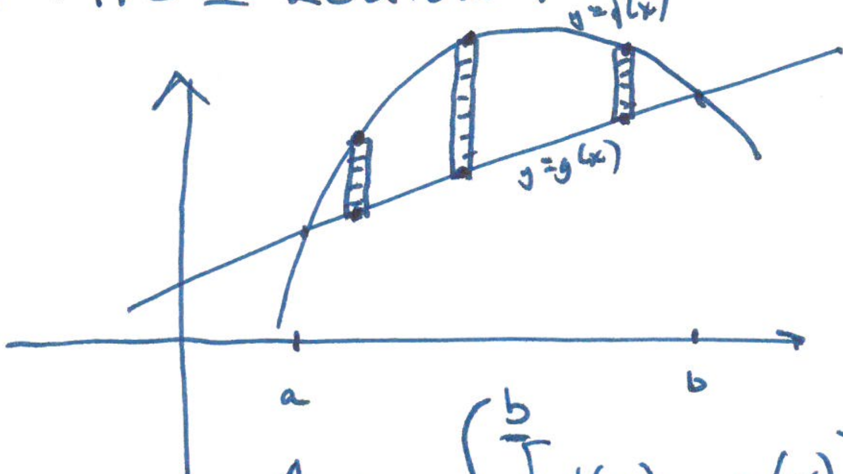
MA141 – 012 (1)

Monday, November 26

- today: finish 5.1
- TEST #4: chapter 4

5.1:

(1) TYPE I REGION:



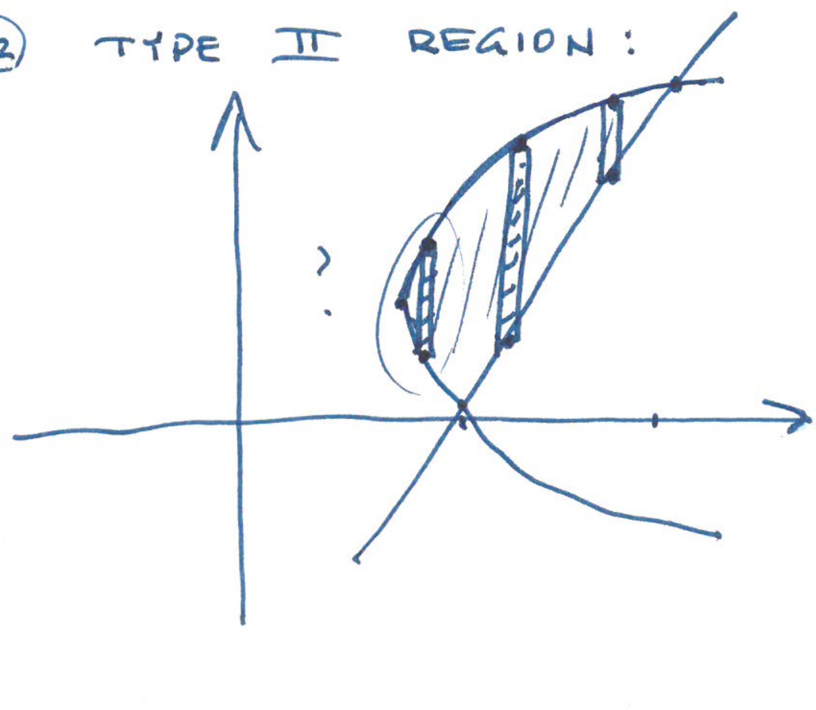
set $f(x) = g(x)$

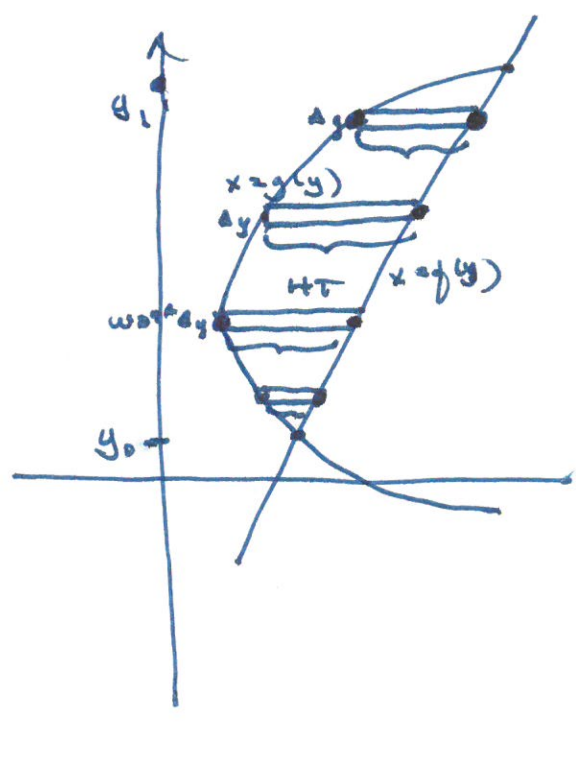
⋮

a, b

$$A = \int_a^b \underbrace{[f(x) - g(x)]}_{HT} \cdot \underbrace{dx}_{WDTH}$$

(2) TYPE II REGION:





$$\sum \left[\begin{pmatrix} \text{x-value} \\ \text{curve on the RT} \end{pmatrix} - \begin{pmatrix} \text{x-value} \\ \text{curve on the LFT} \end{pmatrix} \right] \cdot \Delta y$$

$$\int_{y_0}^{y_1} \left[f(y) - g(y) \right] \cdot dy$$

$x = f(y)$ $\quad$ $x = g(y)$

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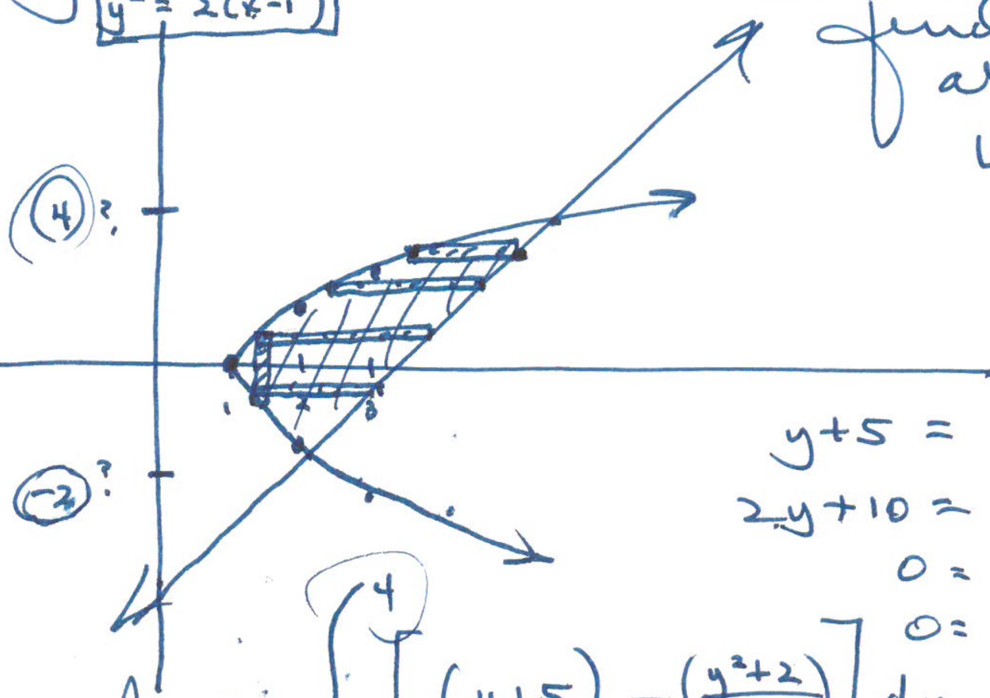
ex: $x = \frac{y^2+2}{2}$ $\qquad$ $x = y+5$

$y^2 = 2x - 2$ and $y = x - 5$

$\boxed{y^2 = 2(x-1)}$

find the area of the bounded region



$$y+5 = \frac{y^2+2}{\cancel{2}} \cancel{2}$$

$$2y + 10 = y^2 + 2$$

$$0 = y^2 - 2y - 8$$

$$0 = (y-4)(y+2)$$

$$y = 4 \quad y = -2$$

$$A = \int_{-2}^{4} \left[(y+5) - \left(\frac{y^2+2}{2}\right) \right] dy$$

$$A = \int_{-2}^{4} \left(-\frac{1}{2}y^2 + y + 4\right) dy$$

$$A = \left[-\frac{1}{2}\cdot\frac{y^3}{3} + \frac{y^2}{2} + 4y\right]_{-2}^{4}$$

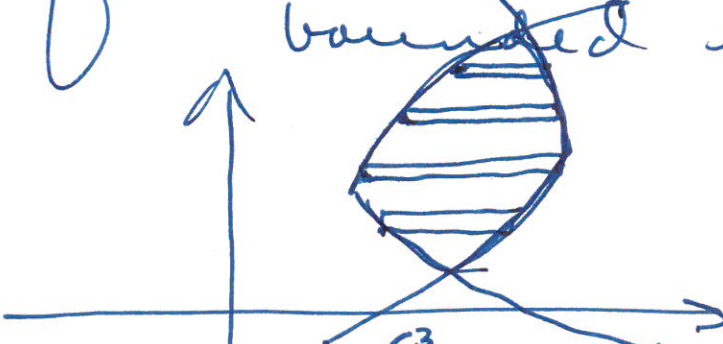
$$A = \left[-\frac{1}{6}(4)^3 + \frac{1}{2}(4)^2 + 4(4)\right] - \left[-\frac{1}{6}(-2)^3 + \frac{1}{2}(-2)^2 + 4(-2)\right]$$

$$A = \left(-\frac{1}{6}(64) + 8 + 16\right) - \left(\frac{4}{3} + 2 - 8\right)$$

$A =$ ______________

$x = y^2 - 4y$ ✓ $\qquad$ $x = 2y - y^2$ ✓

find the area of the bounded region



$$y^2 - 4y = 2y - y^2$$

$$2y^2 - 6y = 0$$

$$2y(y-3) = 0$$

$y = 0 \quad y = 3$

$$\int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right] dy$$

$$A = \int_0^3 (6y - 2y^2)\, dy$$

$$\sum_{i=1}^{n} 1 = \underline{n}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

find the area under the curve $y = x^2 + 2x + 5$ from $x = 1$ to $x = 5$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{5-1}{n} = \frac{4}{n}$

$a = 1$
$b = 5$

$$\to \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x$$

$$f\left(1 + \frac{4i}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + i\left(\frac{4}{n}\right)\right)\cdot\left(\frac{4}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\underbrace{\left(1+\frac{4i}{n}\right)^2 + 2\left(1+\frac{4i}{n}\right) + 5}\right]\left(\frac{4}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\cancel{1} + \cancel{\frac{8i}{n}} + \frac{16i^2}{n^2}\right) \cancel{+2} + \cancel{\frac{8i}{n}} \cancel{+5}\right]\cdot\left(\frac{4}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[8 + \frac{16i}{n} + \frac{16i^2}{n^2}\right]\cdot\left(\frac{4}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{32}{n} + \frac{64i}{n^2} + \frac{64i^2}{n^3}\right)$$

$$\lim_{n\to\infty} \left[\frac{32}{n}\sum_{i=1}^{n} 1 + \frac{64}{n^2}\sum_{i=1}^{n} i + \frac{64}{n^3}\left[\sum_{i=1}^{n} i^2\right]\right]$$

$$\lim_{n\to\infty} \left[\frac{32}{\cancel{n}}(\cancel{n}) + \frac{64}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]\right]$$

$$\lim_{n\to\infty} \left[32 + 32\left[\frac{n(n+1)}{n^2}\right]^{\to 1} + \frac{32}{3}\left[\frac{n(n+1)(2n+1)}{n^3}\right]^{2n^3+\dots \to 2}\right]$$

$$32 + 32(1) + \frac{32}{3}(2) = \underline{\qquad}$$

$$g(x) = \int_4^{8x^3} \sin t \cos t \, dt$$

$u = 8x^3$

$du = 24x^2$

$$g'(x) = \int_4^{u} \sin t \cos t \, dt$$

$$g'(x) = \frac{d\left[\int_4^{u} \sin t \cdot \cos t \, dt\right]}{du} \cdot \frac{du}{dx}$$

$$= (\sin u \cos u) \cdot (24x^2)$$

$$= \left[\sin(8x^3) \cos(8x^3)\right] \cdot (24x^2)$$

MA141 — TEST #4:

Chapter 4:

4.1: AREAS

① approx area

② Riemann Sum:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\cdot\Delta x)\cdot\Delta x$$

$$A = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x} = \int_a^b f(x)\,dx$$

$$\boxed{x_i = a + i\cdot\Delta x}$$

$$\Delta x = \frac{b-a}{n}$$

4.2: properties of the DEFINITE INTEGRAL ✓

4.3: FUNDAMENTAL THEOREM OF CALCULUS

I. $g(x) = \int_a^x f(t)\,dt$ $\quad (g'(x) = f(x))$

therefore .....

$$g'(x) = \frac{d\left[\int_a^x f(t)\,dt\right]}{dx}$$

II. $\int_a^b f(x)\,dx = F(b) - F(a)$

(page 2)

4.4: INTEGRATION USING SUBSTITUTION:

(indef & def integrals)

4.5: INTEGRATION BY PARTS

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

- multiple iterations

$$\int t^2 \cdot \cos t \, dt =$$

$$G(x) = \int_2^x \sin^2 t \, dt$$

find $G'(x)$:

$$G'(x) = \frac{d\left[\int_2^x \sin^2 t \, dt\right]}{dx}$$

$$G'(x) = \sin^2 x$$