= \frac{1}{5} t^2 e^{5t} - \frac{2}{25} t e^{5t} + \frac{2}{125} e^{5t} + C$$

$$\boxed{\int e^x \cdot \sin x \, dx}$$

$u = e^x \qquad v = -\cos x$

$du = e^x \cdot dx \qquad dV = \sin x \, dx$

$$\int e^x \sin x \, dx = (e^x)(-\cos x) - \int (-\cos x)\, e^x \cdot dx$$

$$\int e^x \sin x \, dx = -e^x \cos x + \boxed{\int e^x \cdot \cos x \, dx}$$

$U = e^x \qquad V = \sin x$

$dU = e^x \cdot dx \qquad dV = \cos x \, dx$

$$\int e^x \sin x \, dx = -e^x \cdot \cos x + \left[(e^x)(\sin x) - \int \sin x \cdot e^x \, dx\right]$$

$$\int e^x \sin x \, dx = -e^x \cdot \cos x + e^x \cdot \sin x - \cancel{\int e^x \sin x \, dx}$$

$$+\int e^x \sin x \, dx \qquad\qquad + \cancel{\int e^x \sin x \, dx}$$

$$2\int e^x \cdot \sin x \, dx = -e^x \cdot \cos x + e^x \cdot \sin x$$

$$\int e^x \sin x \, dx = \frac{1}{2}\left[-e^x \cdot \cos x + e^x \cdot \sin x\right] + C$$

$$\int \underbrace{\tan^{-1} x}\, \underbrace{dx}$$

$u = \tan^{-1} x \qquad v = x$

$du = \frac{1}{1+x^2}\,dx \qquad dv = 1\,dx$

$$= (\tan^{-1} x)(x) - \int \boxed{(x) \cdot \frac{1}{1+x^2}}\, dx$$

$$= (\tan^{-1} x)(x) - \frac{1}{2} \int \frac{2 \cdot x}{(1+x^2)}\, dx$$

$\boxed{u = 1+x^2}$

$du = 2x\,dx$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{2x\,dx}{1+x^2}$$

$\int \frac{du}{u} = \int \frac{1}{u}\,du$

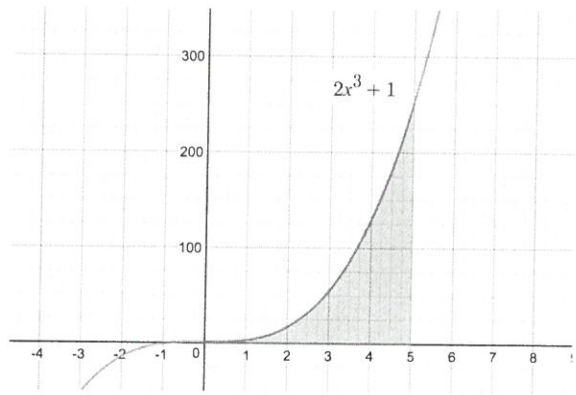
$$= x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{du}{u}$$

$$= \left[x \cdot \tan^{-1} x - \frac{1}{2} \ln|1+x^2|\right] + C$$

MA 141-012 F18 Recitation 12, Nov 13

1. Give a brief explanation for why the following expression gives the exact value of the area of the shaded region below. Compute the exact area. What definite integral can be used to check the answer?

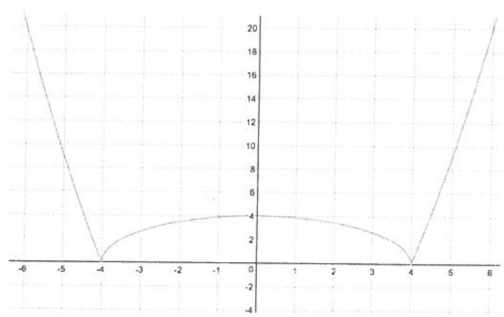
$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(\frac{5(i-1)}{n}\right)^3+1\right]\frac{5}{n}$$



2. Evaluate the definite integral $\int_{-2}^{2}\frac{x^3}{4}dx$ then give a geometric interpretation.

3. Evaluate the definite integral $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt[3]{\sec^4(7\theta)-\tan^7(4\theta)}\,d\theta$.

4. Evaluate the definite integral $\int_{-6}^{6}f(x)dx$ where $f(x)=\begin{cases}\sqrt{16-x^2}, & \text{for } |x|\le 4\\ x^2-16, & \text{for } 4<|x|.\end{cases}$



5. Evaluate the integrals using substitution.

a) $\int \sec(4x-\frac{\pi}{2})\tan(4x-\frac{\pi}{2})dx$ b) $\frac{1}{3}\int\frac{3x^2+1}{x^3+3x}dx$ c) $\int_1^2 x(3x^2-1)^3dx$

$u = x^3+3x$

$du = (3x^2+3)dx = 3.(x^2+1)dx$

Theorem 2. Formulas for Sums

1. $\sum_{i=1}^{n} 1 = n$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Properties of the Definite Integral

Let a, b and c be real numbers with $a < b$ and f and g be continuous functions. Then

1. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
2. $\int_a^a f(x)\,dx = 0$
3. $\int_a^b c\,dx = c(b-a)$
4. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
6. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
7. If $f(x) \geq 0$ for all $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq 0$
8. If $f(x) \geq g(x)$ for all $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
9. If $m \leq f(x) \leq M$ for all $x \in [a,b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$