MA141-012 (1)

Monday, November 19

- no TUESDAY RECITATIONS this week
- finish 4.5
- begin 5.1

$$\int \sec\theta \, d\theta =$$

$$\int \frac{\sec\theta(\sec\theta+\tan\theta)}{(\sec\theta+\tan\theta)} d\theta \quad \checkmark\checkmark$$

$$* \int \frac{\sec^2\theta + \sec\theta\cdot\tan\theta}{(\sec\theta+\tan\theta)} d\theta$$

let $u = \sec\theta + \tan\theta$

$du = (\sec\theta\cdot\tan\theta + \sec^2\theta)\,d\theta$

$$= \int \frac{du}{u} = \int \frac{1}{u}\cdot du$$

$$= \ln|u| + C$$

$$\int \sec\theta \, d\theta = \ln|\sec\theta+\tan\theta| + C \quad *$$

$$-1\int \frac{-1\sin\theta}{\cos\theta}\,d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$\int \csc\theta\, d\theta$$

$$\int \frac{\csc\theta(\csc\theta + \cot\theta)}{(\csc\theta + \cot\theta)}\,d\theta$$

$d(\cot\theta) = -\csc^2\theta$

$d(\csc\theta) = -\csc\theta\cot\theta$

$$-1\int \frac{-1(\csc^2\theta + \csc\theta\cdot\cot\theta)}{(\csc\theta + \cot\theta)}\,d\theta$$

$u = \csc\theta + \cot\theta$

$du = (-\csc\theta\cdot\cot\theta + -\csc^2\theta)\,d\theta$

$$-1\int \frac{du}{u}$$

$$= -1\cdot\ln|\csc\theta + \cot\theta| + C$$

$$\int \sin\theta\, d\theta = -\cos\theta + C$$

$$\int \cos\theta\, d\theta = \sin\theta + C$$

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C$$

$$* \int \cot\theta\, d\theta = -\ln|\csc\theta| + C = \ln|\sin\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \sec^3\theta \, d\theta$$

by parts . . .

$$\int \sec\theta \cdot \sec^2\theta \cdot d\theta$$

$u = \sec\theta \qquad v = \tan\theta$

$du = \sec\theta \cdot \tan\theta \, d\theta \qquad dv = \sec^2\theta \, d\theta$

$$\int \sec^3\theta \, d\theta = (\sec\theta)(\tan\theta) - \int \tan\theta \cdot \sec\theta \cdot \tan\theta \, d\theta$$

$$\boxed{\int \sec^3\theta \, d\theta = \sec\theta \cdot \tan\theta - \int \sec\theta \cdot \tan^2\theta \, d\theta}$$

$$\int \sec^3\theta \, d\theta = \sec\theta \cdot \tan\theta - \left[\int \tan\theta \cdot \sec\theta \cdot \tan\theta \, d\theta\right]$$

$U = \tan\theta \qquad V = \sec\theta$

$du = \sec^2\theta \, d\theta \qquad dV = \sec\theta \tan\theta \, d\theta$

$$\int \sec^3\theta \, d\theta = \sec\theta \cdot \tan\theta - \left[\sec\theta \cdot \tan\theta - \int \sec^3\theta \, d\theta\right]$$

$$\int \sec^3\theta \, d\theta = \sec\theta \cdot \cancel{\tan\theta} - \sec\theta \cdot \cancel{\tan\theta} + \int \sec^3\theta \, d\theta$$

$\frac{s^2}{c^2}+\frac{c^2}{c^2}=\frac{1}{c^2}$ $\quad \tan^2\theta+1=\sec^2\theta$

$$\int \sec^3\theta\,d\theta = \sec\theta\cdot\tan\theta - \int \sec\theta\,\tan^2\theta\,d\theta$$

$$\int \sec^3\theta\,d\theta = \sec\theta\cdot\tan\theta - \int \sec\theta(\sec^2\theta-1)\,d\theta$$

$$\underbrace{\int \sec^3\theta\,d\theta}_{} = \sec\theta\cdot\tan\theta \cancel{- \underbrace{\int \sec^3\theta\,d\theta}_{}} + \int \sec\theta\,d\theta$$

$$+\int \sec^3\theta\,d\theta \qquad\qquad \cancel{+\int \sec^3\theta\,d\theta}$$

$$2\int \sec^3\theta\,d\theta = \sec\theta\cdot\tan\theta + \underbrace{\int \sec\theta\,d\theta}_{}$$

$$2\int \sec^3\theta\,d\theta = \sec\theta\cdot\tan\theta + \ln|\sec\theta+\tan\theta| + C$$

$$\int \sec^3\theta\,d\theta = \frac{1}{2}\left[\sec\theta\cdot\tan\theta + \ln|\sec\theta+\tan\theta|\right] + C$$

$$\int \sec^3\theta \cdot d\theta$$

$u = \sec\theta$

$du = \sec\theta \cdot \tan\theta \, d\theta$

$$\int u^3 \quad ???$$

~~$u = 1 \cdot dt$ $\quad v =$~~

~~$du =$ $\quad dv = \ln t$~~

$d(??) = \ln x$

11.) $\int \ln t \, dt$

$u = \ln t \qquad v = t$

$du = \frac{1}{t} \cdot dt \qquad dv = 1 \cdot dt$

$= t \cdot \ln t - \int \cancel{t} \cdot \frac{1}{\cancel{t}} dt$

$= t \cdot \ln t - \int 1 \cdot dt$

$= t \cdot \ln t - t + C$

5.) $$\int_{1}^{\sqrt{3}} \tan^{-1}(x)\, dx$$

$u = \tan^{-1} x \qquad v = x$

$du = \frac{1}{1+x^2}dx \qquad dv = 1 \cdot dx$

$$= \left[(x)(\tan^{-1} x) - \frac{1}{2}\int \frac{2 \cdot x}{\boxed{1+x^2}} dx \right]_1^{\sqrt{3}}$$

$u = 1+x^2$

$du = 2x\,dx$

$$= x \cdot \tan^{-1} x - \frac{1}{2}\int \frac{du}{u}$$

$$= \left[x \cdot \tan^{-1} x - \frac{1}{2} \ln|1+x^2| \right]_1^{\sqrt{3}}$$

$$= \left(\sqrt{3} \cdot \boxed{\tan^{-1}\sqrt{3}} - \frac{1}{2}\ln|1+(\sqrt{3})^2| \right)$$

$$- \left(1 \cdot \boxed{\tan^{-1} 1} - \frac{1}{2}\ln|1+1^2| \right)$$

$$= \left(\sqrt{3} \cdot \left(\frac{\pi}{3}\right) - \frac{1}{2}\ln 4 \right) - \left(1 \cdot \left(\frac{\pi}{4}\right) - \frac{1}{2}\ln 2 \right)$$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$$= \frac{\sqrt{3} \cdot \pi}{3} - \frac{\pi}{4} + \frac{1}{2}\ln 2 - \frac{1}{2}\ln 4$$

$$= \pi\left[\frac{1}{\sqrt{3}} - \frac{1}{4}\right] + \frac{1}{2}\left[\ln 2 - \ln 4\right]$$

$$= \pi\left[\frac{1}{\sqrt{3}} - \frac{1}{4}\right] + \frac{1}{2}\left[\ln \frac{2}{4}\right]$$

resume 7:00

17.) $\int \frac{\ln x}{3x}\, dx$

$= \int \frac{1}{3} \cdot \frac{1}{x} \cdot \ln x \, dx$

$= \frac{1}{3} \int \underline{\ln x} \cdot \frac{1}{x} \cdot dx$

$u = \underline{\ln x}$

$du = \frac{1}{x} \cdot dx$

$= \frac{1}{3} \int u \, du = \frac{1}{3} \frac{u^2}{2} + C$

$= \frac{1}{6} (\ln x)^2 + C$

19.) $\int \underset{\sim}{x^3} \cdot \cos(\underline{x^2}) \cdot dx$

$\int \underline{x^2} \cdot x \cdot \cos(\underline{x^2}) dx$

$\frac{1}{2} \int x^2 \cos(x^2) \cdot \underline{x \cdot dx (2)}$

$t = x^2$

$dt = 2x dx$

$\frac{1}{2} \int t \cdot \cos t \cdot dt$

$$t = x^2$$

$$\frac{1}{2}\int t\cdot\cos t\cdot dt$$

$$u = t \qquad v = \sin t$$

$$du = 1\cdot dt \qquad dv = \cos t\cdot dt$$

$$= \frac{1}{2}\left[t\cdot\sin t - \int \sin t\, dt\right]$$

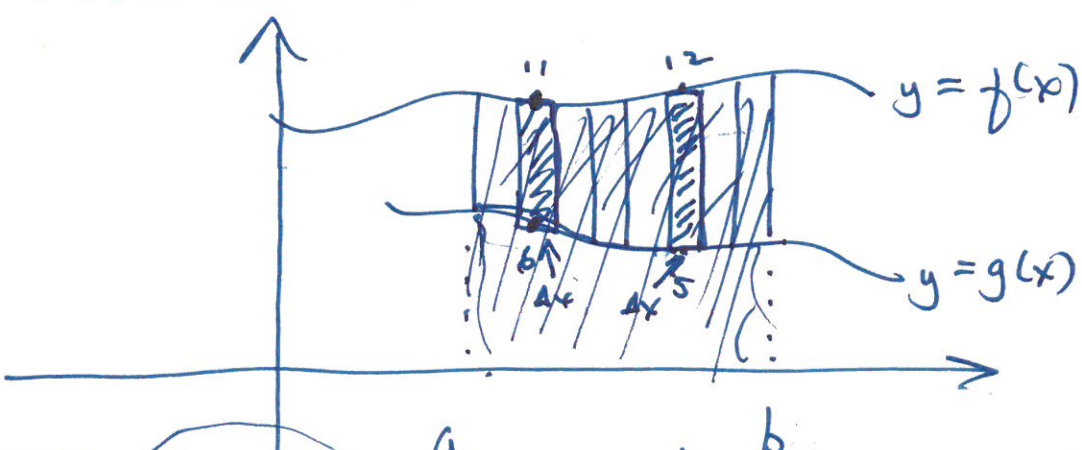
$$= \frac{1}{2}\left[t\cdot\sin t + \cos t\right] + C$$

$$= \frac{1}{2}\left[x^2\cdot\sin x^2 + \cos x^2\right] + C$$

5.1: AREA BETWEEN TWO CURVES

$$\int_a^b \underbrace{f(x)}\cdot\underbrace{dx}$$

TYPE I REGION:



Σ $\quad \Delta y$

$$A = \boxed{\int_a^b f(x)\,dx} - \int_a^b g(x)\,dx$$

$$A = \int_a^b \underbrace{[f(x) - g(x)]}_{\text{HEIGHT}}\cdot\underbrace{dx}_{\text{WIDTH}}$$

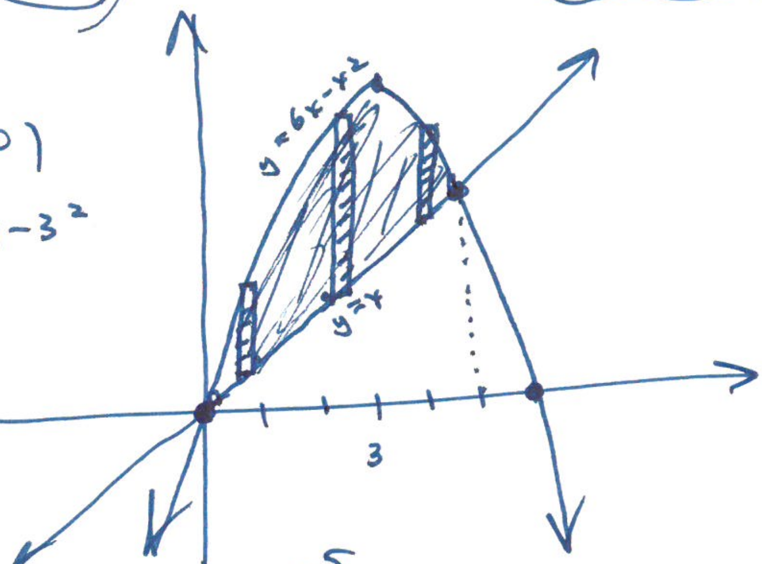
find the area of the bounded region:

$y = 6x - x^2$ and $y = x$

$y = x(6-x)$

$(0,0)$ $(6,0)$

$V(3,9)$ $6(3)-3^2$



$6x - x^2 = x$

$0 = x^2 - 5x$

$0 = x(x-5)$

$0 = x$

$0 = x - 5$ $\quad x = 5$

$$A = \int_0^5 \left[(6x - x^2) - x \right] dx$$

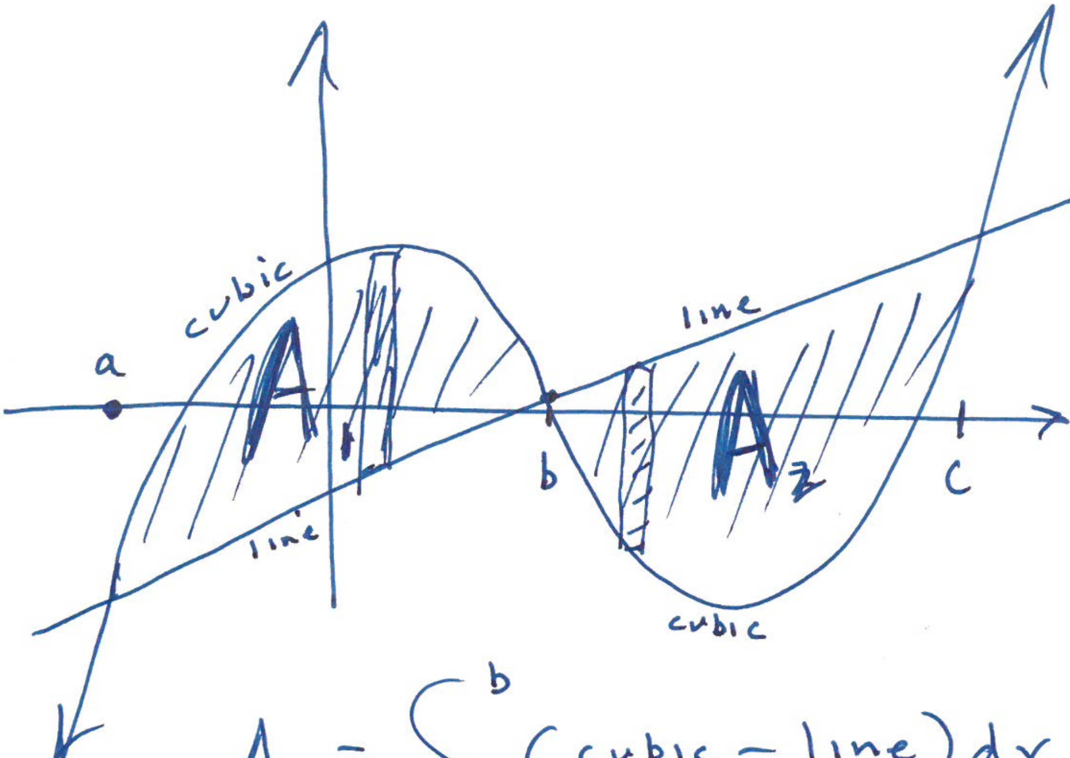
y-value UPPER CURVE

$$A = \int_0^5 (5x - x^2)\, dx$$

$$A = \left[5 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^5$$

$$A = \left(\frac{5}{2} \cdot (5)^2 - \frac{(5)^3}{3} \right) - (0)$$

$$A = \frac{3 \cdot 125}{3 \cdot 2} - \frac{125 \cdot 2}{3 \cdot 2} = \frac{375}{6} - \frac{250}{6} = \frac{125}{6}$$



cubic = line

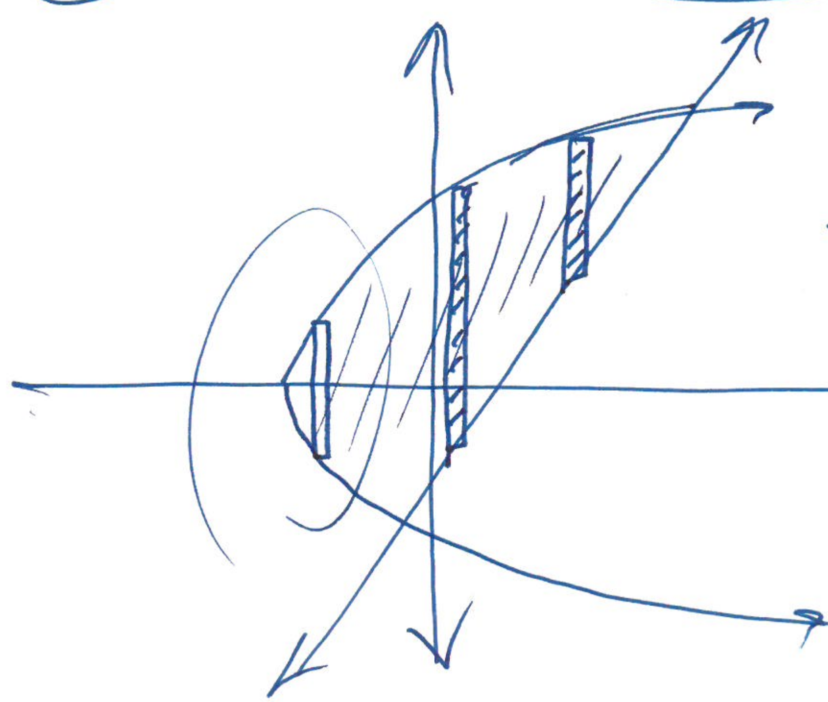
⋮

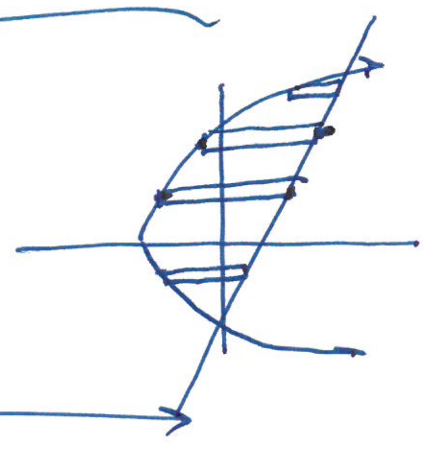
x_1, x_2, x_3

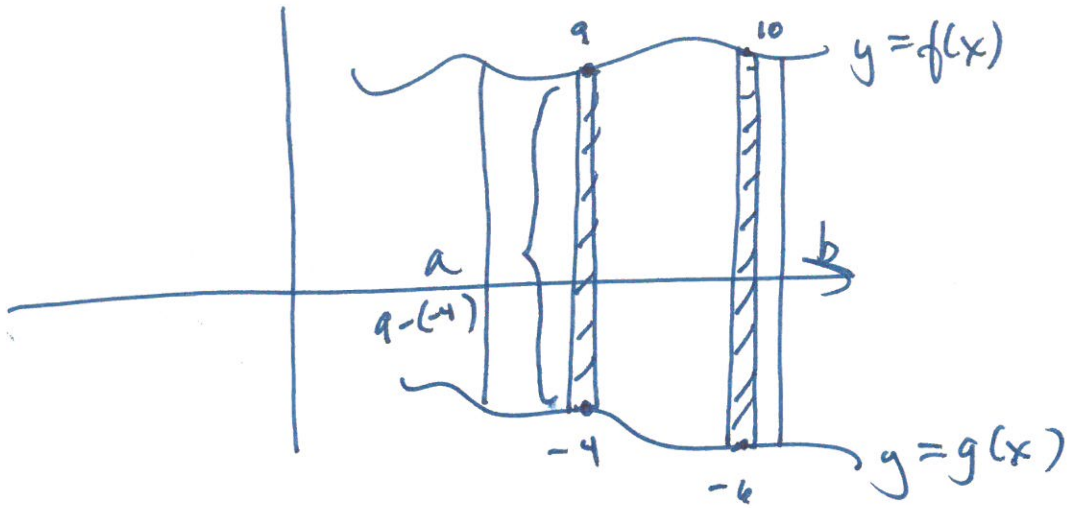
$$A_1 = \int_a^b (\text{cubic} - \text{line})\,dx$$

$$A_2 = \int_b^c (\text{line} - \text{cubic})\,dx$$

$A = A_1 + A_2$







$$A = \int_a^b \underline{\left(f(x) - g(x) \right)} \cdot dx$$